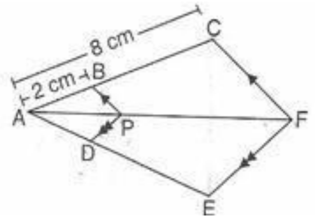
CBSE Test Paper 02

Chapter 6 Triangles

1. In the given figure if $BP||CF$, $DP||\mathrm{E}F$,then AD: DE is equal to **(1)**



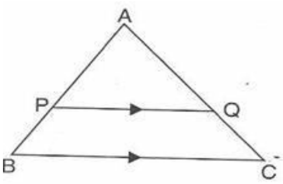
a. 1 : 3.

b. 3 : 4.

c. 2 : 3.

d. 1 : 4.

2. In the given figure $PQ||BC$. $\frac{AP}{PB} = 4$, then the value of $\frac{AQ}{AC}$ is **(1)**



a. 5

b. $\frac{4}{5}$

c. 4

d. $\frac{5}{4}$

3. If $\Delta ABC \sim \Delta PQR$ such that AB = 9.1 cm and PQ = 6.5 cm. If the perimeter of ΔPQR is 25 cm, then the perimeter of ΔABC is **(1)**

a. 34 cm

b. 35 cm

c. 36 cm

d. 30 cm

4. Out of the given statements **(1)**

i. The areas of two similar triangles are in the ratio of the corresponding altitudes.

ii. If the areas of two similar triangles are equal, then the triangles are congruent.

iii. The ratio of areas of two similar triangles is equal to the ratio of their corresponding medians.

iv. The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

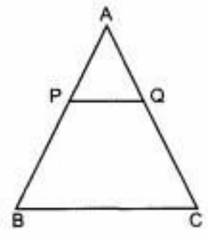
The correct statement is

a. (iii)
b. (ii)
c. (i)
d. (iv)

5. If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$, then **(1)**

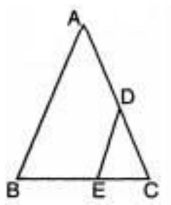
a. $\Delta FDE \sim \Delta ABC$.
b. $\Delta BCA \sim \Delta FDE$.
c. $\Delta FDE \sim \Delta CAB$.
d. $\Delta CBA \sim \Delta FDE$.

6. In the fig PQ $\parallel$ BC and AP: PB = 1:2. Find $\frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)}$. **(1)**

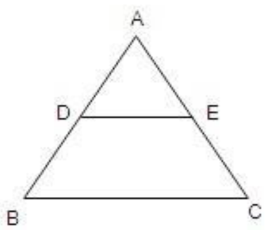


7. If the altitude of two similar triangles are in the ratio 2 : 3, what is the ratio of their areas? **(1)**

8. In the figure of $\triangle$ABC, the points D and E are on the sides CA, CB respectively such that DE $\parallel$ AB, AD = 2x, DC = x + 3, BE = 2 x - 1 and CE = x. Then, find x. **(1)**

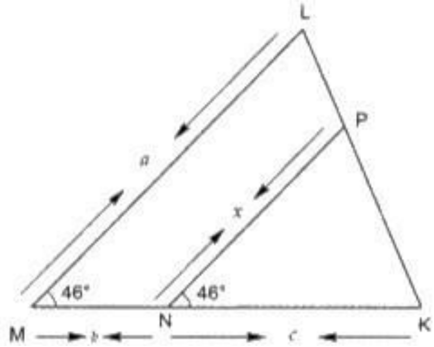


9. In $\triangle ABC$ shown below, DE || BC
If BC = 8 cm , DE = 6 cm and area of $\triangle ADE = 45cm^2$, What is the area of $\triangle ABC$? **(1)**

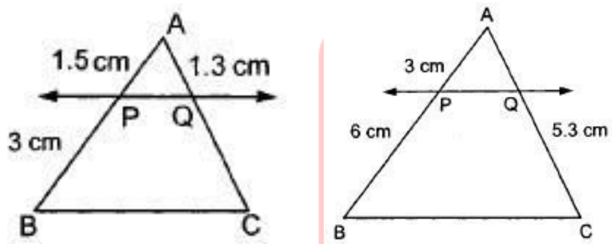


10. In $\triangle$ABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, AY = 5 and YC = 9, then state whether XY and BC are parallel or not. **(1)**

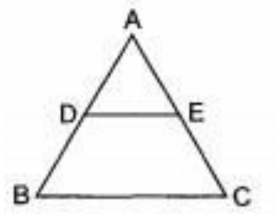
11. In Fig. $\angle$M = $\angle$N = 46°. Express x in terms of a, b and c where a, b, c are lengths of LM, MN and NK respectively. **(2)**



12. In Fig. (i) and (ii), $PQ || BC$. Find QC in (i) and AQ in (ii). **(2)**

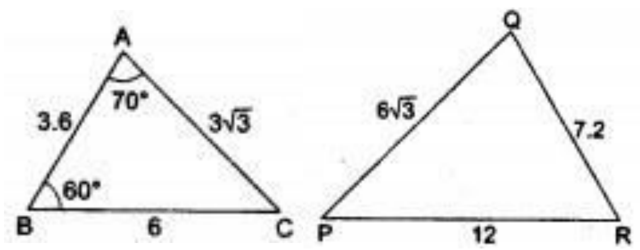


13. In figure, D and E are points on AB and AC respectively, such that DE $||$ BC. If AD = $\frac{1}{3}$ BD, AE = 4.5 cm, find AC. **(2)**



14. In $\Delta ABC, DE || BC$ If AD = x + 2, DB = 3x + 16, AE = x and EC = 3x + 5, then find x. **(3)**

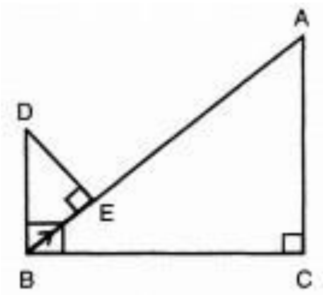
15. Find $\angle P$ in the adjoining figure. **(3)**



16. In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that

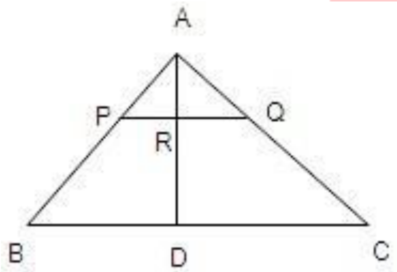
$LM \| AB$ and $MN \| BC$ but neither of L, M, N nor of A, B, C are collinear. Show that $LN \| AC$. **(3)**

17. In the given figure, $DB \perp BC, DE \perp AB$ and AC $\perp$BC. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$ **(3)**

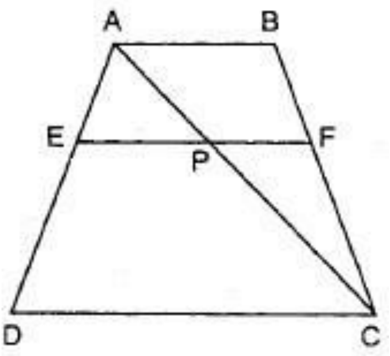


18. For going to a city B from city A, there is a route via city C such that AC $\perp$ CB, AC = 2x km and CB = 2(x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway. **(4)**

19. In the given figure , AP = 3cm , AR = 4.5 cm, AQ = 6 cm ,AB = 5 cm and AC = 10 cm , then find AD and the ratio of areas of $\triangle ARQ$ and $\triangle ADC$. **(4)**



20. In Fig. if $EF \| DC \| AB$. prove that $\frac{AE}{ED} = \frac{BF}{FC}$. **(4)**



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Solution

1. a. 1 : 3.

   Explanation: In $\triangle AFC, BP \parallel FC \Rightarrow \frac{AB}{BC} = \frac{AP}{PF} = \frac{1}{3}$

   In $\triangle AFE, DP \parallel FE \Rightarrow \frac{AD}{DE} = \frac{AP}{PF} = \frac{1}{3}$

   therefore AD:DE = 1:3

2. b. $\frac{4}{5}$

   Explanation: Given: $\frac{\text{AP}}{\text{PB}} = \frac{4}{1}$

   Let AP = 4x and PB = x, then AB = AP + PB = 4x + x = 5x

   Since PQ ∥BC, then

   $\frac{\text{AP}}{\text{AB}} = \frac{\text{AQ}}{\text{AC}}$ [Using Thales theorem]

   $\therefore \frac{\text{AQ}}{\text{AC}} = \frac{\text{AP}}{\text{AB}} = \frac{4x}{5x} = \frac{4}{5}$

3. b. 35 cm

   Explanation:

   $\frac{AB}{PQ} = \frac{7}{5}(cpst) Therefore BC = a \implies QR = \frac{5}{7}a, AC = b \implies PR = \frac{5}{7}$

   $6.5 + \frac{5}{7}a + \frac{5}{7}b = 25 \implies a + b = 25.9$

   Therefore perimeter of $\triangle ABC = 35$

4. b. (ii)

   Explanation: If the areas of two similar triangles are equal, then the triangles are congruent

   Option (i) is wrong since "The areas of two similar triangles are in the ratio of the square of the corresponding altitudes.

   Like that options (iii) and (iv) are also wrong.

5. c. $\Delta FDE \sim \Delta CAB.$

   Explanation: If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}, then$

   $\Delta FDE \sim \Delta CAB$

   because for similarity, all the corresponding sides should be in proportion.

6. In ABC,

$PQ \parallel BC$

$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC}$

Now in ΔAPQ and ΔABC,

$\frac{AP}{AB} = \frac{AQ}{AC}$ (As proved)

$\angle$A = $\angle$A (common angle)

$\triangle APQ \sim \triangle ABC$ (SAS similarity)

Since for similar triangles,the ratio of the areas is the square of the ratio of their corresponding sides. Therefore,

$$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP+PB)^2} = \frac{1^2}{3^2} = \frac{1}{9}$$

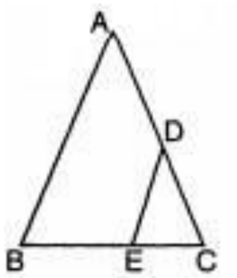
7. We know that the ratio of areas of two similar triangles is equal to the square of the ratio of corresponding altitude.

   Ratio of their areas = (ratio of their altitudes)^2

   = $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

   = 4 : 9

8. $DE \parallel AB$



AD = 2x, DC = x + 3, BE = 2 x - 1 and CE = x

By Basic proportionality theorem

$\frac{CD}{AD} = \frac{CE}{BE}$

$\frac{x+3}{2x} = \frac{x}{2x-1}$

(x + 3) (2x - 1) = x(2x)

$2x^2 - x + 6x - 3 = 2x^2$

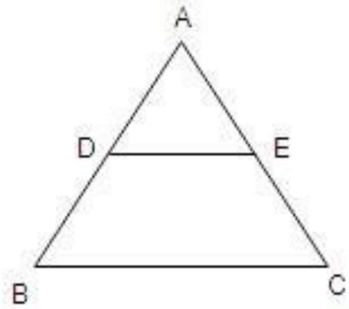
$2x^2 + 5x - 3 = 2x^2$

5x - 3 = 0

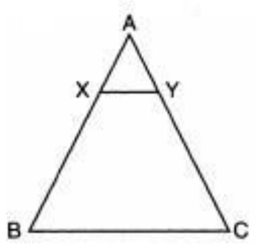
or, 5x = 3

x = $\frac{3}{5}$

9. $\triangle ADE \sim \triangle ABC$



$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = (\frac{DE}{BC})^2$
$\Rightarrow \frac{45}{ar(\triangle ABC)} = (\frac{6}{8})^2$
$\Rightarrow \frac{45}{ar(\triangle ABC)} = \frac{36}{64}$
$\Rightarrow ar(\triangle ABC) = \frac{64(45)}{36}$
$\Rightarrow ar(\triangle ABC) = 80cm^2$

10. 

$\frac{AX}{XB} = \frac{3}{4} \ldots . (i)$
$\frac{AY}{CY} = \frac{5}{9} \ldots . (ii)$
From eqn (i) and (ii)
$\frac{AX}{XB} \neq \frac{AY}{YC}$
So XY and BC are not parallel

11. In ΔKPN and ΔKLM, we have

∠KNP = ∠KML = 46° [Given]

∠NKP = ∠MKL [Common]

Thus, ΔKPN ~ ΔKLM [by AA similarity criterion of triangles]

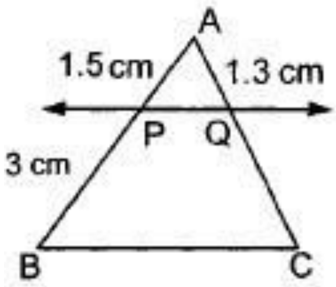
$\frac{\mathrm{KN}}{\mathrm{KM}} = \frac{\mathrm{NP}}{\mathrm{ML}}$ [because we know that corresponding sides of similar triangles are proportional]

$\frac{\mathrm{c}}{\mathrm{b+c}} = \frac{\mathrm{x}}{\mathrm{a}}$ [KM = MN + NK]

$\Rightarrow x(b+c) = ca$

Therefore, x = $\frac{\mathrm{ac}}{\mathrm{b+c}}$

12. According to question



$PQ \| BC$

Therefore, by basic proportionality theorem, we have

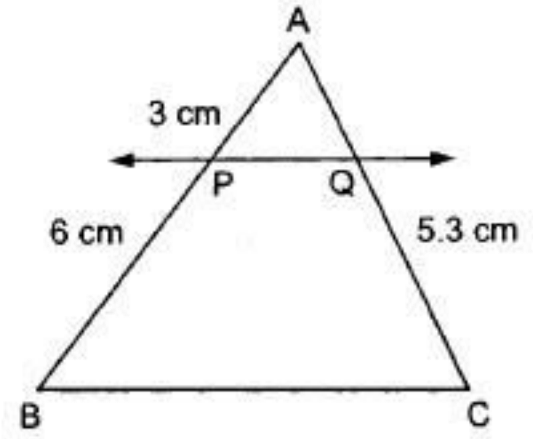
$\frac{AP}{PB} = \frac{AQ}{QC}$

$\Rightarrow \quad \frac{1.5}{3} = \frac{1.3}{QC}$

$\Rightarrow \quad \frac{1}{2} = \frac{1.3}{QC}$

$\Rightarrow$ QC = 2.6 cm

In Fig. (ii)



it is given that $PQ \| BC$.

Therefore, by basic proportionality theorem, we have

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\Rightarrow \quad \frac{3}{6} = \frac{AQ}{5.3}$

$\Rightarrow \quad \frac{1}{2} = \frac{AQ}{5.3}$

$\Rightarrow \quad AQ = \frac{5.3}{2} = 2.65\text{cm}$

Hence QC = 2.6 cm and AQ = 2.65 cm respectively

13. According to question it is given that D and E are the points on sides AB and AC respectively

Also AD = $\frac{1}{3}$ BD,

AE = 4.5 cm, DE $||$ BC

$\therefore \quad \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{EC}}$

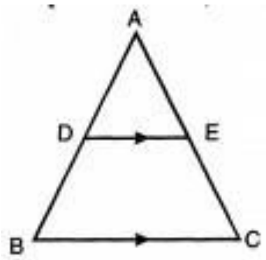
$\Rightarrow \quad \frac{\frac{1}{3}\text{BD}}{\text{BD}} = \frac{4.5}{\text{EC}}$

$\Rightarrow \quad \frac{1}{3} = \frac{4.5}{\text{EC}}$

$\Rightarrow \quad \text{EC} = 4.5 \times 3 \text{ cm}$

$\Rightarrow$ EC = 13.5 cm

Now, AC = AE + EC = 4.5 + 13.5 = 18 cm

14. 

$\therefore \ DE||BC$

$\frac{AD}{DB} = \frac{AE}{EC}$ (by BPT)

or, $\frac{x+2}{3x+16} = \frac{x}{3x+5}$

On cross multiplication we get

(x + 2)(3x + 5) = x(3x + 16)

$3x^2 + 5x + 6x + 10 = 3x^2 + 16x$

5x = 10

x = 2

15. In $\triangle ABC$ and $\angle QRP$, we have

$\frac{AB}{QR} = \frac{3.6}{7.2} = \frac{1}{2}$,

$\frac{BC}{RP} = \frac{6}{12} = \frac{1}{2}$

and $\frac{CA}{PQ} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$

Thus, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$ and so

$\triangle ABC \sim \triangle QRP$ [by SSS-similarly].

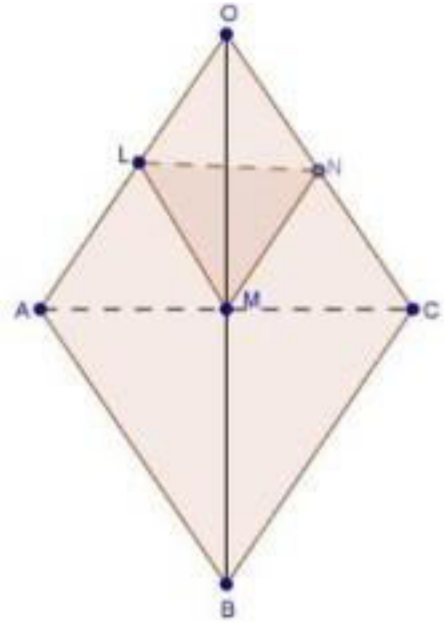
$\therefore \quad \angle C = \angle P$ [corresponding angles of similar triangles].

But, $\angle C = 180^\circ - (\angle A + \angle B)$

= $180^\circ - (70^\circ + 60^\circ) = 50^\circ$

$\therefore \quad \angle P = 50^\circ$.

16. We have,



LM || AB and MN || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{OL}{AL} = \frac{OM}{MB}$ ......(i)

And, $\frac{ON}{NC} = \frac{OM}{MB}$ .... (ii)

Compare equation (i) and equation (ii), we get

$\frac{OL}{AL} = \frac{ON}{NC}$

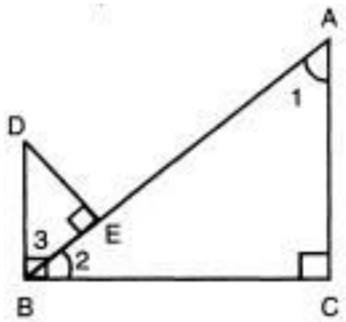
Thus, LN divides sides OA and OC $\triangle$OAC in the same ratio.

Therefore, by the converse of basic proportionality theorem, we have,

LN||AC.

17. Given:

$DB \perp BC, DE \perp AB$ and $AC \perp BC$



To prove:

$\frac{BE}{DE} = \frac{AC}{BC}$

Proof: As per the figure

$\angle 1 + \angle 2 = 90^\circ$

$\angle 2 + \angle 3 = 90^\circ$

So $\angle 1 = \angle 3$

$\triangle ABC$ and $\triangle BDE$

$\angle ACB = \angle DEB = 90^\circ$

$\angle 1 = \angle 3$

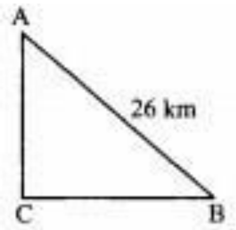
$Hence \triangle ACB \sim \triangle DEB$

Hence $\frac{BE}{DE} = \frac{AC}{BC}$

18. A.T.Q.

Let AC = 2x km

CB = 2(x + 7) km



In right-angled $\triangle$ACB, $AB^2 = AC^2 + CB^2$

Highway is AB = 26 km

$(26)^2 = (2x)^2 + (2(x + 7))^2$

$\Rightarrow 676 = 4x^2 + 4(x + 7)^2$

$\Rightarrow \frac{676}{4} = x^2 + x^2 + 49 + 14x$

$\Rightarrow 169 = 2x^2 + 14x + 49$

$\Rightarrow 2x^2 + 14x + 49 - 169 = 0$

$\Rightarrow 2x^2 + 14x - 120 = 0$

$\Rightarrow x^2 + 7x - 60 = 0$

$\Rightarrow x^2 + 12x - 5x - 60 = 0$

$\Rightarrow x(x + 12) - 5(x + 12) = 0$

$\Rightarrow (x - 5)(x + 12) = 0$

$\Rightarrow x = -12, 5$

So, AC = 2x = 2(5) = 10 km

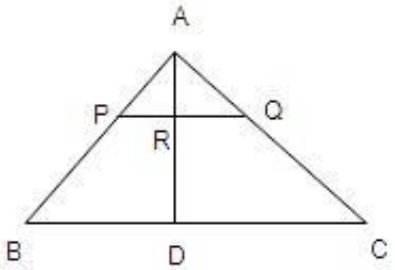
BC = 2(x + 7) = 2(5 + 7) = 24 km

Total distance for A to B via C = 10 + 24 = 34 km

A to B via highway = 26 km

Distance saved = 34 - 26 = 8 km.

19. Given , AP = 3cm, AR = 4.5 cm, AQ = 6 cm, AB = 5 cm and AC = 10 cm



Here , $\frac{AP}{AB} = \frac{3}{5}$ and $\frac{AQ}{AC} = \frac{6}{10} = \frac{3}{5}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Thus, $PQ||BC$ [by converse of basic proportionality theorem]

In $\triangle ARQ$ and $\triangle ADC$

$\angle RAQ = \angle DAC$ [common angle]

$\angle ARQ = \angle ADC$ [corresponding angles]

$\angle RQA = \angle DCA$ [corresponding angles]

So, $\triangle ARQ \sim \triangle ADC$ [By AAA similarity criterion]

$\Rightarrow \frac{AR}{AD} = \frac{AQ}{AC}$ [Since, corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{4.5}{AD} = \frac{6}{10}$

$\Rightarrow AD = \frac{45}{6}$

$\Rightarrow AD = \frac{15}{2} = 7.5cm$ .... (i)

Now, $\frac{ar(\triangle ARQ)}{ar(\triangle ADC)} = (\frac{AQ}{AC})^2$ [By theorem of area of similar triangles]

$\frac{ar(\triangle ARQ)}{ar(\triangle ADC)} = (\frac{6}{10})^2$

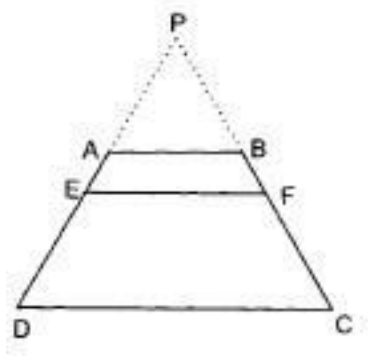
$\frac{ar(\triangle ARQ)}{ar(\triangle ADC)} = \frac{36}{100}$

$\frac{ar(\triangle ARQ)}{ar(\triangle ADC)} = \frac{9}{25}$ .... (ii)

So, AD = 7.5 cm and $\frac{ar(\triangle ARQ)}{ar(\triangle ADC)} = \frac{9}{25}$.

20. Given: According to the question,We have, $EF||DC||AB$ in the given figure.

To prove: $\frac{AE}{ED} = \frac{BF}{FC}$



Construction: Produce DA and CB to meet at P(say).

Proof: In ΔPEF, we have

$AB \| EF$

$\therefore \quad \frac{PA}{AE} = \frac{PB}{BF}$ [By Basic proportionality theorm]

$\Rightarrow \quad \frac{PA}{AE} + 1 = \frac{PB}{BF} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \frac{PA+AE}{AE} = \frac{PB+BF}{BF}$

$\Rightarrow \quad \frac{PE}{AE} = \frac{PF}{BF}$ ...(1)

In ΔPDC, we have,

$EF \| DC$

$\therefore \quad \frac{PE}{ED} = \frac{PF}{FC}$ [By Basic Proportionality Theorem] ...(2)

Therefore, on dividing equation (i) by equation (ii), we get

$\frac{\frac{PE}{AE}}{\frac{PE}{ED}} = \frac{\frac{PF}{BF}}{\frac{PF}{FC}}$

$\Rightarrow \quad \frac{ED}{AE} = \frac{FC}{BF}$

$\Rightarrow \quad \frac{AE}{ED} = \frac{BF}{FC}$