CBSE Test Paper 01

Chapter 4 Quadratic Equation

1. $(x+1)^2 - x^2 = 0$ has **(1)**
    a. no real roots
    b. 1 real root
    c. 2 real roots
    d. 4 real roots
2. $9x^2 + 12x + 4 = 0$ have **(1)**
    a. Real and Distinct roots
    b. No real roots
    c. Distinct roots
    d. Real and Equal roots
3. If the equation $(a^2+b^2)x^2 - 2(ac+bd)x + c^2 + d^2 = 0$ has equal roots, then **(1)**
    a. $ad = bc$
    b. $ab = cd$
    c. $ad = \sqrt{bc}$
    d. $ab = \sqrt{cd}$
4. The ratio of sum and the product of the roots of $7x^2 - 12x + 18 = 0$ is **(1)**
    a. $2 : 3$
    b. $3 : 2$
    c. $7 : 18$
    d. $7 : 12$
5. If y = 1 is the common root of $ly^2 + ly + 3 = 0$ and $y^2 + y + m = 0$, then the value of 'lm' is **(1)**
    a. 3
    b. – 4
    c. 4
    d. – 3
6. Solve the quadratic equations by factorization method: $x^2 - 9 = 0$ **(1)**
7. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots. **(1)**

8. Form a quadratic equation whose roots are -3 and 4. **(1)**

9. If $x = \frac{-1}{2}$ is a solution of the quadratic equation $3x^2 + 2kx + 3 = 0$, find the value of k. **(1)**

10. Write the discriminant of the given quadratic equation $x^2 + x - 12 = 0$ **(1)**

11. Find the values of k for which the given equation has real and equal roots: $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ **(2)**

12. Check, whether the quadratic equation have real roots and if so, then find the roots of equation. $6x^2 + x - 2 = 0$ **(2)**

13. Check whether the given equation is quadratic equation: $(x-3)(2x + 1) = x(x + 5)$ **(2)**

14. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects. **(3)**

15. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find k. **(3)**

16. If p, q, r and s are real numbers such that $pr = 2(q + s)$, then show that at least one of the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots. **(3)**

17. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream. **(3)**

18. A train travelling at a uniform speed for 360 km,would have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train. **(4)**

19. Solve for x: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ **(4)**

20. Solve for x: $2\left(\frac{x+2}{2x-3}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$; given that $x \neq -2, x \neq \frac{3}{2}$ **(4)**

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Solution

1. b. 1 real root

   Explanation: Given: $(x+1)^2 - x^2 = 0$
   $\Rightarrow x^2 + 1 + 2x - x^2 = 0$
   $\Rightarrow 2x + 1 = 0$
   $\Rightarrow x = \frac{-1}{2}$
   Therefore, $(x^2+1)^2 - x^2 = 0$ is a linear polynomial and has one real root.

2. d. Real and Equal roots

   Explanation: Comparing the given equation to the below equation

   $ax^2 + bx + c = 0$

   a = 9, b = 12, c = 4

   $D = b^2 - 4ac$

   $D = 12^2 - 4 \times 9 \times 4$

   D = 144 - 144

   D = 0

   If $b^2-4ac=0$ then equation have equal and real roots.

3. a. ad = bc

   Explanation If the equation $(a^2+b^2)x^2 - 2(ac+bd)x + c^2 + d^2 = 0$ has equal roots, then

   $b^2 - 4ac = 0$
   $\Rightarrow [-2(ac+bd)]^2 - 4 \times (a^2+b^2) \times (c^2+d^2) = 0$
   $\Rightarrow 4\left[a^2c^2 + b^2d^2 + 2abcd\right] - 4\left[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2\right] = 0$
   $\Rightarrow 4\left[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2\right] = 0$
   $\Rightarrow a^2d^2 + b^2c^2 - 2abcd = 0$

   $(ad-bc)^2 = 0$
   $\Rightarrow (ad-bc)^2 = 0$
   $\Rightarrow ad - bc = 0$
   $\Rightarrow ad = bc$

4. a. 2:3

Explanation: Ratio of sum and product of the roots of $7x^2 - 12x + 18 = 0$ is $\frac{\alpha+\beta}{\alpha\beta}$

$\Rightarrow$

$\Rightarrow \frac{-b}{c}$

$\Rightarrow \frac{12}{18} = \frac{2}{3}$ = 2 : 3

5. a. 3

Explanation: In quadratic equation $ly^2 + ly + 3 = 0$,

$l(1)^2 + l(1) + 3 = 0$

$\Rightarrow l + l + 3 = 0$

$\Rightarrow 2l + 3 = 0$

$\Rightarrow l = \frac{-3}{2}$

And $(1)^2 + 1 + m = 0$

$\Rightarrow 1 + 1 + m = 0$

$\Rightarrow 2 + m = 0$

$\Rightarrow m = -2$

$\therefore lm = \frac{-3}{2} \times (-2)$ = 3

6. We have,

$x^2 - 9 = 0$

$\Rightarrow (x-3)(x+3) = 0$

$\Rightarrow x - 3 = 0$ or, $x + 3 = 0$

$\Rightarrow$ x = 3 or, x = -3 $\Rightarrow$ x = ± 3

Thus, x = 3 and x = - 3 are roots of the given equation.

7. $4x^2 + px + 3 = 0$

$a = 4, b = p$ and $c = 3$

As the equation has equal roots

$\therefore D = 0$

$D = b^2 - 4ac = 0$

or, $p^2 - 4 \times 4 \times 3 = 0$

or, $p^2 - 48 = 0$

or, $p^2 = 48$

or, $p = \pm 4\sqrt{3}$

8. We have, x = 4 and x = -3.

Then,

x - 4 = 0 and x + 3 = 0

$\Rightarrow$ (x - 4)(x + 3) = 0

$\Rightarrow$ x^2 + 3x - 4x - 12 = 0

$\Rightarrow$ x^2 - x - 12 = 0

This is the required quadratic equation

9. we have, $3x^2 + 2kx + 3 = 0$ $\frac{-1}{2}$ put, x = (given)

$\Rightarrow$ $3(\frac{-1}{2})^2 + 2k(\frac{-1}{2}) + 3 = 0$

$\Rightarrow$ $3(\frac{1}{4})$ - k + 3 = 0

$\Rightarrow$ $\frac{3}{4}$ - k +3 = 0

$\Rightarrow k = 3 + \frac{3}{4}$

$\therefore k = \frac{15}{4}$

10. The given quadratic equation is $x^2 + x - 12 = 0$,

here a=1, b=1, c=-12

$\therefore D = b^2 - 4ac = (1)^2 - 4((1)(-12) = 1 + 48 = 49$

Hence, the discriminant is 49.

11. We have, $(k+1)x^2 - 2(k - 1)x + 1 = 0$.

a = k + 1, b = -2(k - 1), c = 1.

D = b^2 - 4ac =4$(k-1)^2$ - 4(k + 1) =4(k^2 -3k)

The given equation will have real and equal roots, if

D = 0 $\Rightarrow$ 4 (k^2 - 3k) = 0 $\Rightarrow$ k^2 - 3k = 0 $\Rightarrow$ k (k - 3) = 0 $\Rightarrow$ k = 0, 3

12. The given equation is $6x^2$ + x - 2 = 0

Here, a = 6, b = 1 and, c = - 2

$\therefore$ D = b^2 - 4ac = 1 - 4 $\times$ 6 $\times$ -2 = 49 > 0

So, the given equation has real roots, given by

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-1+\sqrt{49}}{2\times6} = \frac{-1+7}{12} = \frac{6}{12} = \frac{1}{2}$

and, $\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-1-\sqrt{49}}{2\times6} = \frac{-1-7}{12} = \frac{-8}{12} = \frac{-2}{3}$

13. The given equation is (x - 3) (2x +1) = x (x+5)

$\Longrightarrow$ $2x^2 + x - 6x - 3 = x^2 + 5x$

$\Longrightarrow$ $2x^2 - 5x - 3 = x^2 + 5x$

$\Longrightarrow$ $x^2 - 10x - 3 = 0$

It is in the form of $ax^2 + bx + c = 0$, $a \neq 0$

$\therefore$ the given equation is a quadratic equation.

14. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = 30 − x

If, she had got 2 marks more in Mathematics, her marks would be = x + 2

If, she had got 3 marks less in English, her marks in English would be = 30 − x − 3 = 27 − x

According to given condition:

$\Rightarrow$ (x + 2)(27 − x) = 210

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get a = 1, b = −25 and c = 156

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(156)}}{2 \times 1}$

$\Rightarrow \frac{25 \pm \sqrt{625 - 624}}{2}$

$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$

$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$

$\Rightarrow$ x = 13, 12

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = 30 – x = 30 – 13 = 17

Or Shefali's marks in English = 30 – x = 30 – 12 = 18

Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

15. Given, 2 is a root of the equation, $3x^2 + px - 8 = 0$

Putting $x = 2$ in $3x^2 + px - 8 = 0$

$12 + 2p - 8 = 0$

or, $p = -2$

Given, $4x^2 - 2px + k = 0$ has equal roots

$4x^2 + 4x + k = 0$ has equal roots

$D = b^2 - 4ac = 0$

or, $(4)^2 - 4(4)(k) = 0$

or, $16 - 16k = 0$

or, $16k = 16$

$\therefore k = 1$

16. Given quadratic equations are;

$x^2 + px + q = 0$ —(i)

and, $x^2 + rx + s = 0$ ......(ii)

Also given ; $pr = 2(q + s)$........ (iii)

Let D_1 and D_2 be the discriminant of quadratic equations (i) and (ii) respectively.

Then,

$D_1 = p^2 - 4q$ and $D_2 = r^2 - 4s$

$\Rightarrow D_1 + D_2 = p^2 - 4q + r^2 - 4s = (p^2 + r^2) - 4(q + s)$

$\Rightarrow \quad D_1 + D_2 = p^2 + r^2 - 4\left(\frac{pr}{2}\right)$ ([from equation (iii)]

$\Rightarrow \quad D_1 + D_2 = p^2 + r^2 - 2pr = (p - r)^2 \geq 0 \left[\because (p - r)^2 \geq 0 \text{ for all real } p, r\right]$

Now, Since sum of both D_2 & D_1 is greater than or equal to 0. Hence, both can't be negative.

$\Rightarrow$ At least one of D_1 and D_2 is greater than or equal to zero

Case 1. If $D_1 \geq 0$, equation (i) has real roots.

Case 2. If $D_2 \geq 0$, equation (ii) has real roots.

Case 3. If D_1 & D_2 both ≥ 0, then equation (i) & (ii) both have equal roots.

Clearly, from case 1,2 & 3 at least one given quadratic equations has equal roots.

17. Given, speed of boat in still water = 8 Km/hr. Let the speed of the stream be x km/hr. Then,

Speed of boat in downstream = (8 + x) km/hr

Speed of boat in upstream = (8 - x) km/hr

We know that time taken to cover 'd' km with speed 's' km/hr is $\frac{d}{s}$

So,Time taken by the boat to go 15 km upstream $= \frac{15}{8-x}$ hours.

&, Time taken by the boat to 22 km downstream $= \frac{22}{8+x}$ hours.

It is given that the total time taken by boat to go 15 km upstream & 22 km downstream is 5 hours.

$\therefore \frac{15}{8-x} + \frac{22}{8+x} = 5$

$\Rightarrow \frac{15(8+x)+22(8-x)}{(8-x)(8+x)} = 5$

$\Rightarrow \frac{120+15x+176-22x}{8^2-x^2} = 5$

$\Rightarrow \frac{-7x+296}{64-x^2} = 5$

$\Rightarrow$ -7x + 296 = 5(64 - x^2)

$\Rightarrow$ -7x + 296 = 320 - 5x^2

$\Rightarrow$ 5x^2 - 7x + 296 - 320 = 0

$\Rightarrow$ 5x^2 - 7x - 24 = 0

$\Rightarrow$ 5x^2 - 15x + 8x - 24 = 0

$\Rightarrow$ 5x(x - 3) + 8(x - 3) = 0

$\Rightarrow$ (5x + 8)(x- 3) = 0

$\Rightarrow$ x - 3 = 0 [$\because$ Speed can not be negative $\therefore$ 5x + 8 $\neq$ 0]

$\Rightarrow$ x = 3

Hence, the speed of the stream is 3 km/hr.

18. Given that a train travelling at a uniform speed for 360 km

Let the original speed of the train be x km/hr

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}$

Time taken at increased speed $= \frac{360}{x+5}$ hours.

According to the question

$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$

$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$

$or, \frac{360(x+5-x)}{x^2+5x} = \frac{4}{5}$

$or, \frac{1800}{x^2+5x} = \frac{4}{5}$

$\Rightarrow \quad x^2 + 5x - 2250 = 0$

$\Rightarrow \quad x^2 + (50-45)x - 2250 = 0$
$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$
$\Rightarrow \quad (x+50)(x-45) = 0$

Either x = - 50 or x = 45

As speed cannot be negative

∴ Original speed of train = 45 km/hr.

19. We have the following equation,

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Now factorise the equation,

$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$
$\Rightarrow \sqrt{3}x(x+\sqrt{3}) + 7(x+\sqrt{3}) = 0$
$\Rightarrow \quad (x+\sqrt{3})(\sqrt{3}x+7) = 0$
$\Rightarrow \quad x = -\sqrt{3}$ or $x = \frac{-7}{\sqrt{3}}$

If $x = -\frac{7}{\sqrt{3}}$ we need to rationalise it.

$\Rightarrow \quad x = -\frac{7\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = -\frac{7\sqrt{3}}{3}$

Therefore, Roots are $-\sqrt{3}, -\frac{7\sqrt{3}}{3}$

20. Let $\frac{x+2}{2x-3}$ = y ...(i)

∴Given equation becomes,

2y - 9 $\times$ $\frac{1}{y}$ = 3

$\Rightarrow 2y^2 - 3y - 9 = 0$
$\Rightarrow 2y^2 - 6y + 3y - 9 = 0$
$\Rightarrow 2y(y-3) + 3(y-3) = 0$
$\Rightarrow (2y+3)(y-3) = 0$
$\Rightarrow$y = -$\frac{3}{2}$ or y = 3

Putting the value of y in equation (i), we get

$\Rightarrow \frac{x+2}{2x-3}$ = -$\frac{3}{2}$ or $\frac{x+2}{2x-3}$ = 3

$\Rightarrow 2x + 4 = -6x + 9$ or x + 2 = 6x - 9

$\Rightarrow$8x = 5 or -5x = -11

$\Rightarrow$x = $\frac{5}{8}$ or x = $\frac{11}{5}$