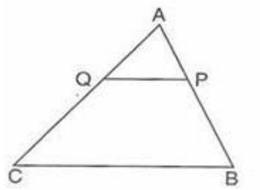
CBSE Test Paper 03

Chapter 6 Triangles

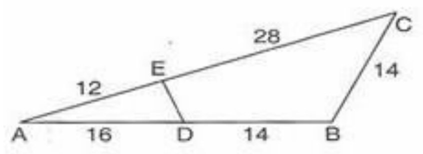
1. If in two triangles ABC and PQR, $\angle A = \angle Q \, and \, \angle R = \angle B$, then which of the following is not true. **(1)**
   a. $\frac{AB}{PQ} = \frac{BC}{RP}$
   b. $\frac{BC}{PR} = \frac{AC}{PQ}$
   c. $\frac{BC}{RP} = \frac{AB}{QR}$
   d. $\frac{AB}{QR} = \frac{AC}{PQ}$

2. In the adjoining figure P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7cm, AQ = 3cm, QC = 6cm and PQ = 4.5cm. The measure of BC is equal to **(1)**



   a. 9 cm
   b. 15 cm
   c. 12.5 cm
   d. 13.5 cm

3. In the given figure if $\Delta AED \sim \Delta ABC$, then DE is equal to **(1)**



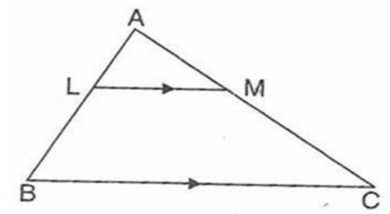
   a. 6.5 cm
   b. 5.6 cm
   c. 5.5 cm
   d. 7.5 cm

4. A semicircle is drawn on AC. Two chords AB and BC of length 8 cm and 6 cm

respectively are drawn in the semicircle. What is the measure of the diameter of the circle? **(1)**

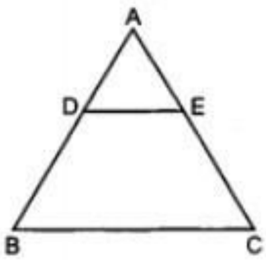
a. 10 cm
b. 12 cm
c. 11 cm
d. 14 cm

5. In the given figure, if $\frac{ar(\Delta ALM)}{ar(trapezium\ LMCB)} = \frac{9}{16}$, Then AL: LB is equal to **(1)**

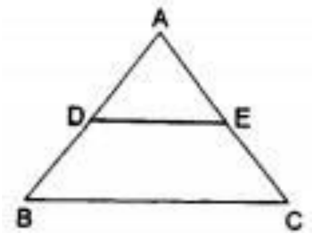


a. it is 3 : 5
b. it is 3 : 4
c. it is 3 : 2
d. it is 2 : 3

6. In figure DE $\|$ BC. If BD = x - 3, AB = 2x. CE = x - 2 and AC = 2x + 3. Find x. **(1)**



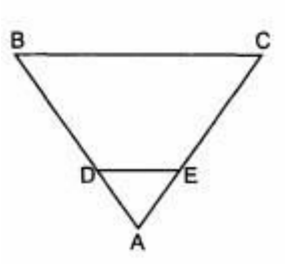
7. D and E are points on the sides AB and AC respectively of a $\triangle ABC$. If AD = 7.2 cm, AE = 6.4 cm, AB = 12 cm and AC = 10 cm then determine whether DE || BC or not. **(1)**



8. In two similar triangles ABC and PQR, if their corresponding altitudes AD and PS are in the ratio 4: 9, find the ratio of the areas of ΔABC and ΔPQR. **(1)**

9. In figure, DE || BC in $\triangle$ABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.

(1)



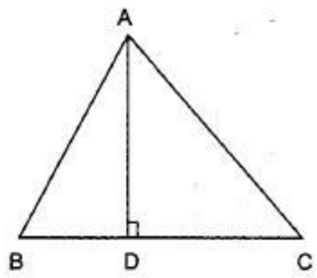
10. If ratio of corresponding sides of two similar triangles is 5 : 6, then find ratio of their areas. **(1)**

11. In $\triangle$ ABC, $\angle$C > 90^o and side AC has produced to D such that segment BD is perpendicular to segment AD. Prove that $AB^2 = BC^2 + AC^2 + 2CA \times CD$. **(2)**

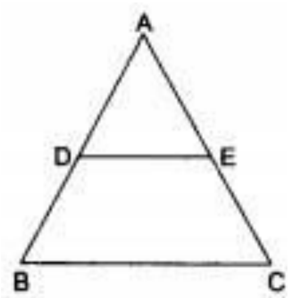
12. Find the altitude of an equilateral triangle when each of its side is 'a' cm. **(2)**

13. Prove that the line segment joining the midpoints of any two sides of a triangle is parallel to the third side. **(2)**

14. In Fig. if $AD \perp BC$ and $\frac{BD}{DA} = \frac{DA}{DC}$, Prove that Δ ABC is a right triangle. **(3)**



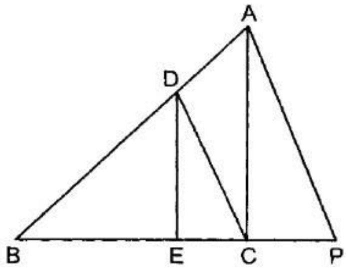
15. In the adjoining figure, ABC is a triangle in which AB = AC. If D and E are points on AB and AC respectively such that AD = AE, show that the points B, C, E and D are concyclic. **(3)**



16. Let s denote the semiperimeter of a triangle ABC in which BC = a, CA = b, AB = c. If a

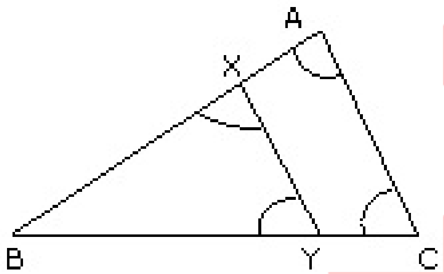
circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = s-b. **(3)**

17. In Fig. DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$. **(3)**



18. In a ΔABC, D and E are points on AB and AC respectively such that DE || BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE. **(4)**

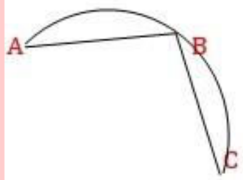
19. In the given figure the line segment XY $\|$ AC and XY divides triangular region ABC into two points equal in area, Determine $\frac{AX}{AB}$. **(4)**



20. In ΔABC, AD is a median. Prove that $AB^2 + AC^2 = 2\ AD^2 + 2\ DC^2$. **(4)**

CBSE Test Paper 03
Chapter 6 Triangles

Solution

1. a. $\frac{AB}{PQ} = \frac{BC}{RP}$
 Explanation: $\triangle ABC \sim \triangle QRP (AA similarity) \implies \frac{AB}{QR} = \frac{AC}{QP} = \frac{BC}{RP}$
 these are the corresponding parts
2. d. 13.5 cm
 Explanation: $\triangle AQP \sim \triangle ACB$(SAS similarity)
 $\frac{AQ}{AC} = \frac{AP}{AB} \implies \frac{3}{6} = \frac{4.5}{BC} (cpst) \Rightarrow$ BC = 13.5
3. b. 5.6 cm
 Explanation: ΔAED $\sim$ ΔABC (SAS Similarly) $\Rightarrow \frac{12}{30} = \frac{ED}{14} \Rightarrow$ ED = 5.6cm
4. a. 10 cm
 Explanation: 

 Here the diameter of circle is AC and $\angle \mathrm{ABC}$ is a semicircle. Therefore
 $\angle \mathrm{ABC}$ = 90^0
 And triangle ABC is a right angled triangle.
 Then, AC = $\sqrt{\mathrm{AB}^2 + \mathrm{BC}^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm
5. c. it is 3 : 2
 Explanation: In triangles ALM and ABC,
 $\angle A = \angle A$ [Common]
 $\angle ALM = \angle ABC$ [Corresponding angles as LM || BC]
 Then $\Delta \mathrm{ALM} \sim \Delta \mathrm{ABC}$ [AA similarity]
 Therefore, $\frac{\text{area}(\Delta \mathrm{ALM})}{\text{area}(\Delta \mathrm{ABC})} = \frac{\mathrm{AL}^2}{\mathrm{AB}^2}$
 Now, $\frac{\text{area}(\text{trap.LMCB})}{\text{area}(\Delta \mathrm{ALM})} = \frac{16}{9}$
 $\Rightarrow \frac{\text{area}(\Delta \mathrm{ABC}) - \text{area}(\Delta \mathrm{ALM})}{\text{area}(\Delta \mathrm{ALM})} = \frac{16}{9} \Rightarrow \frac{\text{area}(\Delta \mathrm{ABC})}{\text{area}(\Delta \mathrm{ALM})} - 1 = \frac{16}{9}$
 $\Rightarrow \frac{\text{area}(\Delta \mathrm{ABC})}{\text{area}(\Delta \mathrm{ALM})} = \frac{25}{9}$

$\Rightarrow \frac{AB^2}{AL^2} = \frac{25}{9} \Rightarrow \frac{AB}{AL} = \frac{5}{3}$

Let AB = 5x and AL = 3x, then LB = AB - AL = 5x - 3x = 2x

Therefore, $\frac{AL}{LB} = \frac{3x}{2x} = \frac{3}{2}$

$\Rightarrow$ AL : LB = 3 : 2

6. In $\triangle$ABC, DE || BC

   Therefore,by basic proportionality theorm,we have,

   $\frac{AD}{BD} = \frac{AE}{CE}$

   $\Rightarrow \frac{AB-BD}{BD} = \frac{AC-CE}{CE}$

   $\Rightarrow \frac{2x-(x-3)}{x-3} = \frac{2x+3-(x-2)}{x-2}$

   $\Rightarrow \frac{x+3}{x-3} = \frac{x+5}{x-2}$

   $\Rightarrow$ (x - 2)(x + 3) = (x + 5)(x - 3)

   $\Rightarrow$ x^2 + x - 6 = x^2 + 2x - 15

   $\Rightarrow$ x = 9 cm

7. Here, we use the converse proportionality theorm.

   Since D and E are points on the sides AB and AC respectively.

   $\frac{AB}{AD} = \frac{AC}{AE}$ [by Thales theorem]

   $\frac{12}{7.2} = \frac{10}{6.4}$

   $\Rightarrow 1.66 \neq 1.56$

   Hence, by the converse of Thales theorem DE is not Parallel to BC.

8. Theorem: ratio of areas of two similar triangles is equal to the ratio of the square of their corresponding altitudes.

   Here it is given that two triangles ABC and PQR are similar.

   $\therefore \quad \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PS^2} \Rightarrow \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$ [$\because$ AD : PS = 4:9]

9. Given DE || BC.

   In $\triangle$ADE and $\triangle$ABC,

   $\angle ADE = \angle ABC$ (corresponding angles)

   $\angle A = \angle A$ (common)

   $\therefore \quad \Delta ADE \sim \triangle ABC$ (AA similarity)

   Since the corresponding sides of similar triangles are proportional, therefore,

   $\frac{AD}{AB} = \frac{DE}{BC}$

Now $\frac{1.5}{6} = \frac{DE}{8}$

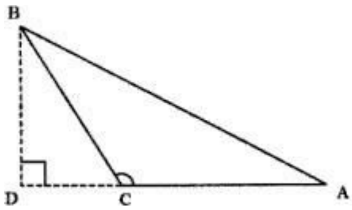
DE = $\frac{1.5 \times 8}{6}$ = 2 cm

10. Let the triangles be $\triangle$ ABC and $\triangle$ DEF

Then the ratio of their area is=

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)}$$
$$= \frac{5^2}{6^2} = \frac{25}{36}$$

11. Given: $\triangle$ ABC in which $\angle$ ACB > 90^o



To prove: $AB^2 = BC^2 + AC^2 + 2CA \times CD$

Construction: Draw BD $\perp$ AC (produced)

proof: In right angled $\triangle$ BDA, we get

$AB^2 = BD^2 + AD^2$ ....(i) [By Pythagoras theorem]

$=BD^2 + (AC + CD)^2$

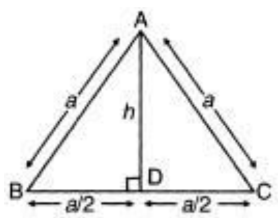
$= BD^2 + AC^2 + CD^2 + 2AC.CD$

$=(BD^2+CD^2) + AC^2 + 2AC.CD$ [$\because$ In right angled $\triangle$BDC,$BD^2 + CD^2 = BC^2$]

$=BC^2 + AC^2 + 2AC.CD$

Hence, $AB^2 = BC^2 + AC^2 + 2CA \times CD$ Hence proved

12.



Let the triangle be ABC.

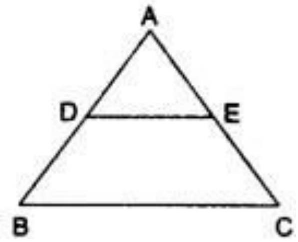
The altitude AD is also the median of equilateral ΔABC.so, BD = DC = $\frac{a}{2}$. Let AD = h cm.

In right-angled $\triangle$ABD,

By Pythagoras theorem,we have

$$AB^2 = BD^2 + AD^2$$
$$(a)^2 = \left(\frac{a}{2}\right)^2 + h^2$$

or, $h^2 = a^2 - \frac{a^2}{4}$
or, $h^2 = \frac{3a^2}{4}$
$\therefore h = \frac{\sqrt{3}a}{2}$ cm

13. 

According to question it is given that ABC is a triangle in which D and E are the midpoints of AB and AC respectively.

To Prove $DE \| BC$

Proof :- Since D and E are the midpoints of AB and AC respectively, we have AD = DB and AE = EC.

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [each equal to 1].

Hence, by the converse of Thales' theorem, $DE \| BC$.

14. In Δ's BDA and ADC, we have

$\frac{DB}{DA} = \frac{DA}{DC}$ [Given]

and, $\angle$ BDA = $\angle$ ADC [Each equal to 90°]

So, by SAS-criterion of similarity, we have

$\Delta BDA \sim \Delta ADC$

$\Rightarrow \angle$ ABD = $\angle$ CAD and $\angle$ BAD= $\angle$ ACD

$\Rightarrow \angle$ ABD + $\angle$ ACD= $\angle$ CAD + $\angle$ BAD

$\Rightarrow \angle$ B + $\angle$ C = $\angle$ A

$\Rightarrow \angle$ A + $\angle$ B + $\angle$ C = 2 $\angle$A [Adding $\angle$ A on both sides]

$\Rightarrow$ 2 $\angle$ A = 180°

$\Rightarrow \angle$ A = 90°

$\Rightarrow \Delta$ABC is a right triangle.

15. It is gIven that:

AD = AE .... (i)

AB = AC................. (ii)

Subtracting AD from both sides, we obtain

AB - AD = AC - AD

$\Rightarrow$ AB - AD = AC - AE (Since, AD = AE)

$\Rightarrow$ BD = EC................(iii)

Dividing equation (i) by equation (iii), we Obtain

AD/DB = AE/EC

Applying the converse of Thales' theorem, we get $DE \| BC$.

$\Rightarrow \angle DEC + \angle ECB = 180^0$ (Sum of interior angle on the same side of a Transversal line 180^0 .)

$\Rightarrow \angle DEC + \angle CBD = 180^0$ (Since, AB = AC $\Rightarrow \angle B = \angle C$)

Hence, quadrilateral BCED is cyclic.

Therefore, B, C, E and D are concyclic points, which completes the proof.

16. According to the question ,

$s = \frac{a+b+c}{2} \Rightarrow 2s = a + b + c$

B is an external point and BD and BF are tangents and from an external point the tangents drawn to a circle are equal in length.

So, BD = BF; AF = AE; CD = CE ... (i)

s = Semi perimeter = $\frac{AB+AC+BC}{2}$

2s = AB + AC + BC

2s = AF + FB + AE + EC + BD + DC

From (i): 2s = 2AE + 2CE + 2BD $\Rightarrow$ s = AE + CE + BD

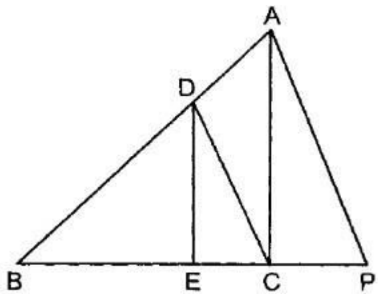
$s = AC + BD$

$\Rightarrow s - b = BD.$

17. In Δ BPA, we have

$DC \| AP$

Therefore, by basic proportionality theorem, we have ,



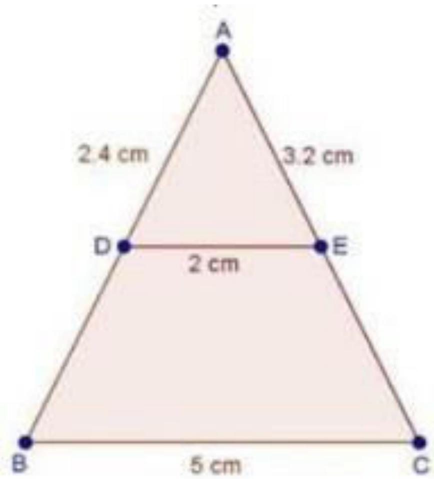
$\frac{BC}{CP} = \frac{BD}{DA}$ ....(1)

In ΔBCA, we have,

$DE \| AC$ [Given]

Therefore, by basic proportionality theorem, we have
$\frac{BE}{EC} = \frac{BD}{DA}$ ....(2)
Comparing (i) and (ii), we get,
$\frac{BC}{CP} = \frac{BE}{EC}$ or, $\frac{BE}{EC} = \frac{BC}{CP}$

18. We have,



DE || BC

Now, In $\triangle$ ADE and $\triangle$ ABC

$\angle A = \angle A$ [common]

$\angle ADE = \angle ABC$ [∵ DE || BC ⇒ Corresponding angles are equal]

$\Rightarrow \triangle ADE = \triangle ABC$ [By AA criteria]

$\Rightarrow \frac{AB}{BC} = \frac{AD}{DE}$ [∵ Corresponding sides of similar triangles are proportional]
$\Rightarrow \frac{AB}{5} = \frac{2.4}{2}$
$\Rightarrow AB = \frac{2.4 \times 5}{2}$

⇒ AB = 1.2 × 5

= 6.0 cm

⇒ AB = 6 cm

∴ BD = AB - AD

= 6 - 2.4

= 3.6 cm

⇒ DB = 3.6 cm

Now,
$\frac{AC}{BC} = \frac{AE}{DE}$ [∵ Corresponding sides of similar triangles are equal]
$\Rightarrow \frac{AC}{5} = \frac{3.2}{2}$
$\Rightarrow AC = \frac{3.2 \times 5}{2}$

= 1.6 $\times$ 5

= 8.0 cm

$\Rightarrow$ AC = 8 cm

$\therefore$ CE = AC - AE

= 8 - 3.2

= 4.8 cm

Hence, BD = 3.6 cm and CE = 4.8 cm

19. Since XY $\parallel$ AC

$\therefore$ $\angle$BXY = $\angle$BAC

$\angle$BYX = $\angle$BCA [Corresponding angles]

$\therefore$ $\triangle$BXY $\cong$ $\triangle$BAC [AA similarity]

$\therefore \frac{ar(\triangle BXY)}{ar(\triangle BAC)} = \frac{BX^2}{BA^2}$

But ar($\triangle$ BXY)=ar(XYCA)

$\therefore$ 2($\triangle$ BXY)=ar($\triangle$ BXY)+ar(XYCA)

= ar($\triangle$BAC)

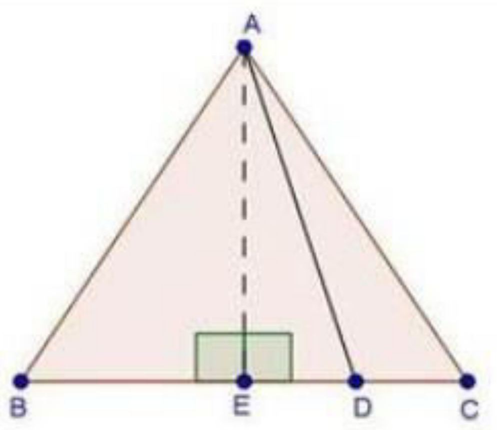
$\therefore \frac{ar(\triangle BXY)}{ar(\triangle BAC)} = \frac{1}{2}$

$\therefore \frac{BX^2}{BA^2} = \frac{1}{2}$

$\Rightarrow \therefore \frac{BX}{BA} = \frac{1}{\sqrt{2}}$

$\therefore \frac{BA-BX}{BA} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

20. 

Construction:- Draw $AE \perp BC$

Given, AD is a median

In $\triangle AED$,

By using pythagoras theorem, we get

$DE^2 = AE^2 + AD^2$

$AE^2 = AD^2 - DE^2$ ... (i)

In $\triangle AEB$,

By using pythagoras theorem, we get

$AB^2 = AE^2 + BE^2$

$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2$ [from (i)]

$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + CD^2 - 2CD \times DE$ ...(i) [BD = CD]

In $\triangle AEC$,

By using pythagoras theorem, we get

$AC^2 = AE^2 + EC^2$

$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + DC)^2$ [from (i)]

$\Rightarrow AC^2 = AD^2 - DE^2 + DE^2 + DC^2 + 2DE \times DC$

$\Rightarrow AC^2 = AD^2 + DC^2 + 2DE \times DC$ ...(ii)

Add equations (i) and (ii)

Therefore, $AB^2 + AC^2 = 2AD^2 + 2CD^2$