CBSE Test Paper 04

Chapter 4 Quadratic Equation

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1. The perimeter of a right triangle is 70cm and its hypotenuse is 29cm. The area of the triangle is **(1)**

a. 210 sq.cm
b. 200 sq.cm
c. 180 sq.cm
d. 250 sq.cm

2. $5x^2 + 8x + 4 = 2x^2 + 4x + 6$ is a **(1)**

a. quadratic equation
b. cubic equation
c. constant
d. linear equation

3. $x^2 - 30x + 225 = 0$ have **(1)**

a. Real roots
b. No real roots
c. Real and Equal roots
d. Real and Distinct roots

4. If the quadratic equation $bx^2 - 2\sqrt{ac}x + b = 0$ has equal roots, then **(1)**

a. $b^2 = -ac$
b. $2b^2 = ac$
c. $b^2 = ac$
d. $b^2 = 2ac$

5. A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if **(1)**

a. $b^2 - 4ac > 0$
b. $b^2 - 4ac < 0$
c. None of these
d. $b^2 - 4ac = 0$

6. Solve: $x^2 + 6x + 5 = 0$ **(1)**

7. Find the roots of the quadratic equation $2x^2 - x - 6 = 0$ **(1)**

8. Without solving, find the nature of the roots of the quadratic equations. $x^2 + x + 1 = 0$. **(1)**

9. Check whether it is quadratic equation: $(x + 1)^3 = x^3 + x + 6$ **(1)**

10. Find the discriminant of equation: $2x^2 - 7x + 6 = 0$. **(1)**

11. Solve the following problem: $x^2 - 45x + 324 = 0$ **(2)**

12. Find the roots of the equation, if they exist, by applying the quadratic formula: $x^2 + 5x - (a^2 + a - 6) = 0$. **(2)**

13. Use factorization method to solve the quadratic equation $ad^2x(\frac{a}{b}x + \frac{2c}{d}) + c^2b = 0$. **(2)**

14. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number. **(3)**

15. Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares. **(3)**

16. Solve: $\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}, x \neq -3, \frac{1}{2}, \frac{-9}{7}$ **(3)**

17. Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3} (x \neq 2, 4)$ **(3)**

18. A man buys a number of pens for Rs. 180. If he had bought 3 more pens for the same amount, each pen would have cost him Rs. 3 less. How many pens did he buy? **(4)**

19. At t minutes past 2 p.m, the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t. **(4)**

20. The hypotenuse of a right triangle is $3\sqrt{10}$ cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}$ cm. How long are the legs of the triangle? **(4)**

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Solution

1. a. 210 sq.cm

   Explanation: Let base of the right triangle be x cm.

   Given: Perpendicular = x + 29 = 70 $\Rightarrow$ Perpendicular = $(41 - x)$ cm

   Now, using Pythagoras theorem,

   $(29)^2 = x^2 + (41 - x)^2$
   $\Rightarrow 841 = 1681 + x^2 - 82x + x^2$
   $\Rightarrow 2x^2 - 82x + 840 = 0$
   $\Rightarrow x^2 - 41x + 420 = 0$
   $\Rightarrow x^2 - 20x - 21x + 420 = 0$
   $\Rightarrow x(x - 20) - 21(x - 20) = 0$
   $\Rightarrow (x - 20)(x - 21) = 0$
   $\Rightarrow x - 20 = 0$ and $x - 21 = 0$
   $\Rightarrow x = 20 \quad x =$ and 21

   Therefore, the two sides other than hypotenuse are of 20 cm and 21 cm.

   $\therefore$ Area of right triangle = $\frac{1}{2} \times$ Base $\times$ Perpendicular = $\frac{1}{2} \times 20 \times 21$ = 210 sq. cm

2. a. quadratic equation

   Explanation: Given: $5x^2 + 8x + 4 = 2x^2 + 4x + 6$
   $\Rightarrow 5x^2 - 2x^2 + 8x - 4x + 4 - 6 = 0$
   $\Rightarrow 3x^2 + 4x - 2 = 0$

   Here, the degree is 2, therefore it is a quadratic equation.

3. c. Real and Equal roots

   Explanation: D = $(-30)^2 - 4 \times 1 \times 225$

   D = 900 - 900

   D = 0. Hence Real and Equal roots.

4. c. $b^2 = ac$

   Explanation: If the quadratic equation $bx^2 - 2\sqrt{ac}x + b = 0$ has equal roots,

then $b^2 - 4ac = 0$
$\Rightarrow (-2\sqrt{ac})^2 - 4 \times b \times b = 0$
$\Rightarrow 4ac - 4b^2 = 0$
$\Rightarrow b^2 = ac$

5. a. $b^2 - 4ac > 0$

Explanation: A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if $b^2 - 4ac > 0$.

6. Given, $x^2 + 6x + 5 = 0$

Splitting middle term,

$\Rightarrow x^2 + 5x + x + 5 = 0$

$\Rightarrow x(x + 5) + 1(x + 5) = 0$

$\Rightarrow (x + 5)(x + 1) = 0$

$\Rightarrow x + 5 = 0$ or $x + 1 = 0$

Therefore, x = -5 or -1

7. Given, $2x^2 - x - 6 = 0$

Splitting the middle term of the equation,

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (x - 2)(2x + 3) = 0$

$\Rightarrow x - 2 = 0$ or $2x + 3 = 0$

Therefore, x = 2 or x = $-\frac{3}{2}$

8. $x^2 + x + 1 = 0$. Here $a = 1,\ b = 1,\ c = 1$

$D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

$\therefore$ equation has no real roots.

9. We have the following equation,

$(x + 1)^3 = x^3 + x + 6$

$\Rightarrow x^3 + 1 + 3x(x + 1) = x^3 + x + 6$

$\Rightarrow 3x^2 + 2x - 5 = 0.$

This is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

10. Given, $2x^2 - 7x + 6 = 0$

$a = 2$, $b = -7$ and $c = 6$

$\therefore D = b^2 - 4ac$

$= (-7)^2 - 4(2)(6)$

$= 49 - 48$

$= 1$

11. $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 9)(x - 36) \Rightarrow x = 9, 36$

12. The given equation is $x^2 + 5x - (a^2 + a - 6) = 0$

Comparing it with $Ax^2 + Bx + C = 0$, we get

$A = 1, B = 5 \text{ and } C = -(a^2 + a - 6)$

$\therefore D = B^2 - 4AC = (5)^2 - 4(1)(-(a^2 + a - 6))$

$= 25 + 4a^2 + 4a - 24 = 4a^2 + 4a + 1 = (2a + 1)^2 > 0$

So, the given equation has real roots, given by

$$\alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-5+\sqrt{(2a+1)^2}}{2\times1} = \frac{-5+(2a+1)}{2} = \frac{2a-4}{2} = a - 2$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-5-\sqrt{(2a+1)^2}}{2\times1} = \frac{-5-(2a+1)}{2} = \frac{-2a-6}{2} = -(a + 3)$$

Hence, $(a - 2)$ and $-(a + 3)$ are the roots of the given equation.

13. We have, $ad^2x(\frac{a}{b}x + \frac{2c}{d}) + c^2b = 0$

$$\implies \frac{a^2d^2}{b}x^2 + 2acdx + c^2b = 0$$

$$\implies \frac{a^2d^2}{b}x^2 + acdx + acdx + c^2b = 0$$

$$\implies adx(\frac{ad}{b}x + c) + bc(\frac{ad}{b}x + c) = 0$$

$$\implies (adx + bc)(\frac{ad}{b}x + c) = 0$$

Either $adx + bc = 0$ or $(\frac{ad}{b}x + c) = 0$

$$\implies x = -\frac{bc}{ad}$$

Hence, $x = -\frac{bc}{ad}$ is the requirreed solution.

14. Let the ten's place digit be y and unit's place be x.

Therefore, number is 10y + x.

According to given condition,

10y + x = 4(x + y) and 10y + x = 2xy

$\Rightarrow$ x = 2y and 10y + x = 2xy

Putting x = 2y in 10y + x = 2xy

10y + 2y = 2.2y.y

12y = $4y^2$

$4y^2$ - 12y = 0 $\Rightarrow$ 4y(y - 3) = 0

$\Rightarrow$ y - 3 = 0 or y = 3

Hence, the ten's place digit is 3 and units digit is 6 (2y = x)

Hence the required number is 36.

15. Let the sides of two squares be a and b,

then $a^2 + b^2 = 400$ ...(i)

and 4(a -b) = 16

or, a -b = 4:

or, a =4 + b (ii)

From equations (i) and (ii), we get

$(4 + b)^2 + b^2 = 400$

or, $16 + b^2 + 8b + b^2 = 400$

or, $2b^2 + 8b - 384 = 0$

or, $b^2 + 4b - 192 = 0$

or, $b^2 + 16b - 12b - 192 = 0$

or, $b(b + 16) - 12(b + 16) = 0$

or, $(b + 16)(b - 12) = 0$

b = - 16 (Rejecting the negative value)

So, b=12 cm

then a=16 cm

16. Given,

$\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}$

Taking LCM, we get

$\Rightarrow \frac{(2x-1)+(x+3)}{(x+3)(2x-1)} = \frac{11}{(7x+9)} \Rightarrow \frac{(3x+2)}{2x^2+5x-3} = \frac{11}{(7x+9)}$

Now cross multiply

$\Rightarrow$ (3x + 2)(7x + 9) = 11(2x^2 + 5x - 3)

$\Rightarrow$ 21x^2 + 41x +18 = 22x^2 + 55x - 33

$\Rightarrow$ x^2 + 14x - 51 = 0

$\Rightarrow$ x^2 + 17x - 3x - 51 = 0

$\Rightarrow$ x(x + 17) - 3(x + 17) = 0

$\Rightarrow$ (x + 17)(x - 3) = 0

$\Rightarrow$ x + 17 = 0 or x - 3 = 0

$\Rightarrow$ x = -17 or x = 3.

Therefore, -17 and 3 are the roots of the given equation.

17. The given equation is

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}(x\neq 2,4)$

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-4x-x+4+x^2-2x-3x+6}{x^2-4x-2x+8}=\frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$

$\Rightarrow 3\left(2x^2-10x+10\right)=10\left(x^2-6x+8\right)$

$\Rightarrow 6x^2-30x+30=10x^2-60x+80$

$\Rightarrow 4x^2-30x+50=0$

$\Rightarrow (2x)^2-2(2x)\left(\frac{15}{2}\right)+\left(\frac{15}{2}\right)^2-\left(\frac{15}{2}\right)^2+50=0$

$\Rightarrow \left(2x-\frac{15}{2}\right)^2-\frac{225}{4}+50=0$

$\Rightarrow \left(2x-\frac{15}{2}\right)^2-\frac{25}{4}=0$

$\Rightarrow 2x-\frac{15}{2}=\pm\frac{5}{2}\Rightarrow 2x=\frac{15}{2}\pm\frac{5}{2}$

$\Rightarrow 2x=\frac{15}{2}+\frac{5}{2},\frac{15}{2}-\frac{5}{2}$

$\Rightarrow 2x=10,5\Rightarrow x=5,\frac{5}{2}$

Hence, the solutions of the given equation and 5 and $\frac{5}{2}$.

18. Let the number of pens purchased be x.

Cost of 1 pen = Rs. $\frac{180}{x}$

If number of pens increase by 3. Then,

Cost of one pen = Rs. $\frac{180}{x+3}$

According to question,

$\frac{180}{x} - \frac{180}{x+3} = 3$

$\Rightarrow \frac{180x+540-180x}{x^2+3x} = 3$

$\Rightarrow 540 = 3x^2 + 9x$

$\Rightarrow 3x^2 + 9x - 540 = 0$

$\Rightarrow x^2 + 3x - 180 = 0$

$\Rightarrow x^2 + 15x - 12x - 180 = 0$

$\Rightarrow x(x + 15) - 12(x + 15) = 0$

$\Rightarrow x + 15 = 0$ or $x - 12 = 0$

$\Rightarrow x = -15$ or $x = 12$

As number of pens can't be negative.

$\Rightarrow x = 12$

Therefore, he bought 12 pens.

19. Total time taken by minute hand from 2 p.m. to 3 p.m. is 60 min.

According to question,

$t + \left(\frac{t^2}{4} - 3\right) = 60$

$\Rightarrow 4t + t^2 - 12 = 240$

$\Rightarrow t^2 + 4t - 252 = 0$

$\Rightarrow t^2 + 18t - 14t - 252 = 0$

$\Rightarrow t(t + 18) - 14(t + 18) = 0$

$\Rightarrow (t + 18)(t - 14) = 0$

$\Rightarrow t + 18 = 0$ or $t - 14 = 0$

$\Rightarrow t = -18$ or $t = 14$ min.

As time can't be negative. Therefore, t = 14 min.

20. Suppose, the smaller side of the right triangle be x cm and the larger side be y cm.

Then,

$\therefore x^2 + y^2 = (3\sqrt{10})^2$ [Using pythagoras theorem]

$\Rightarrow x^2 + y^2 = 90$ ....(i)

If the smaller side is tripled and the larger side be doubled, the new hypotenuse is $9\sqrt{5}$ cm.

$\therefore (3x)^2 + (2y)^2 = \left(9\sqrt{5}\right)^2$ [Using pythagoras theorem]

$\Rightarrow 9x^2 + 4y^2 = 405$ ....(ii)

Putting $y^2 = 90 - x^2$ in equation (ii), we get

$9x^2 + 4(90 - x^2) = 405$

$\Rightarrow 9x^2 + 360 - 4x^2 = 405$

$\Rightarrow 5x^2 = 405 - 360$

$\Rightarrow 5x^2 = 45$

$\Rightarrow x^2 = 9$

$\Rightarrow$ x = $\pm$3

But, length of a side can not be negative. Therefore, $x = 3$

Putting $x = 3$ in (i), we get

$(3)^2 + y^2 = 90$

$\Rightarrow y^2 = 90 - 9$

$\Rightarrow y^2 = 81$

$\Rightarrow$ y = $\pm$9

But, length of a side can not be negative. Therefore, $y = 9$

Hence, the length of the smaller side is 3 cm and the length of the larger side is 9 cm.