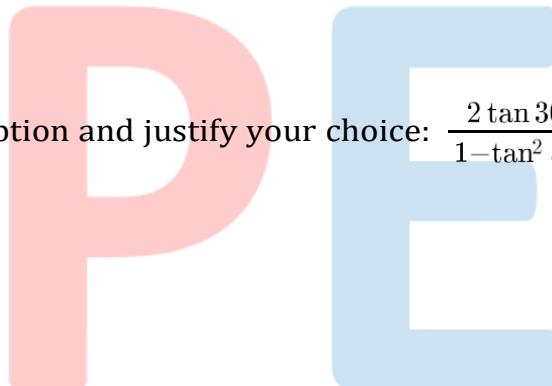


CBSE Test Paper 05
Chapter 8 Introduction to Trigonometry

1. The value of $\cos 48^\circ - \sin 42^\circ$ is **(1)**
 - a. 0
 - b. 1
 - c. $\sqrt{2}$
 - d. $\frac{1}{2}$
2. If $2\sin 2\theta = \sqrt{3}$, then the value of θ **(1)**
 - a. 60°
 - b. 45°
 - c. 0°
 - d. 30°
3. Choose the correct option and justify your choice: $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$ **(1)**
 - a. $\cos 60^\circ$
 - b. $\sin 30^\circ$
 - c. $\sin 60^\circ$
 - d. $\tan 60^\circ$
4. If A, B and C are interior angles of a triangle ABC, then the value of $\tan\left(\frac{B+C}{2}\right)$ is **(1)**
 - a. $\cot \frac{A}{2}$
 - b. None of these
 - c. $\tan \frac{A}{2}$
 - d. $\sin \frac{A}{2}$
5. If ΔPQR is right angled at Q, then the value of $\sin(P + R)$ is **(1)**
 - a. $\frac{1}{2}$
 - b. 1
 - c. $\sqrt{2}$
 - d. 0
6. If $\sec \theta \sin \theta = 0$, then find the value of θ . **(1)**



7. Solve $2\sin^2 \theta = \frac{1}{2}$ when $0^\circ < \theta < 90^\circ$. (1)

8. Prove the trigonometric identity: $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta = 1$ (1)

9. Prove the trigonometric identity: (1)

$$\cos^2 \theta + \frac{1}{1+\cot^2 \theta} = 1$$

10. If $\tan A = \cot B$, prove that $A + B = 90^\circ$. (1)

11. Prove that: $\frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} = 1 + \tan \theta + \cot \theta$ (2)

12. Prove the trigonometric identity: (2)

If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

13. Evaluate 2 $- \sqrt{3} \left(\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ} \right) \left(\frac{\cos 58^\circ}{\sin 32^\circ} \right)$. (2)

14. If $\sin \theta = \frac{3}{4}$, show that $\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$. (3)

15. In $\triangle PQR$, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$. (3)

16. If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$. (3)

17. Prove the trigonometric identity: (3)

$$(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2 \tan A$$

18. Prove the trigonometric identity: (4)

$$\frac{\cot^2 A (\sec A - 1)}{1 + \sin A} = \sec^2 A \left(\frac{1 - \sin A}{1 + \sec A} \right)$$

19. If $\sec \theta = \frac{13}{5}$, show that $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3$. (4)

20. Evaluate without using trigonometric tables: (4)

$$\frac{\sec^2(90^\circ - \theta) - \cot^2 \theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} + \frac{2 \cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

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Solution

1. a. 0

Explanation: Here, $\cos 48^\circ - \sin 42^\circ$

$$= \cos 48^\circ - \sin(90^\circ - 48^\circ)$$

$$= \cos 48^\circ - \cos 48^\circ = 0$$

2. d. 30°

Explanation: Given: $2 \sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

but we know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\sin 2\theta = \sin 60^\circ$$

on comparing on both sides

$$2\theta = 60^\circ$$

$$\theta = 30^\circ$$

3. d. $\tan 60^\circ$

Explanation: $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60^\circ$$

4. a. $\cot \frac{A}{2}$

Explanation: According to question,

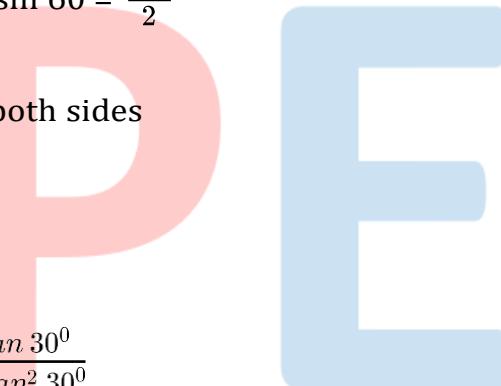
$$A + B + C = 180^\circ$$

$$\Rightarrow \frac{A + B + C}{2} = 90^\circ$$

$$\Rightarrow \frac{A}{2} + \frac{B + C}{2} = 90^\circ$$

$$\Rightarrow \frac{B + C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \tan\left(\frac{B + C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right)$$



$$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$$

5. b. 1

Explanation: If in right angled triangle PQR, right angled at Q, then P and R are acute angles.

Let $\angle P = \theta$, then $\angle R$

$$= 90^\circ - \theta$$

$$\text{Now, } \sin(P + R) = \sin(\theta + 90^\circ - \theta) = \sin 90^\circ = 1$$

6. Given $\sec\theta \cdot \sin\theta = 0$

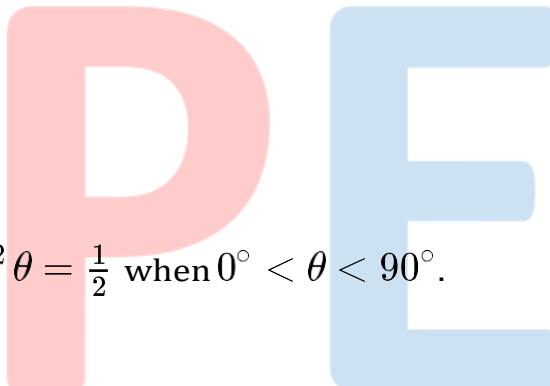
$$\frac{1}{\cos\theta} \times \sin\theta = 0$$

$$\text{or, } \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 0$$

$$\tan\theta = \tan 0^\circ$$

$$\theta = 0^\circ$$



7. Here it is given $2 \sin^2 \theta = \frac{1}{2}$ when $0^\circ < \theta < 90^\circ$.

We have,

$$2 \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{2} [\because \sin\theta > 0 \text{ for } 0^\circ < \theta < 90^\circ]$$

$$\Rightarrow \theta = 30^\circ$$

8. LHS = $(1 - \cos^2\theta) \operatorname{cosec}^2\theta$

$$= \sin^2\theta (\operatorname{cosec}^2\theta) [(1 - \cos^2\theta) = \sin^2\theta]$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta} = 1 = \text{RHS}$$

9. We have,

$$\text{LHS} = \cos^2\theta + \frac{1}{1 + \cot^2\theta}$$

$$\Rightarrow \text{LHS} = \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} [\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$$

$$\Rightarrow \text{LHS} = \cos^2\theta + \sin^2\theta = 1 = \text{RHS} \left[\because \frac{1}{\operatorname{cosec}\theta} = \sin\theta \right]$$

10. Given: $\tan A = \cot B$

$$\Rightarrow \cot(90^\circ - A) = \cot B$$

$$\Rightarrow 90^\circ - A = B$$

$$\Rightarrow 90^\circ = A + B$$

$$\Rightarrow A + B = 90^\circ$$

Hence proved

$$\begin{aligned}
 11. & + \frac{\tan \theta}{1-\cot \theta} - \frac{\cot \theta}{1-\tan \theta} \tan \theta + \cot \theta \\
 \text{L.H.S.} &= \frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} \\
 &= \frac{\frac{\sin \theta}{\cos \theta}}{1-\frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1-\frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta(\sin \theta - \cos \theta)} \\
 &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\
 &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} [\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
 &= \frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \\
 &= \tan \theta + \cot \theta + 1 = 1 + \tan \theta + \cot \theta = \text{R.H.S. proved}
 \end{aligned}$$

$$\begin{aligned}
 \text{Since, } \tan A &= \frac{\sin A}{\cos A} \\
 \cot A &= \frac{\cos A}{\sin A}
 \end{aligned}$$

12. We have,

$$\begin{aligned}
 x &= a \cos^3 \theta, y = b \sin^3 \theta \\
 \frac{x}{a} &= \cos^3 \theta \text{ and } \frac{y}{b} = \sin^3 \theta \\
 \text{L.H.S.} &= \left[\frac{x}{a} \right]^{2/3} + \left[\frac{y}{b} \right]^{2/3} \\
 &= [\cos^3 \theta]^{2/3} + [\sin^3 \theta]^{2/3} [\because (a^m)^n = a^{m \times n}] \\
 &= \cos^2 \theta + \sin^2 \theta \\
 &= 1 [\because \cos^2 \theta + \sin^2 \theta = 1] \\
 &= \text{R.H.S.}
 \end{aligned}$$

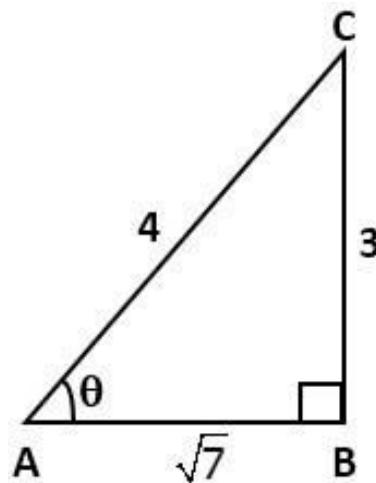
Hence proved.

13. We have,

$$\begin{aligned}
 & 2 \left(\frac{\cos 58^\circ}{\sin 32^\circ} \right) - \sqrt{3} \left(\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ} \right) \\
 &= 2 \left\{ \frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ} \right\} - \sqrt{3} \left\{ \frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 15^\circ \tan 60^\circ \tan(90^\circ - 15^\circ)} \right\} \\
 &= 2 \left(\frac{\sin 32^\circ}{\sin 32^\circ} \right) - \sqrt{3} \left\{ \frac{\cos 38^\circ \sec 38^\circ}{\tan 15^\circ \times \sqrt{3} \times \cot 15^\circ} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & [\because \cos(90 - \theta) = \sin\theta, \csc(90 - \theta) = \sec\theta, \tan(90 - \theta) = \cot\theta] \\
 & = 2 - \sqrt{3} \left\{ \frac{\cos 38^\circ \times \frac{1}{\cos 38^\circ}}{\tan 15^\circ \times \sqrt{3} \times \frac{1}{\tan 15^\circ}} \right\} = 2 - \frac{\sqrt{3}}{\sqrt{3}} = 2 - 1 = 1 \\
 & \left[\sec\theta = \frac{1}{\cos\theta}, \cot\theta = \frac{1}{\tan\theta} \right] \\
 & \text{therefore, } 2 \left(\frac{\cos 58^\circ}{\sin 32^\circ} \right) - \sqrt{3} \left(\frac{\cos 38^\circ \csc 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ} \right) = 1
 \end{aligned}$$

14. Let us draw a triangle ABC in which $\angle B = 90^\circ$.



Let $\angle A = \theta^\circ$.

Given,

$$\sin\theta = \frac{3}{4}$$

$$\Rightarrow \csc\theta = \frac{4}{3} \dots\dots(1)$$

$$\text{But, } \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{4}$$

Let $BC = 3$ and $AC = 5$,

By Pythagoras' theorem in ΔABC , we have :-

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2$$

$$= 4^2 - 3^2 = 16 - 9 = 7$$

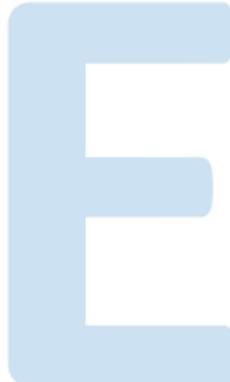
$$\Rightarrow AB = \sqrt{7}$$

$$\text{Now, } \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{\sqrt{7}}{3} \dots\dots(2)$$

$$\cot\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{4}{\sqrt{7}} \dots\dots(3)$$

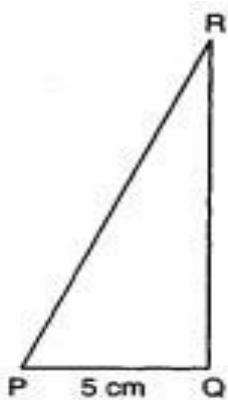
$$\sec\theta =$$

LHS



$$\begin{aligned}
 &= \sqrt{\frac{\cos ec^2\theta - \cot^2\theta}{\sec^2\theta - 1}} \\
 &= \sqrt{\frac{(4/3)^2 - (\sqrt{7}/3)^2}{(4/\sqrt{7})^2 - 1}} \quad [\text{from (2) \& (3)}] \\
 &= \sqrt{\frac{16/9 - 7/9}{12}} \\
 &= \sqrt{\frac{1}{9/7}} \\
 &= \sqrt{\frac{7}{9}} \\
 &= \frac{\sqrt{7}}{3} \\
 &= \text{RHS (Hence Proved).}
 \end{aligned}$$

15.



In $\triangle PQR$ by Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 \quad [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - QR]$$

$$625 - 50QR + QR^2 = 25 + QR^2$$

$$\Rightarrow 600 - 50QR = 0$$

$$\Rightarrow QR = \frac{600}{50} = 12 \text{ cm}$$

$$\text{Now, } PR + QR = 25 \text{ cm}$$

$$\Rightarrow PR = 25 - QR = 25 - 12 = 13 \text{ cm}$$

$$\text{Hence, } \sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13} \text{ and, } \tan P = \frac{QR}{PQ} = \frac{12}{5}$$

16. We have

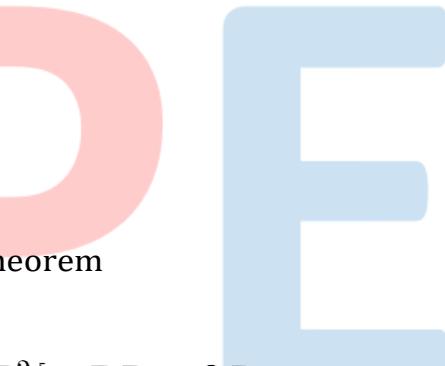
$$\cos ec A = \sqrt{2}$$

$$\Rightarrow \cos ec^2 A = (\sqrt{2})^2 = 2$$

$$\cot^2 A = \cos ec^2 A - 1$$

$$\Rightarrow \cot^2 A = 2 - 1$$

$$\Rightarrow \cot^2 A = 1$$



$$\tan^2 A = \frac{1}{\cot^2 A} = \frac{1}{1} = 1$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\sec^2 \theta = 1 + 1 = 2$$

$$\cos^2 \theta = \frac{1}{\sec^2 \theta} = \frac{1}{2} \quad [\because \cos \theta = \frac{1}{\sec \theta}]$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin^2 \theta = 1 - \frac{1}{2}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{2}$$

$$\text{Now, } \frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$$

$$= \frac{2 \times \frac{1}{2} + 3 \times 1}{4 \left(1 - \frac{1}{2}\right)}$$

$$= \frac{1+3}{4 \times \frac{1}{2}}$$

$$= \frac{4}{2}$$

$$= 2$$

$$\text{Hence } \frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)} = 2$$

17. We have, $(\sec A + \tan A - 1)(\sec A - \tan A + 1)$

$$= (\sec A + \tan A - (\sec^2 A - \tan^2 A)) (\sec A - \tan A + (\sec^2 A - \tan^2 A))$$

$$= (\sec A + \tan A - (\sec A + \tan A)(\sec A - \tan A)) (\sec A - \tan A + (\sec A + \tan A)(\sec A - \tan A))$$

$$= \{(\sec A + \tan A) (1 - (\sec A - \tan A))\} \{(\sec A - \tan A) (1 + (\sec A + \tan A))\}$$

$$= \{(\sec A + \tan A)(1 - \sec A + \tan A)\} \{(\sec A - \tan A)(1 + \sec A + \tan A)\}$$

$$= (\sec A + \tan A) (\sec A - \tan A) (1 - \sec A + \tan A) (1 + \sec A + \tan A)$$

$$= (\sec^2 A - \tan^2 A) (1 - \sec A + \tan A) (1 + \sec A + \tan A)$$

$$= 1 \left(1 - \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) \left(1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= 1 \left(\frac{\cos A - 1 + \sin A}{\cos A}\right) \left(\frac{\cos A + 1 + \sin A}{\cos A}\right)$$

$$= \left(\frac{\cos A + \sin A - 1}{\cos A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\cos A + \sin A)^2 - 1}{\cos^2 A}$$

$$= \frac{\cos^2 A + \sin^2 A + 2 \cos A \sin A - 1}{\cos^2 A + \sin^2 A + 2 \cos A \sin A - 1}$$

$$= \frac{\cos^2 A}{1 + 2 \cos A \sin A - 1} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{\cos^2 A}{2 \cos A \sin A}$$

$$= \frac{1}{\cos A \sin A}$$

$$= 2 \frac{\sin A}{\cos A} = 2 \tan A = RHS.$$

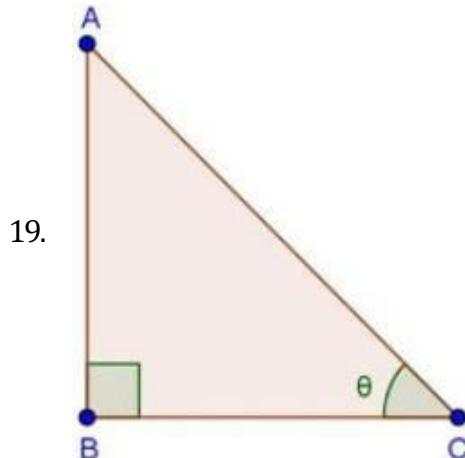
Hence proved.

$$\begin{aligned}
 18. \quad LHS &= \frac{\cot^2 A (\sec A - 1)}{1 + \sin A} \\
 &= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1}{\cos A} - 1 \right)}{1 + \sin A} \\
 &= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1 - \cos A}{\cos A} \right)}{1 + \sin A} \\
 &= \frac{\frac{1 + \sin A}{\cos A \times \cos A} \left(\frac{1 - \cos A}{\cos A} \right)}{1 - \cos^2 A} \quad [\because \sin^2 A + \cos^2 A = 1] \\
 &= \frac{1 + \sin A}{\cos A} (1 - \cos A) \\
 &= \frac{1 + \sin A}{(1 + \cos A)(1 - \cos A)} (1 - \cos A) \\
 &= \frac{(1 + \sin A)}{\cos A} \\
 &= \frac{(1 + \sin A)}{(1 + \cos A)(1 + \sin A)} \\
 \text{RHS} &= \sec^2 A \left[\frac{1 - \sin A}{1 + \sec A} \right] \\
 &= \frac{1}{\cos^2 A} \left[\frac{1 - \sin A}{1 + \frac{1}{\cos A}} \right] \\
 &= \frac{1}{\cos^2 A} \left[\frac{1 - \sin A}{\frac{\cos A + 1}{\cos A}} \right] \\
 &= \frac{1}{\cos^2 A} \left[\frac{(1 - \sin A) \cos A}{(1 + \cos A)} \right] \\
 &= \frac{1}{\cos A \times \cos A} \left[\frac{(1 - \sin A) \cos A}{(1 + \cos A)} \right] \\
 &= \frac{1 - \sin A}{\cos A(1 + \cos A)}
 \end{aligned}$$



Multiplying numerator and denominator by $(1 + \sin A)$

$$\begin{aligned}
 &= \frac{(1 - \sin A)}{\cos A(1 + \cos A)} \times \frac{(1 + \sin A)}{1 + \sin A} \\
 &= \frac{(1)^2 - \sin^2 A}{\cos A(1 + \cos A)(1 + \sin A)} \\
 &= \frac{\cos^2 A}{\cos A(1 + \cos A)(1 + \sin A)} \\
 &= \frac{\cos A \times \cos A}{\cos A(1 + \cos A)(1 + \sin A)} \\
 &= \frac{\cos A}{(1 + \cos A)(1 + \sin A)} \\
 \therefore LHS &= RHS
 \end{aligned}$$



$$\text{Given, } \sec \theta = \frac{13}{5} = \frac{AC}{BC}$$

Let $AC = 13K$

and $BC = 5K$

In $\triangle ABC$, By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + (5K)^2 = (13K)^2$$

$$AB^2 + 25K^2 = 169K^2$$

$$AB^2 = 169K^2 - 25K^2 = 144K^2$$

$$AB = \sqrt{144K^2} = 12K$$

$$\therefore \sin \theta = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$$

$$\cos \theta = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$$

$$\text{LHS} = \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$$

$$= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} + 9 \times \frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} + \frac{45}{13}}$$

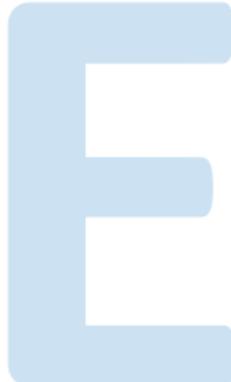
$$= \frac{\frac{9}{13}}{\frac{93}{13}}$$

$$= \frac{9}{13} \times \frac{13}{9}$$

$$= 3 = \text{R.H.S.}$$

$$20. + \frac{\sec^2(90^\circ - \theta) - \cot^2 \theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} - \frac{2 \cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

$$+ \frac{\cosec^2 \theta - \cot^2 \theta}{2\{\sin^2 25^\circ + \sin^2 (90^\circ - 25^\circ)\}} + \frac{2\left(\frac{1}{2}\right)^2 \tan^2 28^\circ \tan^2 (90^\circ - 28^\circ)}{3\{\sec^2 43^\circ - \cot^2 (90^\circ - 43^\circ)\}} \because \sec(90^\circ - \theta) = \cosec \theta$$



$$\begin{aligned} &= \frac{1}{2(\sin^2 25^\circ + \cos^2 25^\circ)} + \frac{2 \cdot \frac{1}{4} \cdot \tan^2 28^\circ \cdot \cot^2 28^\circ}{3(\sec^2 43^\circ - \tan^2 43^\circ)} \\ &\because \csc^2 \theta - \cot^2 \theta = 1 \\ &\sin(90^\circ - \theta) = \cos \theta \\ &\tan(90^\circ - \Theta) = \cot \theta \\ &\cot(90^\circ - \theta) = \tan \theta \\ &= \frac{1}{2(1)} + \frac{2 \cdot \frac{1}{4} \cdot \tan^2 28^\circ \cdot \frac{1}{\tan^2 28^\circ}}{3(1)} \\ &\therefore \sin^2 \theta + \cos^2 \theta = 1 \\ &\cot \theta = \frac{1}{\tan \theta} \\ &\sec^2 \theta - \tan^2 \theta = 1 \\ &= \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3} \end{aligned}$$

