CBSE Test Paper 02

Chapter 1 Real Number

1. _______is neither prime nor composite. **(1)**
   a. 4
   b. 1
   c. 2
   d. 3
2. All non-terminating and non-recurring decimal numbers are **(1)**
   a. rational numbers
   b. irrational numbers
   c. integers
   d. natural numbers
3. The HCF of two consecutive odd numbers is **(1)**
   a. 2
   b. 0
   c. 1
   d. 3
4. The decimal expansion of 'π': **(1)**
   a. is non-terminating and non-recurring
   b. is terminating
   c. does not exist
   d. is non-terminating and recurring
5. If a is rational and $\sqrt{b}$ is irrational, then $a+\sqrt{b}$ is: **(1)**
   a. an irrational number
   b. an integer
   c. a natural number
   d. a rational number
6. Find the simplest form of $\frac{69}{92}$. **(1)**
7. State whether the given rational number will have a terminating decimal expansion or a nonterminating repeating decimal expansion. **(1)**

$\frac{13}{3125}$

8. What can you say about the prime factorisations of the denominators of 43.123456789. **(1)**

9. Find the LCM and HCF of 24, 15 and 36 by applying the prime factorization method. **(1)**

10. For any integer a and 3, there exists unique integers q and r such that a = 3q + r. Find the possible values of r. **(1)**

11. If α and β are zeroes of x^2- (k - 6)x + 2(2k - 1), find the value of k: if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **(2)**

12. Find the prime factorization of 1296. **(2)**

13. Without actual division, show that rational number $\frac{33}{50}$ is a terminating decimal. Express decimal form. **(2)**

14. Show that one and only one out of n, (n + 2) or (n + 4) is divisible by 3, where n EN. **(3)**

15. Wrtie the HCF and LCM of smallest odd composite number and the smallest odd prime number. If an odd number p divides q^2, then will it divide q^3 also? Explain. **(3)**

16. The HCF and LCM of two polynomials P(x) and Q(x) are (2x−1) and $\left(6x^3 + 25x^2 - 24x + 5\right)$ respectively. If $P(x) = 2x^2 + 9x - 5$, determine Q(x). **(3)**

17. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 a.m. then at what time will they again change simultaneously? **(3)**

18. Show that the cube of any positive integer is of the form 4m, 4m+1 or 4m+3, for some integer m. **(4)**

19. Find the HCF of 256 and 36 using Euclid's Division Algorithm. Also, find their LCM and verify that HCF $\times$ LCM = Product of the two numbers. **(4)**

20. Use Euclid's division algorithm, to find the largest number, which divides 957 and 1280 leaving remainder 5 in each case. **(4)**

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Solution

1. b. 1

   Explanation: 1 is neither prime nor composite.

   A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself

   e.g. 5 is prime because 1 and 5 are its only positive integers factors but 6 is composite because it has divisors 2 and 3 in addition to 1 and 6.

2. b. irrational numbers

   Explanation: All non-terminating and non-recurring decimal numbers are irrational numbers. A number is rational if and only if its decimal representation is repeating or terminating.

3. c. 1

   Explanation: The HCF of two consecutive odd numbers is 1.(e.g the HCF of 25, 27 is 1)

4. a. is non-terminating and non-recurring

   Explanation: The decimal expansion of 'π' is non-terminating and non-recurring.

   The value of π = 3.141592653589.........

   $\therefore$ Value of π is not-repeating decimal, non-terminating and non-recurring number.

5. a. an irrational number

   Explanation: Let a be rational and $\sqrt{b}$ is irrational.

   If possible let $a + \sqrt{b}$ be rational.

   Then $a + \sqrt{b}$ is rational and a is rational.

   $\Rightarrow \left[\left(a + \sqrt{b}\right) - a\right]$ is rational [Difference of two rationals is rational]

   $\Rightarrow \sqrt{b}$ is rational.

   This contradicts the fact that $\sqrt{b}$ is irrational.

   The contradiction arises by assuming that $a + \sqrt{b}$ is rational.

   Therefore, $a + \sqrt{b}$ is irrational.

6. The prime factors of 69 and 92 are:

   69= 3 $\times$ 13

   $92=4\times23=2^2\times23$

   $Hence\ \frac{69}{92}=\frac{3\times23}{2\times2\times23}=\frac{3}{4}$

7. $\frac{13}{3125}=\frac{13}{5^5}$ Here, q = 5^5,

   which is of the form 2^n5^m (n = 0, m = 5).

   So the rational number $\frac{13}{3125}$ has a terminating decimal expansion.

8. $43.12456789=\frac{43123456789}{1000000000}=\frac{43123456789}{10^9}$

   $=\frac{43123456789}{(2\times5)^9}=\frac{43123456789}{2^9\times5^9}$

   Prime factorization of the denominator of 43.123456789 are $2^9\ \times5^9$

   and are of the form, $2^m\ \times5^n$

   where m=9 and n=9

9. 24, 15 and 36

   Let us first find the factors of 24, 15 and 36

   24 = $2^3\times$ 3

   15 = 3 $\times$ 5

   36 = 2 $\times$ 2 $\times$ 3 $\times$ 3

   LCM of 24, 15 and 36 = 2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 3 $\times$ 5

   LCM of 24, 15 and 36 = 360

   HCF of 24, 15 and 36 = 3

10. According to Euclid's division lemma for two positive number a and b there exist integers q and r such that a = b × q + r where 0 $\leq$ r < b.

    Here b = 3

    Therefore, $0\leq r<3$

    So, the possible values of r can be 0, 1, 2 because as per Euclid's division lemma r is greater then or equal to zero and smaller then b

11. we are given that α and β are zeroes of x^2- (k - 6)x + 2(2k - 1),

    Given, α,β are the zeroes of polynomial

x^2 - (k - 6)x + 2(2k - 1)

$\therefore \quad \alpha+\beta$ = -[-(k - 6)] = k- 6

$\alpha\beta$= 2(2k - 1)

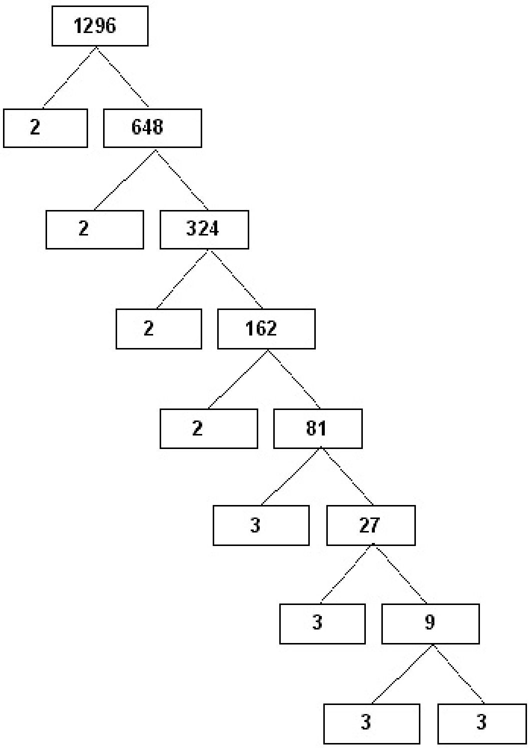
$\alpha+\beta=\frac{1}{2}\alpha\beta$

or, k + 6 = $\frac{2(2k-1)}{2}$

or, k - 6 = 2k - 1

or k = - 5

Hence the value of k = - 5.

12. 

So, 1296 = 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 3 $\times$ 3 $\times$ 3 = $2^4 \times 3^4$

Hence the prime factors of 1296 are 2, 2, 2, 2, 3, 3, 3, 3.

13. The given number is $\frac{33}{50}$.

The denominator 50 = 2 $\times$ 25

= 2 $\times$ 5^2 = $2^1 \times 5^2$

So the denominator is in the form of $2^m \times 5^n$ where m = 1 and n = 2.

Hence the given number is a terminating decimal.

Now, $\frac{33}{50}=\frac{33}{(2\times5^2)}=\frac{33\times2}{(2^2\times5^2)}=\frac{66}{(2\times5)^2}$

$=\frac{66}{(10)^2}=\frac{66}{100}=0.66.$

14. Let the number be (3q + r)

$$n = 3q + r \quad 0 \leq r < 3$$

or 3q, 3q + 1, 3q + 2

If n = 3 q then, numbers are 3 q , (3q + 1), (3q + 2)

3q is divisible by 3.

If n = 3q + 1 then, numbers are (3q + 1), (3q + 3), (3q + 4)

(3q + 3) is divisible by 3

If n = 3q + 2 then, numbers are (3q + 2), (3q + 4), (3q + 6)

(3q + 6) is divisible by 3.

∴ out of n, (n + 2) and (n + 4) only one is divisible by 3.

15. Smallest odd composite number = 9

and smallest odd prime number = 3

HCF of 9 and 3 = 3 and LCM of 9 and 3 = 9

Now, if an odd number p divides q^2, then p is one of the factors of q^2,

i.e. $q^2 = pm$, for some integer m .... (i)

Now, $q^3 = q^2 \times q$

$\Rightarrow q^3 = pm \times q$

$\Rightarrow q^3 = p(mq)$[from Eq(i)]

$\Rightarrow$ p is a factor of q^3 also $\Rightarrow$ p divides q^3.

16. It is given that $P(x) = 2x^2 + 9x - 5$

$$= 2x^2 + 10x - x - 5 = (x + 5)(2x - 1)$$

HCF of P(x) and Q(x) = (2x - 1) and

LCM of p(x) and Q(x) = $6x^3 + 25x^2 - 24x + 5$

= $(2x - 1)(3x^2 + 14x - 5)$ [Applying factor theorem]

= $(2x - 1)(3x^2 + 15x - x - 5)$

= (2x - 1) (x + 5) (3x - 1)

Now, P(x) × Q(x) = [HCF of P(x) and Q(x)] × [LCM of P(x) and Q(x)]

$\Rightarrow$ (x + 5) (2x - 1) $\Rightarrow$ Q(x) = (2x - 1) (2x - 1) (x + 5) (3x - 1)

Q(x) = (2x - 1) (3x - 1) = $6x^2 - 5x + 1$

17. We have to find Prime Factors of the following numbers

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

so the LCM of 48, 72 and 108is

$LCM = 2^4 \times 3^3$

$LCM = 16 \times 27 = 432$

432 seconds = $\frac{432}{60}$ mins

432 seconds = 7.2 mins

So the time it will change together again is

8 am + 7 mins 12 seconds = 8 : 07 : 12 am

18. Let a be an arbitrary positive integer.

Then, by Euclid's division Lemma, corresponding to the positive integers a and 4, there exist non - negative integers q and r such that

a = 4q+r , where $0 \leqslant r < 4$

⇒ $a^3 = (4q+r)^3 = 64q^3 + r^3 + 12\ qr^2 + 48q^2r$ $[(A+B)^3 = A^3 + B^3 + 3AB^2 + 3A^2B]$

⇒ $a^3 = 64\ q^3 + 48\ q^2r + 12\ qr^2 + r^3$ where $0 \leqslant r < 4$........(i)

The possible values of r are 0,1,2,3.

Case I: If r=0 then from Eq.(i) we get

$a^3 = 64\ q^3 + 48\ q^2(0) + 12\ q(0)^2 + (0)^3$

$a^3 = 64\ q^3 = 4(16\ q^3)$

⇒ $a^3 = 4m$

where, m = $16\ q^3$ is an integer.

Case II: If r = 1, then from Eq.(i), we get

$a^3 = 64\ q^3 + 48\ q^2r + 12\ q + 1$

$a^3 = 64\ q^3 + 48\ q^2(1) + 12\ q(1)^2 + (1)^3$

$= 4(\ 16\ q^3 + 12\ q^2 + 3\ q) + 1 = 4m + 1$

where, m = $(16\ q^3 + 12\ q^2 + 3\ q)$ is an integer.

Case III: If r = 2, then from Eq.(i), we get

$a^3 = 64\ q^3 + 48\ q^2(2) + 12\ q(2)^2 + (2)^3$

$a^3 = 64q^3 + 96q^2 + 48q + 8$

$= 4(16q^3 + 24q^2 + 12q + 2) = 4m$

where, $m = (16q^3 + 24q^2 + 12q + 2)$ is an integer.

Case IV: If r = 3, then from Eq.(i), we get

$a^3 = 64q^3 + 48q^2(3) + 12q(3)^2 + (3)^3$

$a^3 = 64q^3 + 144q^2 + 108q + 27$

$= 64q^3 + 144q^2 + 108q + 24 + 3$

$= 4(16q^3 + 36q^2 + 27q + 6) + 3 = 4m + 3$

where, $m = (16q^3 + 36q^2 + 27q + 6)$ is an integer.

Hence, the cube for any positive integer is of the form 4m, 4m+ 1o 4m +3 for some integer m.

19. Since 256 > 36, we apply the division lemma to 256 and 36, to get

$256 = 36 \times 7 + 4$

Again on applying the division lemma to 36 and 4, to get

$36 = 4 \times 9 + 0$

Hence, the HCF of 256 and 36 is 4

$$256 = 16 \times 16 = 2^4 \times 2^4 = 2^8$$
$$36 = 4 \times 9 = 2^2 \times 3^2$$

So LCM (36,256) = $2^8 \times 3^2 = 256 \times 9 = 2304$

HCF $\times$ LCM= $4 \times 2304 = 9216$

and $36 \times 256 = 9216$

So HCF $\times$ LCM = 36×256

Hence HCF $\times$ LCM =Product of two numbers

20. Given numbers are 957 and 1280 and remainder is 5 in each case.Then , new numbers after subtracting remainders are

957 – 5 = 952 and 1280 – 5 = 1275

Now, by using Euclid's Division lemma , we get

1275 = (952 × 1) + 323

Here remainder = 323

So, on taking 952 as dividend and 323 as new divisor and then apply Euclid's Division lemma, we get

952 = (323 × 2) + 306

Again, remainder = 306.

So, on taking 323 as dividend and 306 as new divisor and then apply Euclid's Division lemma, we get

323 = (306 × 1) + 17

Again, remainder = 17.

So, on taking 306 as dividend and 17 as new divisor and then apply Euclid's Division lemma, we get

306 = (17 × 18) + 0

Here, remainder = 0.

Since, remainder has now become zero and the last divisor is 17.

Therefore, HCF of 952 and 1275 is 17.