CBSE Test Paper 04

Chapter 13 Surface Area and Volume

1. The maximum volume of a cone that can be carved out of a solid hemisphere of radius ‘r’ is **(1)**

   a. πr^3
   b. $\frac{2}{3}\pi r^3$
   c. $\frac{1}{3}\pi r^3$
   d. $\frac{1}{3}\pi r^2 h$

2. The longest rod that can be placed inside the cube is **(1)**

   a. 3 edge
   b. $\sqrt{2}$ edge
   c. $\sqrt{3}$ edge
   d. $\sqrt{5}$ edge

3. The perpendicular distance between the two parallel circular bases is called the _______ of the frustum of the cone. **(1)**

   a. radius
   b. height
   c. slant height
   d. volume

4. If three metallic spheres of radii 6cm, 8cm and 10cm are melted to form a single sphere, the diameter of the sphere is **(1)**

   a. 26cm
   b. 30cm
   c. 24cm
   d. 36cm

5. The edge of the cube whose volume is 1728 cu.cm is **(1)**

a. 72cm
b. 18cm
c. 17cm
d. 12cm

6. Find the area of a parallelogram with base equal to 25 cm and the corresponding height measuring 16.8 cm. **(1)**

7. Two cylindrical cans have equal base areas. If one of the can is 15 cm high and other is 20 cm high, find the ratio of their volumes. **(1)**

8. The parallel sides of a trapezium are 12 cm and 9 cm and the distance between them is 8 cm. Find the area of the trapezium. **(1)**

9. The height of a right circular cone is 12 cm and the radius of its base is 4.5 cm. Find its slant height. **(1)**

10. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius. **(1)**

11. The largest sphere is to be curved out of a right circular cylinder of radius 7 cm. and height 14 cm. Find the volume of the sphere. **(2)**

12. The base of a right-angled triangle measures 48 cm and its hypotenuse measures 50 cm. Find the area of the triangle. **(2)**

13. How many lead balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm? **(2)**

14. A tent is in the shape of a right-circular cylinder upto a height of 3 m and conical above it . The total height of the tent is 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of 2 Rs/m^2, if the radius of the base is 14 m. **(3)**

15. Two solid right circular cones have the same height. The radii of their bases are r_1 and r_2. They are melted and recast into a cylinder of same height. Show that the

radius of the base of the cylinder is $\sqrt{\frac{r_1^2+r_2^2}{3}}$. **(3)**

16. An open metal bucket is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at Rs.30 per litre. **(3)**

17. A milk container of height 16 cm is made of metal sheet in the form of a frustrum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs.22 per L, which the container can hold. **(3)**

18. A right circular cone is divided into three parts trisecting its height by two planes drawn parallel to the base. Show that volumes of the three portions starting from the top are in the ratio 1 : 7 : 19. **(4)**

19. A container, open from the top, made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs 15 per litre and the cost of metal sheet used, costs Rs 5 per 100 cm^2. (Use π = 3.14) **(4)**

20. The radius of a solid iron sphere is 8 cm. Eight rings of iron plate of external radius $6\frac{2}{3}$ cm and thickness 3 cm are made by melting this sphere. Find the internal diameter of each ring. **(4)**

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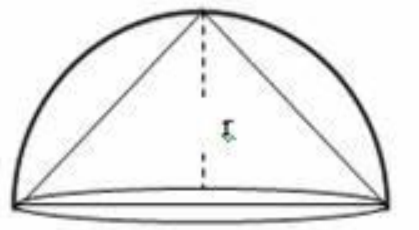
Solution

1. c. $\frac{1}{3}\pi r^3$

Explanation:



Volume of cone = $\frac{1}{3}\pi r^2 h$

Here height of the carved out cone = Radius of the hemisphere

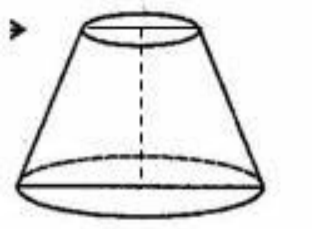
$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 \times r$ = $\frac{1}{3}\pi r^3$

2. c. $\sqrt{3}$ edge

Explanation: The longest rod that can be placed inside the cube is $\sqrt{3}$ times the edge of the cube, i.e., $\sqrt{3}$ edge

3. b. height

Explanation: The perpendicular distance between the two parallel circular bases is called the height of the frustum of the cone.



Frustum of cone

4. c. 24 cm

Explanation: Let r_1, r_2 and r_3 be the radii of three metallic spheres and r be the radius of a single sphere.

Therefore, according to the question,

$\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right) = \frac{4}{3}\pi r^3$

$\Rightarrow r_1^3 + r_2^3 + r_3^3 = r^3$

$\Rightarrow (6)^3 + (8)^3 + (10)^3 = r^3$

$\Rightarrow 216+512+1000=r^3$
$\Rightarrow r^3=1728$
$\Rightarrow r=12$ cm
$\therefore$ Diameter of the sphere = $2 \times 12 = 24$ cm

5. d. 12cm

Explanation: Given: Volume of cube = 1728 cu.cm
$\Rightarrow a^3=1728$
$\Rightarrow a^3=(12)^3$
$\Rightarrow a=12$ cm

6. Area of the ||gm = (base $\times$ height) sq. unit

= (25×16.8) cm^2 = 420 cm^2

7. Let the base area of first cylinder is πr^2.

$\therefore$ Base area of second cylinder is also πr^2.

h_1 = 15 cm, h_2 = 20 cm

Ratio of volumes = $\frac{\pi r^2 h_1}{\pi r^2 h_2}=\frac{15}{20}=\frac{3}{4}$

Volume of first cylinder: Volume of second cylinder = 3 : 4.

8. Area of a trapezium = $\frac{1}{2} \times$ (sum of parallel sides) $\times$ (distance between them)

$\therefore$ Area of a trapezium = $\frac{1}{2}\times(12+9)\times 8=\frac{1}{2}\times 21\times 8$ = 84 cm^2

9. h = 12 cm, r = 4.5 cm

Slant height $l=\sqrt{r^2+h^2}=\sqrt{(4.5)^2+12^2}$
$=\sqrt{20.25+144}=\sqrt{164.25}$
= 12.816 (approx)

10. Let r_1 , r_2, r_3 be the radii of the given 3 spheres.

So, r_1 = 3, r_2 = 4, r_3= 5

Again, let radius of the new sphere is R

Now, given three solid metallic spherical balls are melted into a single spherical ball

So, sum of volume of the three sphere = volume of the new sphere

$\frac{4}{3}\pi R^3=\frac{4}{3}\pi\left[(3)^3+(4)^3+(5)^3\right]$

or, $R^3 = 27 + 64 + 125$
or, $R^3 = 216$
or, R = 6 cm

11. According to the question,we are given that,
Radius of cylinder = 7 cm
Height of cylinder = 14 cm
The largest sphere is carved out from cylinder,
Then, Diameter of sphere = Diameter of cylinder
= 2 $\times$7 = 14 cm
Volume of sphere = $\frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$
$= \frac{4312}{3}$ = 1437.3 cm^3

12. Base of right-angled triangle = 48 cm
hypotenuse= 50 cm
Height of the right angled triangle = $\sqrt{(\text{ hypotenuse})^2 - (\text{ base })^2}$
height = $\sqrt{(50)^2 - (48)^2}\text{cm}$
=$\sqrt{2500 - 2304}\text{cm}$ = $\sqrt{196}$ cm = 14 cm
Area of triangle = $\left(\frac{1}{2} \times \text{ Base } \times \text{ Height }\right) \text{cm}^2$
=$\left(\frac{1}{2} \times 48 \times 14\right) \text{cm}^2$= 336$cm^2$

13. Radius of sphere = 8 cm
Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 = \frac{2048\pi}{3}\text{cm}^3$
Radius of each ball = 1 cm
Volume of each ball = $\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi\text{cm}^3$
Number of lead balls = $\frac{\text{Volume of sphere}}{\text{Volume of a ball}}$
$= \frac{\frac{2048\pi}{3}\text{cm}^3}{\frac{4}{3}\pi\text{cm}^3} = \frac{2048}{4} = 512$
Hence, the required number of balls made is 512.

14. Radius of cylinder = Radius of cone = r = 14m
Height of Cylinder = h = 3m
Total height of tent = 13.5 m

Therefore, Height of Cone = H = height of tent - height of cylinder = 13.5 - 3 = 10.5m

Required Curvded surface area= CSA of cylinder + CSA of cone

$= 2\pi rh + \pi rl$

Where, l = $\sqrt{r^2 + H^2}$

$= 2\pi rh + \pi r\sqrt{r^2 + H^2}$

$= 2 \times \pi \times 14 \times 3 + \pi \times 14 \times \sqrt{(14)^2 + (10.5)^2}$

= 264 + 44 x 17.5

= 1034 m^2

Cost of painting inner side of tent is Rs 2 per m^2

Therefore, cost of painting inner side of the tent = 2×1034 = 2068.

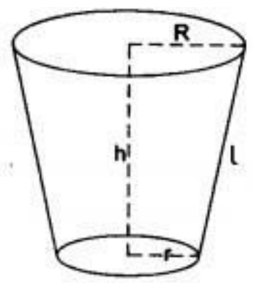
Therefore total cost of painting is Rs 2068.

15. Let h be the height of two given cones of base radii r_1 and r_2 respectively. Further, let R be the radius of the cylinder. It is given that the cylinder is also of height h.

Volume of the cylinder = Sum of the Volumes of two cones

$$\Rightarrow \quad \pi R^2 h = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$$
$$\Rightarrow \quad \pi R^2 h = \frac{1}{3}\pi h\left(r_1^2 + r_2^2\right)$$
$$\Rightarrow \quad R^2 = \frac{1}{3}\left(r_1^2 + r_2^2\right)$$
$$\Rightarrow \quad R = \sqrt{\frac{r_1^2 + r_2^2}{3}}$$

16. Let R and r be the radii of the top and the base of the bucket respectively, and let h be its height.



Then, R = 20 cm, r = 10 cm and h = 21 cm.

Capacity of the bucket

= volume of frustum of the cone

$= \frac{1}{3}\pi h\left[R^2 + r^2 + Rr\right]$

$= \left[\left(\frac{1}{3} \times \frac{22}{7} \times 21\right) \cdot \left\{(20)^2 + (10)^2 + 20 \times 10\right\}\right] \text{cm}^3$

= (22 $\times$ 700) cm^3 = 15400 cm^3.

Volume of milk that the bucket can hold

$= \left(\frac{15400}{1000}\right)$ litres $= 15.4$ litres. cost

$\therefore$ of milk = Rs(15.4 $\times$ 30) = Rs462.

17. In order to find cost of milk which can completely fill container, we need to find volume in litres.

Here r_1 = radius of lower end = 8 cm

r_2 = radius of upper end = 20 cm

h = height = 16 cm

$\therefore$ Volume of milk = Volume of frustrum as it is filled completely

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 16\left[8^2 + 20^2 + 8 \times 20\right]$

$= \frac{22 \times 16}{21}[64 + 400 + 160]$

$= \frac{352}{21} \times 624 = \frac{352 \times 208}{7} = \frac{73216}{7}$

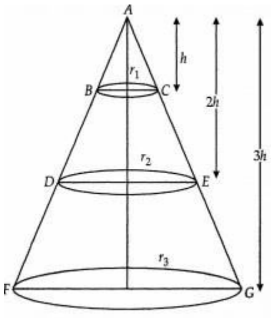
= 10459.428 cm^3 = 10.459 litre

Volume of milk = 10.459 litre

$\therefore$ Cost of milk = Rs.22 $\times$ 10.459 = Rs.230.098

Hence, the cost of milk = Rs.230.098

18. According to the question, A right circular cone is divided into three parts trisecting its height by two planes drawn parallel to the base.



Let the radii of three cones from top be r_1, r_2 and r_3 respectively.

Let the height of given cone be 3 h.

So, the height of cone ADE = 2h and height of cone ABC = h.

$\therefore \triangle ABC \sim \triangle ADE, \frac{r_1}{r_2} = \frac{h}{2h}$

$\Rightarrow \quad 2r_1 = r_2$

$\triangle ABC \sim \triangle AFG \frac{1}{r_3} = \frac{h}{3h}$

$3r_1 = r_3$

Volume of cone ABC $= \frac{1}{3}\pi r_1^2 h$

Volume of cone ADE $= \frac{1}{3}\pi (r_2)^2 2h$

$= \frac{1}{3}\pi (2r_1)^2 .2h$

Volume of frustum BCED $= \frac{1}{3}\pi 4r_1^2 .2h - \frac{1}{3}\pi r_1^2 h$

$= \frac{7}{3}\pi r_1^2 h$

Volume of frustum DEGF $= \frac{1}{3}\pi r_3^2 .3h - \frac{1}{3} r_2^2 \cdot 2h$

$= \frac{1}{3}\pi (3r_1)^2 3h - \frac{1}{3}(2r_1)^2 \cdot 2h$

$= \frac{1}{3}\pi r_1^2 h(27 - 8)$

$= \frac{19}{3}\pi r_1^2 h$

Ratio $= \frac{1}{3}\pi r_1^2 h : \frac{7}{3}\pi r_1^2 h : \frac{19}{3}\pi r_1^2 h$

Hence, required ratio = 1 : 7 : 19.

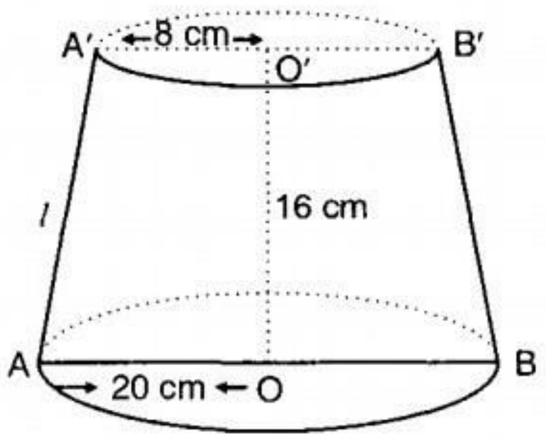
19. Radius (r_1) of upper end of container = 20 cm

Radius (r_2) of lower end container = 8 cm

height (h) of container = 16 cm

slant height (l) of frustum $= \sqrt{(r_1 - r_2)^2 + h^2}$

= $\sqrt{144 + 256} = \sqrt{400} = 20\text{cm}$



Let V be the volume of the container. Then,

$\therefore \quad V=\frac{\pi}{3}\left\{r_1^2+r_2^2+r_1 r_2\right\} h$

$\Rightarrow \quad V=\frac{\pi}{3}\left\{20^2+8^2+20 \times 8\right\} \times 16 \mathrm{~cm}^3$

$\Rightarrow \quad V=\frac{3.14 \times 624 \times 16}{3} \mathrm{~cm}^3$

$=10449.92 \mathrm{~cm}^3=\frac{10449.92}{1000}$ litres

= 10.45 litres approx.

$\therefore$ Cost of milk at the rate of RS 15 per litre = Rs $(10.45 \times 15)=Rs 156.75$

Let S be the surface area of the frustum. Then,

$S=\pi\left(r_1+r_2\right) l+\pi r_2^2$

$\Rightarrow \quad S=\left\{3.14(20+8) \times 20+3.14 \times 8^2\right\} \mathrm{cm}^2$

$\Rightarrow \quad S=3.14 \times(560+64) \mathrm{cm}^2=3.14 \times 624 \mathrm{~cm}^2$ $=1959.36 \mathrm{~cm}^2$

$\therefore$ Cost of metal used = Rs $\left(\frac{1959.36 \times 5}{100}\right)$ = Rs 97.96 (Approx)

20. We have,

Radius of the iron sphere = 8 cm

External radius of the 8 rings of iron plates = 20/3 cm

Thickness of the 8 rings of iron plates 3 cm

Let internal radius of the ring be r cm

Volume of solid iron sphere = $\frac{4}{3} \pi \times 8^3 \mathrm{~cm}^3=\frac{2048}{3} \pi \mathrm{cm}^3$

External radius of each iron ring = $6 \frac{2}{3} \mathrm{~cm}=\frac{20}{3} \mathrm{~cm}$

Now, it can be assumed that each ring of iron plate is a hollow cylindrical shell having internal radius 'r' cm and external radius 20/3 cm along with the height 3 cm

$\therefore$ Volume of each ring = $=\pi\left\{\left(\frac{20}{3}\right)^2-r^2\right\} \times 3 \mathrm{~cm}^3$

Volume of 8 such rings = $8 \pi\left(\frac{400}{9}-r^2\right) \times 3 cm^3=24 \pi\left(\frac{400}{9}-r^2\right) cm$

Clearly, Volume of 8 rings = Volume of the sphere

$\Rightarrow \quad 24 \pi\left(\frac{400}{9}-r^2\right)=\frac{2048}{3} \pi$

$\Rightarrow \quad \frac{400}{9}-r^2=\frac{2048}{3} \pi \times \frac{1}{24 \pi}$

$\Rightarrow \quad r^2=\frac{400}{9}-\frac{256}{9}=\frac{144}{9}=16$

$\Rightarrow$ r = 4 cm

Hence, internal radius of each ring is 4 cm.