CBSE Test Paper 05

CH-1 Number Systems

1. The value of $\frac{2}{\sqrt{5}-\sqrt{3}}$ is
   a. $\frac{1}{\sqrt{5}-\sqrt{3}}$
   b. $\sqrt{5}\ -\sqrt{3}$
   c. $\sqrt{5}\ +\sqrt{3}$
   d. $\frac{1}{\sqrt{5}+\sqrt{3}}$
2. A terminating decimal is
   a. a natural number
   b. a rational number
   c. a whole number
   d. an integer.
3. The decimal form of $\frac{1}{999}$ is
   a. 0.999
   b. $0.\overline{001}$
   c. $0.00\overline{1}$
   d. $0.0\overline{01}$
4. The value of $\sqrt[3]{1000}$ is
   a. 3
   b. 10
   c. 1
   d. 0
5. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ =
   a. 8
   b. -10
   c. 10
   d. -8
6. Fill in the blanks:

   The value of $\frac{1+\sqrt{2}}{1-\sqrt{2}} =$ _______.

7. Fill in the blanks:

   The smallest natural number is________.

8. Solve the equation: $3(2^x + 1) - 2^{x+2} + 5 = 0$

9. Is $(1+\sqrt{5})-(4+\sqrt{5})$ a rational number?

10. Simplify :$256^{3/4}$

11. Express $2.417\overline{8}$ in the form $\frac{a}{b}$

12. Express 0.404040.... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

13. If both a and b are rational numbers, find the values of a and b of the following equality: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15}$

14. State whether the following statements are true or false. Give reasons for your answers.
    (i) Every natural number is a whole number.
    (ii) Every integer is a whole number.
    (iii) Every rational number is a whole number.

15. If $a = \frac{1}{7-4\sqrt{3}}$ and $b = \frac{1}{7+4\sqrt{3}}$, then find the value of:

    i. $a^2 + b^2$
    ii. $a^3 + b^3$

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Solution

1. (c) $\sqrt{5}+\sqrt{3}$

   Explanation:

   $\frac{2}{\sqrt{5}-\sqrt{3}}$

   multiplying nu nominator and denominator by

   $\sqrt{5}+\sqrt{3}$, we get

   $\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

   $=\frac{2(\sqrt{5}+\sqrt{3})}{5-3}=\sqrt{5}+\sqrt{3}$

2. (b) a rational number

   Explanation: a rational number because it can be written in fraction

3. (b) $0.\overline{001}$

   Explanation: When we divide 1 by 999 it result is 0.001001001001001........

   So, $0.\overline{001}=\frac{1}{999}$

4. (b) 10

   Explanation:

   $(10)^3=1000$

   So, $\sqrt[3]{1000}=1000^{\frac{1}{3}}=\left(10^3\right)^{\frac{1}{3}}$

   =10

5. (c) 10

   Explanation:

   $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

   $\Rightarrow \frac{(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

   $\Rightarrow \frac{(3+2+2\sqrt{6})+3+2-2\sqrt{6}}{3-2}$

   $\Rightarrow 10$

6. $-\left(3+2\sqrt{2}\right)$

7. 1

8. We have,

$3(2^x + 1) - 2^{x+2} + 5 = 0$

$\Rightarrow 3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$

$\Rightarrow 3 \times 2^x - 4 \times 2^x + 8 = 0$

$\Rightarrow (3 - 4)2^x + 8 = 0$

$\Rightarrow -2^x + 8 = 0 \Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$

9. $(1+\sqrt{5})-(4+\sqrt{5})=-3$, which is a rational number.

10. $256^{3/4} = (4^4)^{3/4}$

$= 4^{4 \times 3/4} = 4^3 = 64$

11. $x = 2.417\overline{8}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

multiply eq(1) by 1000

$1000x = 2417.\overline{8}$

$1000x = 2417.88888888888888888\ldots\ldots\ldots\ldots(2)$

now multiply eq(2) by 10

$10000x = 24178.888888888888888\ldots\ldots\ldots\ldots\ldots(3)$

Subtract eq(2) from eq(3)

$10000x - 1000x = (24178.888888\ldots\ldots) - (2417.88888888\ldots\ldots)$

9000 x=21761

$x = \frac{21761}{9000}$

12. Let x = 0.404040.... = $0.\overline{40}$. ...(1)

$\therefore\ 100x = 40.\overline{40}$ ...(2)

Subtracting (1) from (2), we get

99x = 40

$\therefore\ x = \frac{40}{99}$

13. To find the value of a and b, we will first rationalise the L.H.S of the given expression,

$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$

$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\times\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{5+3+2\sqrt{5\times3}}{5-3} = \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$

$\therefore \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15} \Rightarrow 4 + \sqrt{15} = a + b\sqrt{15}$

On comparing both sides, we get

$\Rightarrow$ a = 4 and b = 1

14. (i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5.....$

We know that natural number series is $1, 2, 3, 4, 5.....$

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$,where $q = 1$

Now, considering the series of integers, we have $....-4, -3, -2, -1, 0, 1, 2, 3, 4.....$

We know that whole number series is $0, 1, 2, 3, 4, 5.....$

We can conclude that all the numbers of whole number series lie in the series of integers. But every

number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$ where $q \neq 0$

We know that whole number series is $0, 1, 2, 3, 4, 5.....$

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as

$\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}.....$

We conclude that every number of the whole number series is a rational number. But, every rational number does not appear in the whole number series. like $\frac{2}{3}, \frac{5}{6}$
Therefore, we conclude that every rational number is not a whole number.

15. Given, $a = \frac{1}{7-4\sqrt{3}}$ and $b = \frac{1}{7+4\sqrt{3}}$,

Now, $a = \frac{1}{7-4\sqrt{3}} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{7^2-(4\sqrt{3})^2}$

$= \frac{7+4\sqrt{3}}{49-16\times3} = \frac{7+4\sqrt{3}}{49-48}$

$\therefore a = \frac{1}{7-4\sqrt{3}} = 7 + 4\sqrt{3}$

Now, $b = \frac{1}{7+4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{7^2-(4\sqrt{3})^2}$

$= \frac{7-4\sqrt{3}}{49-16\times3} = \frac{7-4\sqrt{3}}{49-48}$

$\therefore b = \frac{1}{7+4\sqrt{3}} = 7 - 4\sqrt{3}$

$a + b = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$

$ab = (7 + 4\sqrt{3})(7 - 4\sqrt{3})$

$= 7^2 - (4(\sqrt{3}))^2$

$= 49 - 16 \times 3 = 49 - 48$

$\Rightarrow ab = 1$

Now, $a^2 + b^2 = (a + b)^2 - 2ab$

$= (14)^2 - 2 \times 1$

$= 196 - 2$

$\therefore a^2 + b^2 = 194$

Also, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (14)^3 - 3 \times 1 (14)$

$= 2744 - 42$

$= 2702$