CBSE Test Paper 04
CH-1 Number Systems

1. If $x = 4 - \sqrt{15}$, then the value of $\left(x + \frac{1}{x}\right)$ is
   a. 7
   b. 6
   c. 8
   d. 10
2. $\left(\frac{125}{216}\right)^{\frac{-1}{3}}$ =
   a. $\frac{6}{5}$
   b. 125
   c. $\frac{5}{6}$
   d. 216
3. $\sqrt{8}$ is an
   a. irrational number
   b. natural number
   c. rational number
   d. integer
4. $16\sqrt{13} \div 9\sqrt{52}$ is equal to
   a. $\frac{3}{9}$
   b. $\frac{9}{8}$
   c. $\frac{8}{9}$

d. None of these

5. The value of $x^{a-b} \times x^{b-c} \times x^{c-a}$ is

   a. 1

   b. 2

   c. x

   d. 0

6. Fill in the blanks:

   $\frac{5}{6}$ in the decimal form is________.

7. Fill in the blanks:

   Rational number $\frac{42}{100}$ in decimal form is________.

8. Rationalise the denominator of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

9. Classify the following number as rational or irrational.0.3796

10. Prove $\sqrt{5}$ - 2 is an irrational.

11. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

12. Find the two rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$

13. Prove that 1.101001000100001... is an irrational number.

14. Find the values of a and b in each of $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

15. If x = 2 + $\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

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Solution

1. (c) 8

   Explanation:

   $x + \frac{1}{x} = \frac{x^2+1}{x}$

   Now , put x=4- $\sqrt{15}$

   $\Rightarrow \frac{(4-\sqrt{15})^2+1}{4-\sqrt{15}}$

   $\Rightarrow \frac{16+15-8\sqrt{15}+1}{4-\sqrt{15}}$

   $\Rightarrow \frac{32-8\sqrt{15}}{4-\sqrt{15}}$

   $\Rightarrow 8$

2. (a) $\frac{6}{5}$

   Explanation:

   $\left(\frac{125}{216}\right)^{\frac{-1}{3}}$

   $\Rightarrow \left(\frac{5}{6}\right)^{3\times\frac{-1}{3}}$

   $\Rightarrow \left(\frac{5}{6}\right)^{-1}$

   $\Rightarrow \frac{6}{5}$

3. (a) irrational number

   Explanation:

   $\sqrt{8}$ is an irrational number

   $\because \sqrt{4\times 2} = 2\sqrt{2}$

4. (c) $\frac{8}{9}$

   Explanation:

   $16\sqrt{13} \div 9\sqrt{52}$

   $\frac{16\sqrt{13}}{9\sqrt{52}} = \frac{16}{9} \times \sqrt{\frac{13}{52}} = \frac{16}{9} \times \frac{1}{2}$

   $= \frac{8}{9}$

5. (a) 1

Explanation:

$x^{a-b} \times x^{b-c} \times x^{c-a}$

$\Rightarrow x^{a-b+b-c+c-a}$

$\Rightarrow x^{0}$

=1

6. 0.8333

7. $0.\underline{42}$

8. $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = \frac{4+3+4\sqrt{3}}{4-3}$

$= \frac{7+4\sqrt{3}}{1} = 7 + 4\sqrt{3}$

9. $\therefore$ The decimal expansion is terminating.

$\therefore$ 0.3796 is a rational number.

10. $\because$ $\sqrt{5}$ is irrational and 2 is rational.

$\therefore$ $\sqrt{5}$ - 2 is irrational.

($\because$ The difference of a rational number and an irrational number is irrational.)

11. Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5} \ldots\ldots$

So, we arrive at the conclusion that 0 can be written in form of $\frac{p}{q}$, where q is any integer. Therefore, zero is a rational number.

12. First rational number between $\frac{1}{2}$ and $\frac{1}{3}$

$= \frac{1}{2}\left[\frac{1}{2} + \frac{1}{3}\right] \Rightarrow \frac{1}{2}\left[\frac{3+2}{6}\right] \Rightarrow \frac{5}{12}$

$= \frac{1}{2}, \frac{5}{12}$ and $\frac{1}{3}$

Second rational number between $\frac{1}{2}$ and $\frac{1}{3}$

$= \frac{1}{2}\left[\frac{1}{2} + \frac{5}{12}\right] \Rightarrow \frac{1}{2}\left[\frac{6+5}{12}\right] \Rightarrow \frac{11}{24}$

$= \frac{5}{12}$ and $\frac{11}{24}$ are two rational number between $\frac{1}{2}$,and $\frac{1}{3}$

13. We can observe that the number 1.101001000100001.... is a non-terminating on recurring decimal.

We know that non terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001..... is an irrational number.

14. LHS $= \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2}$

$= \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20} = \frac{19-9\sqrt{5}}{-11}$

Now, $\frac{19-9\sqrt{5}}{-11} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{-19}{11} + \frac{9}{11}\sqrt{5} = a\sqrt{5} - \frac{19}{11}$

$\Rightarrow \frac{9}{11}\sqrt{5} - \frac{19}{11} = a\sqrt{5} - \frac{19}{11}$

Hence, $a = \frac{19}{11}$.

15. We have,

$x = 2 + \sqrt{3}$

$\therefore \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1}$

$\Rightarrow \frac{1}{x} = 2 - \sqrt{3}$

Now, $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})\,[x^2 - x \times \frac{1}{x} + (\frac{1}{x})^2]$

$= (2 + \sqrt{3} + 2 - \sqrt{3})\,[(2 + \sqrt{3})^2 - 1 + (2 - \sqrt{3})^2]$

$= 4[(2)^2 + (\sqrt{3})^2 + 2 \times 2 \times \sqrt{3} - 1 + (2)^2 + (-\sqrt{3})^2 - 2 \times 2 \times \sqrt{3}]$

$= 4[4 + 3 + 4\sqrt{3} - 1 + 4 + 3 - 4\sqrt{3}]$

$= 4[13]$

$= 52$