CBSE Test Paper 02
Chapter 2 polynomials

1. If 'α' and 'β' are the zeroes of the polynomial x^2 -6x + 8, then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is **(1)**
   a. 8
   b. 6
   c. 12
   d. 9
2. A polynomial of degree_____is called a linear polynomial. **(1)**
   a. 1
   b. 3
   c. 2
   d. 0
3. If $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, then the other zeroes are **(1)**
   a. – 2 and $-\frac{1}{2}$
   b. 2 and- $\frac{1}{2}$
   c. $\frac{1}{2}$ and – $\frac{1}{2}$
   d. 1 and $\frac{1}{2}$
4. If 'α' and 'β' are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha+\beta=$ **(1)**
   1. $\frac{-b}{a}$
   2. $\frac{-c}{a}$
   3. $\frac{c}{a}$
   4. $\frac{b}{a}$
5. The degree of a biquadratic polynomial is **(1)**
   1. 2
   2. 4
   3. 3
   4. 1
6. Find a quadratic polynomial, whose sum and product of zeros are -5 and 6 respectively. **(1)**

7. Sum and product of zeroes of a quadratic polynomial are 0 and $\sqrt{15}$ respectively. Find the quadratic polynomial. **(1)**

8. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 - 5x + 4, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$. **(1)**

9. If one zero of $2x^2-3x+k$ is reciprocal to the other, then find the value of k **(1)**

10. If α and β are the zeroes of the quadratic polynomial p(x) = x^2 - p(x + 1) - c such that $(\alpha+1)(\beta+1)$= 0, what is the value of c? **(1)**

11. Find the value of b for which the polynomial $2x^3+9x^2-x-b$ is divisible by $2x+3$ **(2)**

12. α, β are zeroes of the quadratic polynomial x^2 - (k + 6)x + 2(2k - 1). Find the value of k if $\alpha+\beta=\frac{1}{2}\alpha\beta$. **(2)**

13. If α and β are the zeros of the polynomial f(x) = $5x^2$ - 7x + 1, find the value of $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$. **(2)**

14. If α and β are the zeros of the quadratic polynomial f(x) = $3x^2$ - 4x + 1, find a quadratic polynomial whose zeros are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. **(3)**

15. Obtain all zeros of the polynomial ($2x^3$ - 4x - x^2 + 2), if two of its zeros are $\sqrt{2}$ and -$\sqrt{2}$ **(3)**

16. Find the zeroes of the quadratic polynomial $5x^2$ + 8x - 4 and verify the relationship between the zeroes and the coefficients of the polynomial. **(3)**

17. Find the zeroes of the polynomial y^2 + $\frac{3}{2}\sqrt{5}y$ - 5 by factorisation method and verify the relationship between the zeroes and coefficient of the polynomials. **(3)**

18. If two zeroes of the polynomial p(x) = x^4 - $6x^3$ - $26x^2$ + 138x - 35 are 2 ± $\sqrt{3}$. Find the other zeroes. **(4)**

19. Given that x - $\sqrt{5}$ is a factor of the polynomial x^3 - 3$\sqrt{5}$ x^2 - 5x + 15$\sqrt{5}$, find all the zeroes of the polynomial. **(4)**

20. If the polynomial x^4 - $6x^3$ + $16x^2$ - 25x + 10 is divided by (x^2 - 2x + k), the remainder comes out to be x + a, find k and a. **(4)**

CBSE Test Paper 02

Chapter 2 polynomials

Solution

1. d. 9

   Explanation: Here $a = 1, b = -6, c = 8, \alpha + \beta = 6, \alpha\beta = 8$

   Since $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)\left[\alpha^2+\beta^2-\alpha\beta\right]}{\alpha\beta} = \frac{(\alpha+\beta)\left[\alpha^2+\beta^2+2\alpha\beta-3\alpha\beta\right]}{\alpha\beta}$

   = $\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{\alpha\beta}$

   = $\frac{6[6^2-3\times8]}{8}$ = 9

2. a. 1

   Explanation: A polynomial of degree 1 is called a linear polynomial. Example 4x + 3, 65y are linear polynomials.

3. d. 1 and $\frac{1}{2}$

   Explanation: Since $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, then $\left(x - \sqrt{2}\right)$ and $\left(x + \sqrt{2}\right)$ are the factors of given polynomial i.e., $\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = \left(x^2 - 2\right)$ is a factor of given polynomial.

   $\therefore p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2 \Rightarrow p(x) = \left(x^2 - 2\right)\left(2x^2 - 3x + 1\right)$

   $$
   \begin{array}{r|l}
    & 2x^2 - 3x + 1 \\
   x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
    & 2x^4 \quad\quad - 4x^2 \\
    & - \quad\quad\quad + \\
    & \quad -3x^3 + x^2 + 6x - 2 \\
    & \quad -3x^3 \quad\quad + 6x \\
    & \quad + \quad\quad\quad - \\
    & \quad\quad x^2 - 2 \\
    & \quad\quad x^2 - 2 \\
    & \quad\quad - \quad + \\
    & \quad\quad\quad 0
   \end{array}
   $$

   $\Rightarrow p(x) = \left(x^2 - 2\right)\left[2x^2 - 2x - x + 1\right] \Rightarrow$

   $p(x) = \left(x^2 - 2\right)\left[2x(x - 1) - 1(x - 1)\right] \Rightarrow$

   $p(x) = \left(x^2 - 2\right)(x - 1)(2x - 1)$

   $\therefore$ Other zeroes are x - 1 = 0 and 2x - 1 = 0 $\Rightarrow$ x = 1 and $x = \frac{1}{2}$

4. a. $\frac{-b}{a}$

Explanation: If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$,

$\because$ Sum of the zeroes of a quadratic polynomial $ax^2 + bx + c$= $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

then $\alpha + \beta = \frac{-b}{a}$

5. b. 4

Explanation: Biquadratic polynomial is a polynomial of the fourth degree.

Biquadratic polynomial = $a\left(x^2\right)^2 + b(x)^2 + c$ = $ax^4 + bx^2 + c$

6. Let α and β be the zeros of the required polynomial.

Then, (α + β) = -5 and $\alpha\beta$ = 6

f(x) = x^2- (α + β)x + $\alpha\beta$ = x^2 - (-5)x + 6

= x^2 + 5x + 6.

Hence, the required polynomial is f(x) = x^2 + 5x + 6.

7. Here sum of zeroes, $S = 0$

Product of zeroes, $P = \sqrt{15}$

Quadratic polynomial $p(x) = x^2 - (S)x + P$

= $x^2 - 0x +$ $\sqrt{15}$

= $x^2 +$ $\sqrt{15}$

8. We have, α and β are the roots of the quadratic polynomial. f(x) =x^2 - 5x + 4

Sum of zeros: $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

product of zeros: $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

We have a=1,b=-5 and c= 4.

Sum of the roots = α + β = 5

Product of the roots = αβ = 4

So,

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

$5/4 - 2 \times 4 = 5/4 - 8$ =$(5 - 32)/4$=$-27/4$

Hence,we get the result of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ = $-\frac{27}{4}$

9. Let be the two zeroes of the given polynomial.

Then, $\alpha \times \frac{1}{\alpha} = \frac{Constant\_term}{Coefficient(x^2)}$

$\Rightarrow 1 = \frac{k}{2}$
$\Rightarrow k = 2$

10. It is given that:

p(x) = x^2 - px - p - c

Here a = 1, b = - p and c = -p - c

$\therefore \quad \alpha + \beta = p$ and $\alpha\beta = (-p - c)$

$\because \quad (\alpha + 1)(\beta + 1) = 0$

$\Rightarrow \quad \alpha\beta + \alpha + \beta + 1 = 0$

$\Rightarrow$ - p - c + p + 1 = 0

$\Rightarrow$ c = 1

11.

$$
\begin{array}{r|l}
 & x^2 + 3x - 5 \\
\hline
2x + 3 & 2x^3 + 9x^2 - x - b \\
 & 2x^3 + 3x^2 \\
 & - \quad - \\
\hline
 & \quad 6x^2 - x - b \\
 & \quad 6x^2 + 9x \\
 & \quad - \quad - \\
\hline
 & \quad\quad -10x - b \\
 & \quad\quad -10x - 15 \\
 & \quad\quad + \quad + \\
\hline
 & \quad\quad 15 - b
\end{array}
$$

If the polynomial $2x^3 + 9x^2 - x - b$ is divisible by 2x + 3, then the remainder must be zero.

So, 15 - b = 0, b = 15

12. Polynomial is $x^2 - (k + 6)x + 2(2k - 1)$.

$\alpha + \beta = -\frac{b}{a} = \frac{k+6}{1} = k + 6$

and $\alpha\beta = \frac{c}{a} = \frac{2(2k-1)}{1} = 4k - 2$

Now, $\alpha + \beta = \frac{1}{2}\alpha\beta$

k + 6 = $\frac{1}{2}$(4k - 2)

k+6 = 2k -1

k = 7

13. Compare f(x) = $5x^2 - 7x + 1$ with $ax^2 + bx + c$ we get,

a = 5, b = - 7 and c= 1

Since α and β are the zeros of $5x^2 - 7x + 1$, we have

$\alpha + \beta = -\frac{(b)}{a} = -\frac{(-7)}{5} = \frac{7}{5}$

$\alpha\beta = \frac{c}{a} = \frac{1}{5}$

$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{\frac{7}{5}}{\frac{1}{5}}$

$= \frac{7}{5} \times \frac{5}{1}$

= 7

14. Here it is given that the zeros of f(x)= $3x^2-4x+1$ are $\alpha and \beta$

Here a=3, b=-4 and c=1

$\alpha + \beta = -\frac{\mathrm{b}}{\mathrm{a}} = -\left(-\frac{4}{3}\right) = \frac{4}{3}$ and $\alpha\beta = \frac{\mathrm{c}}{\mathrm{a}} = \frac{1}{3}$

Let S and P denote respectively the sum and product of the zeros of the polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$, then

S = $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(\frac{4}{3}\right)^3-3\times\frac{1}{3}\times\frac{4}{3}}{\frac{1}{3}} = \frac{28}{9}$...........(1)

and, P = $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = \frac{1}{3}$................(2)

Hence the polynomial with zeros $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ is

g(x)=x^2-Px+S=0

putting values of P and S from (1) and (2) we get the polynomial

$\mathrm{g(x)}=\mathrm{x}^2 - \frac{28}{9}\mathrm{x} + \frac{1}{3}$

or $\mathrm{g(x)}=9\mathrm{x}^2\text{-}28\mathrm{x}+3$

15. The given polynomial is:

f(x) = $2x^3 - x^2 - 4x + 2$.

It is given that the two zeroes of the above polynomial are $\sqrt{2}$ and -$\sqrt{2}$

Therefore, (x - $\sqrt{2}$)(x + $\sqrt{2}$) = (x^2 - 2) is a factor of f(x).

Now we divide (x) = $2x^3 - x^2 - 4x + 2$ by (x^2- 2), we obtain

$x^2 - 2)\, 2x^3 - x^2 - 4x + 2\, (2x - 1$
$2x^3 \quad - 4x$
$-x^2 \quad + 2$
$-x^2 \quad + 2$
$\times$

Where quotient = (2x - 1)

$\therefore$ f(x) = 0 $\Rightarrow$ $(x^2 - 2)(2x - 1) = 0$

$\Rightarrow (x - \sqrt{2})\ (x + \sqrt{2})(2x - 1) = 0$

$\Rightarrow (x - \sqrt{2}) = 0$ or $(x + \sqrt{2}) = 0$ or $(2x - 1) = 0$

$\Rightarrow x = \sqrt{2}$ or $x = -\sqrt{2}$ or $x = \frac{1}{2}$.

Hence, all zeros of f(x) are $\sqrt{2}$, $-\sqrt{2}$ and $\frac{1}{2}$.

16. $p(x) = 5x^2 + 8x - 4 = 0$

$= 5x^2 + 10x - 2x - 4 = 0$

$= 5x(x + 2) - 2(x + 2) = 0$

$= (x + 2)(5x - 2) = 0$

Hence, zeroes are -2 and $\frac{2}{5}$

Verification: Sum of zeroes = $-2 + \frac{2}{5} = \frac{-8}{5}$

Product of zeroes = $(-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5}$

Again sum of zeroes = $-\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2} = \frac{-8}{5}$

Product of zeroes = $\frac{\text{Constant term}}{\text{Coeff. of } x^2} = \frac{-4}{5}$

Verified.

17. $y^2 + \frac{3}{2}\sqrt{5}y - 5 = \frac{1}{2}\left(2y^2 + 3\sqrt{5}y - 10\right)$

$= \frac{1}{2}\left(2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10\right)$

$= \frac{1}{2}[2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5})]$

$= \frac{1}{2}(y + 2\sqrt{5})(2y - \sqrt{5})$

$\Rightarrow \quad y = -2\sqrt{5}, \frac{\sqrt{5}}{2}$ are zeroes of the polynomial.

If given polynomial is $y^2 + \frac{3}{2}\sqrt{5}y - 5$ then a = 1, b=$\frac{3}{2}\sqrt{5}$ and c = -5

Sum of zeroes = $-2\sqrt{5}$ $\frac{\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}$ .... (i)

Also, $-\frac{-b}{a} = \frac{-3\sqrt{5}}{2}$------- (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $-2\sqrt{5} \times \frac{\sqrt{5}}{2} = -5$............(iii)

Also, $\frac{c}{a} = \frac{-5}{1} = -5$........... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

18. As 2 ± $\sqrt{3}$ are the zeroes of p(x), so x - (2 ± $\sqrt{3}$) are the factors of p(x) and the product of factors,

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$$
$$=\{(x-2)-\sqrt{3}\}\{(x-2)+\sqrt{3}\}$$
$$=(x-2)^2-(\sqrt{3})^2$$

= x^2 - 4x + 1

Dividing p(x) by x^2 - 4x + 1

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & -\quad+\quad- \\
\hline
 & -2x^3-27x^2+138x \\
 & -2x^3+8x^2-2x \\
 & +\quad-\quad+ \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & +\quad-\quad+ \\
\hline
 & 0
\end{array}
$$

Factorising (x^2 - 2x - 35) we get

= (x + 5)(x - 7)

x = -5, 7

Hence, other two zeroes of p(x) are - 5 and 7.

19.

$$
\begin{array}{r|l}
 & x^2-2\sqrt{5}x-15 \\
x-\sqrt{5} & x^3-3\sqrt{5}x^2-5x+15\sqrt{5} \\
 & x^3-\sqrt{5}x^2 \\
 & -\quad+ \\
\hline
 & -2\sqrt{5}x^2-5x \\
 & -2\sqrt{5}x^2+10x \\
 & +\quad- \\
\hline
 & -15x+15\sqrt{5} \\
 & -15x+15\sqrt{5} \\
 & +\quad- \\
\hline
 & 0
\end{array}
$$

On factorising the quotient, we get

$$x^2 - 2\sqrt{5}x - 15 = x^2 - 3\sqrt{5}x + \sqrt{5}x - 15$$
$$= x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5})$$
$$= (x + \sqrt{5})(x - 3\sqrt{5})$$
$$\therefore \ (x + \sqrt{5})(x - 3\sqrt{5}) = 0$$
$$\Rightarrow \ x = -\sqrt{5}, 3\sqrt{5}$$

Therefore, all the zeroes are $\sqrt{5}, -\sqrt{5}$ and $3\sqrt{5}$.

20.

$$\begin{array}{l}
\qquad\qquad\qquad x^2 - 4x + (8 - k) \\
x^2 - 2x + k\,)\; x^4 - 6x^3 + 16x^2 - 25x + 10 \\
\qquad\qquad\quad x^4 - 2x^3 + kx^2 \\
\qquad\qquad\quad - \quad + \quad - \\
\hline
\qquad\qquad\quad -4x^3 + (16 - k)x^2 - 25x + 10 \\
\qquad\qquad\quad -4x^3 + 8x^2 - 4kx \\
\qquad\qquad\quad + \quad - \quad + \\
\hline
\qquad\qquad\quad (8 - k)x^2 - (25 - 4k)x + 10 \\
\qquad\qquad\quad (8 - k)x^2 - (16 - 2k)x + (8k - k^2) \\
\qquad\qquad\quad - \quad + \quad - \\
\hline
\qquad\qquad\qquad\qquad (2k - 9)x + (10 - 8k + k^2)
\end{array}$$

Given, remainder = x + a

On comparing the multiples of x

(2k - 9)x = 1

or, 2k - 9 = 1 or k = $\frac{10}{2}$ = 5

On putting this value of k into other portion of remainder, we get

and a = 10 - 8k + k^2 = 10 - 40 + 25 = -5