CBSE Test Paper 04

Chapter 1 Real Numbers

1. The largest number which divides 245 and 1029 leaving remainder 5 in each case is **(1)**
   a. 8
   b. 12
   c. 4
   d. 16
2. Every positive odd integer is of the form__________where ‘q’ is some integer. **(1)**
   a. 2q + 2
   b. 5q + 1
   c. 3q + 1
   d. 2q + 1
3. Which of the following is a rational number? $\sqrt{15}$, $\sqrt{9}$, $\sqrt{10}$, $\sqrt{12}$. **(1)**
   a. $\sqrt{12}$
   b. $\sqrt{9}$
   c. $\sqrt{10}$
   d. $\sqrt{15}$
4. What is a lemma? **(1)**
   a. contradictory statement
   b. proven statement
   c. no statement
   d. None ot these
5. If HCF(a, b) = 12 and $a \times b$ = 1800, then LCM(a, b) is **(1)**
   a. 150
   b. 90
   c. 900
   d. 1800
6. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other. **(1)**

7. What is the HCF of the smallest composite number and the smallest prime number? **(1)**

8. Find the HCF of the following polynomials: $x^8 - y^8$; $\left(x^4 - y^4\right)(x + y)$. **(1)**

9. An army contingent of 1000 members is to march behind an army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **(1)**

10. If a and b are prime numbers, then what is their L.C.M.? **(1)**

11. Find the HCF of 1,656 and 4,025 by Euclid's division algorithm. **(2)**

12. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468. **(2)**

13. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **(2)**

14. If p is a prime number, then prove $\sqrt{p}$ that is an irrational. **(3)**

15. Find the HCF of 180, 252 and 324 by using Euclid's division lemma. **(3)**

16. Prove that 15 + 17$\sqrt{3}$ is an irrational number. **(3)**

17. Find the LCM and HCF of 336 and 54 and verify that LCM $\times$ HCF = product of two numbers. **(3)**

18. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial g(x), the quotient is $x^2 - 3x - 5$ and the remainder is $- 5x + 8$. Find the polynomial g(x). **(4)**

19. If the HCF of 152 and 272 is expressible in the form 272 $\times$ 8+ 152x, then find x. **(4)**

20. Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4. **(4)**

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Solution

1. d. 16

   Explanation: Let us subtract 5 (the remainder) from each number in order to find their HCF.

   245 - 5 = 240

   1029 - 5 = 1024

   Now, Let us find HCF of 240 , 1024

   1024 = 240 $\times$ 4 + 64

   240 = 64 $\times$ 3 + 48

   64 = 48 $\times$ 1 + 16

   48 = 16 $\times$ 3 + 0

   The largest number which divides 245 and 1029 leaving remainder 5 in each case is 16.

2. d. 2q + 1

   Explanation: Let a be any positive integer and b = 2

   Then by applying Euclid's Division Lemma,

   we have, a = 2q +r,

   where $0 \leqslant r < 2 \Rightarrow r$ = 0 or 1 $\therefore$ a = 2q or 2q + 1 .

   Therefore, it is clear that a = 2q i.e., a is an even integer.

   Also, 2q and are two 2q + 1 consecutive integers, therefore, 2q + 1 is an odd integer.

3. b. $\sqrt{9}$

   Explanation: $\sqrt{9}$ is an irrational number but because $\sqrt{9} = \sqrt{3^2}$ =3 and 3 is a rational number.

4. b. proven statement

   Explanation: A lemma is a proven statement that is used to prove another statement.

5. a. 150

Explanation: Using the result,

$HCF \times LCM$ = Product of two natural numbers

$\Rightarrow$ LCM (a, b) = $\frac{1800}{12}$ = 150

6. We are given that:

   HCF =145,LCM=2175 a=725 and b=?

   We know that

   LCM $\times$ HCF = a $\times$ b

   $\mathrm{b} = \frac{\mathrm{HCF} \times \mathrm{LCM}}{\mathrm{a}}$

   $= \frac{145 \times 2175}{725} = 435$

   Therefore the second number is 435

7. The smallest prime number is 2 and the smallest composite number is $4 = 2^2$

   Hence the required HCF (4 , 2) = 2

8. $P(x) = x^8 - y^8$

   $= (\mathrm{x}-\mathrm{y})(\mathrm{x}+\mathrm{y})\left(\mathrm{x}^2+\mathrm{y}^2\right)\left(\mathrm{x}^4+\mathrm{y}^4\right)$ Using Identity $\mathrm{a}^2-\mathrm{b}^2=(\mathrm{a}+\mathrm{b})(\mathrm{a}-\mathrm{b})$

   $Q(x) = \left(x^4 - y^4\right)(x+y)$

   $= (\mathrm{x}-\mathrm{y})(\mathrm{x}+\mathrm{y})\left(\mathrm{x}^2+\mathrm{y}^2\right)(\mathrm{x}+\mathrm{y})$ Using ldentity $\mathrm{a}^2-\mathrm{b}^2=(\mathrm{a}+\mathrm{b})(\mathrm{a}-\mathrm{b})$

   $\therefore \mathrm{HCF} = (\mathrm{x}-\mathrm{y})(\mathrm{x}+\mathrm{y})\left(\mathrm{x}^2+\mathrm{y}^2\right) = \mathrm{x}^4 - \mathrm{y}^4$

   Using Identity $\mathrm{a}^2-\mathrm{b}^2=(\mathrm{a}+\mathrm{b})(\mathrm{a}-\mathrm{b})$

9. 1000 = $2 \times 2 \times 2 \times 5 \times 5 \times 5$

   56 = $2 \times 2 \times 2 \times 7$

   HCF of 1000 and 56 = 8

   $\therefore$ Maximum number of columns = 8

10. a = 1 $\times$ a

    b = 1 $\times$ b

    HCF of a and b = 1

    Their LCM = 1 $\times$ a $\times$ b

    LCM of a and b = ab

11. AS 4025 > 1656 So applying Euclid's division algorithm on 4025 and 1656 we get

$$4025 = 1656 \times 2 + 713$$
$$1656 = 713 \times 2 + 230$$
$$713 = 230 \times 3 + 23$$
$$230 = 23 \times 10$$

Hence, HCF(1656, 4025) = 23

12. The smallest number divisible by 520 and 468 = LCM(520,468)

Prime factors of 520 and 468 are :

$520 = 2^3 \times 5 \times 13$

$468 = 2 \times 2 \times 3 \times 3 \times 13$

Hence LCM(520,468) = $2^3 \times 3^2 \times 5 \times 13 = 8 \times 9 \times 5 \times 13 = 4680$

Now the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

= LCM(520,468)-17

= 4680-17

= 4663

13. We can write the two given irrational numbers as:

$(\sqrt{2})^2 = 2$ and $(\sqrt{3})^2 = 3$

Let p be any rational number between $\sqrt{2}$ and $\sqrt{3}$.

$\Rightarrow (\sqrt{2})^2 < (p)^2 < (\sqrt{3})^2$

$\Rightarrow 2 < (p)^2 < 3$

One possible value of $(p)^2 = 2.25$

$\Rightarrow p = 1.5$

14. Let p be a prime number and if possible, let $\sqrt{p}$ be rational.

$\therefore \sqrt{p} = \frac{m}{n}$, where m and n are co-primes and $n \neq 0$

Squaring on both sides, we get

$\frac{(\sqrt{p})^2}{1} = \left(\frac{m}{n}\right)^2$

or, $p = \frac{m^2}{n^2}$

or, $pn^2 = m^2$ ....(I)

$\therefore$Hence p is a factor of m^2, So p is a factor of m...............(1)

So let m=pt ,t is any integer

On putting m=pt in.(i), we get

$pn^2 = p^2t^2$

$n^2 = pt^2$

Hence p is a factor of n^2, So p is a factor of n .... (2)

From (1) and (2) p is a common factor of m and n

Thus this contradicts the fact that m and n are co-primes.

The contradiction arises by assuming that $\sqrt{p}$ is rational.

Hence, $\sqrt{p}$ is irrational

15. Given numbers are 180, 252 and 324.

324 > 252 >180

On applying Euclid's Division lemma for 324 and 252, we get

324 = (252 × 1) + 72

Here, remainder = 72 ≠ 0

So, again applying Euclid's Division lemma with new dividend 252 and new divisor 72, we get

252 = (72 × 3) + 36

Here, remainder =36 ≠ 0

So, again applying Euclid's Division lemma with new dividend 72 and new divisor 36, we get

72 = (36 × 2) + 0

Here, remainder = 0 and divisor is 36

So, HCF of 324 and 252 is 36

Now, applying Euclid's Division lemma for 180 and 36, we get

180 = (36 × 5) + 0

Here, remainder = 0

So HCF of 180 and 36 is 36.

Hence, HCF of 180,252 and 324 is 36.

16. Suppose $\sqrt{3} = \frac{a}{b}$, where a and b are co-prime integers, $b \neq 0$

Squaring both sides,

$\Rightarrow 3 = \frac{a^2}{b^2}$

Multiplying with b on both sides,

$\Rightarrow 3b = \frac{a^2}{b}$

LHS = $3 \times b = Integer$

RHS = $\frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}} = Rational\ Number$

$\Rightarrow \text{LHS} \neq \text{RHS}$

$\therefore$ Our supposition is wrong.

$\Rightarrow \sqrt{3}$ is irrational.

Suppose $15 + 17\sqrt{3}$ is a rational number.

$\therefore 15 + 17\sqrt{3} = \frac{a}{b}$, where $a\ and\ b$ are co-prime, $b \neq 0$

$\Rightarrow \quad 17\sqrt{3} = \frac{a}{b} - 15$

$\sqrt{3} = \frac{a-15b}{17b}$

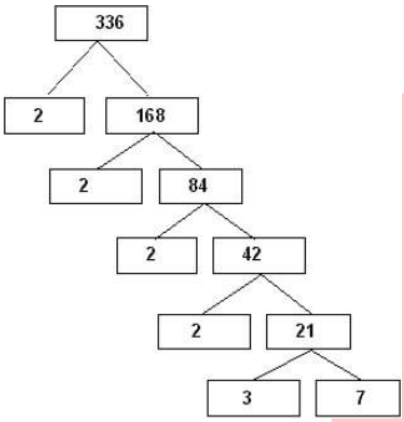
$\frac{a-15b}{17b}$ is rational number,

$\sqrt{3}$ is irrational.

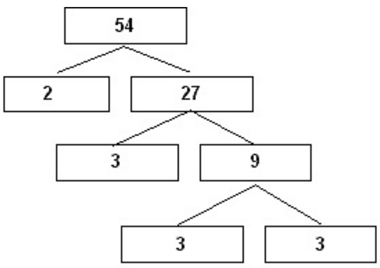
$\therefore \quad \sqrt{3} \neq \frac{a-15b}{17b}$

$\therefore$ Our supposition is wrong.

$\Rightarrow \quad 15 + 17\sqrt{3}$ is irrational.

17. 

So, 336 = 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 7 = 2^4 $\times$ 3 $\times$ 7



So, 54 = $2 \times 3 \times 3 \times 3 = 2 \times 3^3$

Therefore,

LCM (336, 54) = $2^4 \times 3^3 \times 7 = 3024$

HCM (336, 54) = $2 \times 3 = 6$.

Verification:

LCM $\times$ HCF = 3024 $\times$ 6 = 1814 and 336 $\times$ 54 = 18144

i.e. LCM $\times$ HCF = product of two numbers

18. It is given that on dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $- 5x + 8$. We have to find the polynomial $g(x)$.

Now, we know that

Dividend = (Divisor $\times$ Quotient) + Remainder

$4x^4 - 5x^3 - 39x^2 - 46x - 2 = g(x)(x^2 - 3x - 5) + (-5x + 8)$

or, $4x^4 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8 = g(x)(x^2 - 3x - 5)$

or, $4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$

$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$

$$
\begin{array}{r|l}
 & 4x^2 + 7x + 2 \\
\hline
x^2 - 3x - 5 & 4x^4 - 5x^3 - 39x^2 - 41x - 10 \\
 & 4x^4 - 12x^3 - 20x^2 \\
 & - \quad + \quad + \\
\hline
 & 7x^3 - 19x^2 - 41x - 10 \\
 & 7x^3 - 21x^2 - 35x \\
 & - \quad + \quad + \\
\hline
 & 2x^2 - 6x - 10 \\
 & 2x^2 - 6x - 10 \\
 & - \quad + \quad + \\
\hline
 & 0
\end{array}
$$

Hence, $g(x) = 4x^2 + 7x + 2$

19. On applying the Euclid's division lemma to find HCF of 152, 272, we get

$$
\begin{array}{r|l}
152 & 272 \ (1 \\
 & 152 \\
\hline
 & 120
\end{array}
\qquad
\begin{array}{r|l}
120 & 152 \ (1 \\
 & 120 \\
\hline
 & 32
\end{array}
$$

$272 = 152 \times 1 + 120$

Here the remainder = 0.

Using Euclid's division lemma to find the HCF of 152 and 120, we get

$$152 = 120 \times 1 + 32$$

Again the remainder = 0.

Using division lemma to find the HCF of 120 and 32, we get

$$\begin{array}{r} 32\,\overline{)\,120\,}\,(3 \\ 96 \\ \hline 24 \\ \hline \end{array}$$

$$120 = 32 \times 3 + 24$$

Similarly,

$$\begin{array}{r} 8\,\overline{)\,24\,}\,(3 \\ 24 \\ \hline 0 \\ \hline \end{array}$$

$$32 = 24 \times 1 + 8$$

$$24 = 8 \times 3 + 0$$

HCF of 272 and 152 is 8.

272×8 + 152x = H.C.F. of the numbers

$$\Rightarrow 8 = 272 \times 8 + 152x$$

$$\Rightarrow \ 8 - 272 \times 8 = 152x$$

$$\Rightarrow 8(1 - 272) = 152x$$

$$\Rightarrow x = \frac{-2168}{152} = \frac{-271}{19}$$

20. Let x = 2p + 1 and y = 2q + 1

$$\therefore \quad x^2 + y^2 = (2p+1)^2 + (2q+1)^2$$

$$= 4p^2 + 4p + 1 + 4q^2 + 4q + 1$$

$$= 4\left(p^2 + q^2 + p + q\right) + 2$$

$$= 2\left(2p^2 + 2q^2 + 2p + 2q + 1\right)$$

$$= 2m \quad \text{where } m = \left(2p^2 + 2q^2 + 2p + 2q + 1\right)$$

$\therefore \quad x^2 + y^2$ is an even number but not divisible by 4.