

CBSE Test Paper 04
Chapter 8 Introduction to Trigonometry

1. $\sin^2 A + \sin^2 A \tan^2 A = \mathbf{(1)}$

a. $\tan^2 A$

b. $\cos^2 A$

c. None of these

d. $\sin^2 A$

2. If $\sin \theta = \frac{5}{13}$ then $\cos \theta = \mathbf{(1)}$

1. $\frac{13}{12}$

2. $\frac{\sqrt{5}}{13}$

3. $\frac{12}{5}$

4. $\frac{12}{13}$

3. If $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is **(1)**

1. -1

2. 2

3. 0

4. 1

4. If $\cot \theta = \frac{7}{8}$, then the value of $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ is **(1)**

1. $\frac{64}{49}$

2. $\frac{7}{8}$

3. $\frac{8}{7}$

4. $\frac{49}{64}$

5. If $\sin \alpha = \frac{1}{\sqrt{2}}$ and $\tan \beta = 1$, then the value of $\cos(\alpha + \beta)$ is **(1)**

1. 3

2. 1

3. 2

4. 0

6. Evaluate: $\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ} . \mathbf{(1)}$

7. Prove the trigonometric identity: $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$ **(1)**
8. In $\triangle ABC$, right angle at B, if $AB = 12$ cm and $BC = 5$ cm, find $\sin C$ and $\cot C$. **(1)**
9. Evaluate: $\frac{\sec 49^\circ}{\operatorname{cosec} 41^\circ}$ **(1)**
10. Without using trigonometric tables, prove that: $\cos 81^\circ - \sin 9^\circ = 0$ **(1)**
11. If $5 \sin \theta + 3 \cos \theta = 4$, find the value of $3 \sin \theta - 5 \cos \theta$. **(2)**
12. Prove that: $\frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta} = \frac{\sec \theta + 1}{\sec \theta - 1}$ **(2)**
13. Solve the equation for θ : $\frac{\cos^2 \theta}{\cot^2 \theta - \cos^2 \theta} = 3$. **(2)**
14. If $\cos \theta = \frac{12}{13}$, show that $\sin \theta (1 - \tan \theta) = \frac{35}{156}$ **(3)**
15. Find the acute angle θ , when $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$. **(3)**
16. If $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$, then prove that $\tan \theta = 1$ or $\frac{1}{2}$. **(3)**
17. Evaluate: $\frac{\sec 41^\circ \sin 49^\circ + \cos 29^\circ \operatorname{cosec} 61^\circ - \frac{2}{\sqrt{3}} (\tan 20^\circ \tan 60^\circ \tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$. **(3)**
18. The angle of elevation of the top of a tower from a certain point is 30° . If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15° . Find the height of the tower. **(4)**
19. Prove the trigonometric identity:

$$\left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}$$
 (4)
20. Evaluate without using trigonometric tables:

$$\frac{\operatorname{cosec}^2 (90^\circ - \theta) - \tan^2 \theta}{4(\cos^2 48^\circ + \cos^2 42^\circ)} - \frac{2 \tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\operatorname{cosec}^2 70^\circ - \tan^2 20^\circ}$$
 (4)

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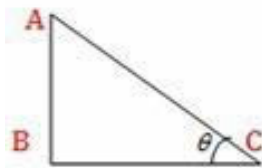
Solution

1. a. $\tan^2 A$

Explanation: Given: $\sin^2 A + \sin^2 A \tan^2 A$
 $= \sin^2 A (1 + \tan^2 A)$
 $= \sin^2 A (\sec^2 A)$
 $= \sin^2 A \times \frac{1}{\cos^2 A}$
 $= \frac{\sin^2 A}{\cos^2 A}$
 $= \tan^2 A$

2. d. $\frac{12}{13}$

Explanation: Let $AB = 5k$ and $AC = 13k$



$\therefore BC = \sqrt{(13k)^2 - (5k)^2} = \sqrt{169k^2 - 25k^2}$
 $\Rightarrow BC = \sqrt{144k^2} = 12k$
 $\therefore \cos \theta = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

3. d. 1

Explanation: Given: $\sin A + \sin^2 A = 1$
 $\Rightarrow \sin A = 1 - \sin^2 A$
 $\Rightarrow \sin A = \cos^2 A$
 Squaring both sides, we get
 $\Rightarrow \sin^2 A = \cos^4 A$
 $\Rightarrow 1 - \cos^2 A = \cos^4 A$
 $\Rightarrow \cos^2 A + \cos^4 A = 1$

4. d. $\frac{49}{64}$

Explanation: Given: $\cot \theta = \frac{7}{8}$

Now, $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

$$\begin{aligned}
 &= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} \\
 &= \left(\frac{7}{8}\right)^2 \\
 &= \frac{49}{64}
 \end{aligned}$$

5. d. 0

Explanation: Given: $\sin \alpha = \frac{1}{\sqrt{2}}$

$$\Rightarrow \sin \alpha = \sin 45^\circ$$

$$\Rightarrow \alpha = 45^\circ$$

$$\text{And } \tan \beta = 1$$

$$\Rightarrow \tan \beta = \tan 45^\circ$$

$$\Rightarrow \beta = 45^\circ$$

$$\therefore \cos(\alpha + \beta) = \cos(45^\circ + 45^\circ) = \cos 90^\circ = 0$$

6. We have,

$$\begin{aligned}
 &\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\frac{\tan^2 60^\circ}{(\sin 45^\circ)^2 + (\cos 45^\circ)^2}} \\
 &= \frac{(\tan 60^\circ)^2}{(1/\sqrt{2})^2 + (1/\sqrt{2})^2} \\
 &= \frac{(\sqrt{3})^2}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{3}
 \end{aligned}$$

7. LHS = $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

$$= \sin^2 A \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \frac{\sin^2 A}{\cos^2 A} \left[\because \cot^2 A = \frac{\cos^2 A}{\sin^2 A}, \tan^2 A = \frac{\sin^2 A}{\cos^2 A} \right]$$

$$= \cos^2 A + \sin^2 A \left[\because \cos^2 A + \sin^2 A = 1 \right]$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

8. $\sin C = \frac{AB}{AC} = \frac{12}{13}$

and $\cot C = \frac{BC}{AB} = \frac{5}{12}$

9. $\frac{\sec 49^\circ}{\cos 41^\circ} = \frac{\sec(90^\circ - 41^\circ)}{\cos 41^\circ} = \frac{\sec 41^\circ}{\cos 41^\circ} = 1 \left[\because \sec(90^\circ - \theta) = \sec \theta \right].$

10. LHS = $\cos 81^\circ - \sin 9^\circ$

$$= \cos(90^\circ - 9^\circ) - \sin 9^\circ$$

$$= \sin 9^\circ - \sin 9^\circ = 0 = \text{RHS}$$

$$11. 5\sin\theta + 3\cos\theta = 4$$

$$\Rightarrow (5\sin\theta + 3\cos\theta)^2 = (4)^2$$

$$\Rightarrow 25\sin^2\theta + 9\cos^2\theta + 30\sin\theta\cos\theta = 16$$

$$\Rightarrow 25(1 - \cos^2\theta) + 9(1 - \sin^2\theta) + 30\sin\theta\cos\theta = 16$$

$$\because \sin^2\theta = 1 - \cos^2\theta \text{ and } \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow (3\sin\theta - 5\cos\theta)^2 = 18$$

$$\Rightarrow 3\sin\theta - 5\cos\theta = \pm 3\sqrt{2}$$

$$12. \text{L.H.S.} = \frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$$

$$= \frac{\sin\theta\left(\frac{1}{\cos\theta} + 1\right)}{\sin\theta\left(\frac{1}{\cos\theta} - 1\right)} = \frac{\sec\theta + 1}{\sec\theta - 1} = \text{R.H.S.}$$

$$13. \frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$$

$$\Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$$

$$\Rightarrow 4\cos^2\theta = 3\cot^2\theta$$

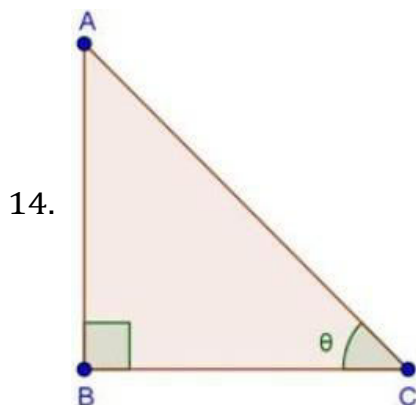
$$\Rightarrow 4\cos^2\theta = \frac{3\cos^2\theta}{\sin^2\theta}$$

$$\Rightarrow 4\sin^2\theta = \frac{3\cos^2\theta}{\cos^2\theta}$$

$$\Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 60^\circ$$



$$\text{Given } \cos \theta = \frac{12}{13} = \frac{BC}{AC}$$

$$\text{Let } BC = 12K$$

$$\text{and, } AC = 13K$$

In $\triangle ABC$, By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + (12K)^2 = (13K)^2$$

$$AB^2 + 144K^2 = 169K^2$$

$$AB^2 = 169K - 144K^2 = 25K^2$$

$$AB = \sqrt{25K^2} = 5K$$

$$\therefore \sin \theta = \frac{AB}{AC} = \frac{5K}{13K} = \frac{5}{13}$$

$$\tan \theta = \frac{AB}{BC} = \frac{5K}{12K} = \frac{5}{12}$$

$$\therefore LHS = \sin \theta (1 - \tan \theta)$$

$$= \frac{5}{13} \left(1 - \frac{5}{12} \right)$$

$$= \frac{5}{13} \left(\frac{12-5}{12} \right)$$

$$= \frac{5}{13} \times \frac{7}{12}$$

$$= \frac{35}{156} = RHS$$

15. According to question

$$\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

$$\Rightarrow \frac{(\cos \theta - \sin \theta) + (\cos \theta + \sin \theta)}{(\cos \theta - \sin \theta) - (\cos \theta + \sin \theta)} = \frac{(1 - \sqrt{3}) + (1 + \sqrt{3})}{(1 - \sqrt{3}) - (1 + \sqrt{3})}$$

[Applying componendo and dividendo]

$$\Rightarrow \frac{2 \cos \theta}{-2 \sin \theta} = \frac{2}{-2\sqrt{3}}$$

$$\Rightarrow \cot \theta = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

16. Given,

$$1 + \sin^2 \theta = 3 \sin \theta \cos \theta$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + \sin^2 \theta = 3 \sin \theta \cos \theta$$

$$\Rightarrow 2 \sin^2 \theta + \cos^2 \theta = 3 \sin \theta \cos \theta$$

$$\Rightarrow 2 \sin^2 \theta - 3 \sin \theta \cos \theta + \cos^2 \theta = 0$$

$$\Rightarrow 2 \sin^2 \theta - 2 \sin \theta \cos \theta - \sin \theta + \cos^2 \theta = 0$$

$$\Rightarrow (\sin\theta - \cos\theta)(2\sin\theta - \cos\theta) = 0$$

$$\text{either, } \sin\theta - \cos\theta = 0 \text{ or, } 2\sin\theta - \cos\theta = 0$$

$$\Rightarrow \sin\theta = \cos\theta. \text{ or, } 2\sin\theta = \cos\theta$$

$$\Rightarrow \tan\theta = 1 \text{ or, } \tan\theta = \frac{1}{2} \cdot \left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta \right]$$

Hence, proved.

17. We have,

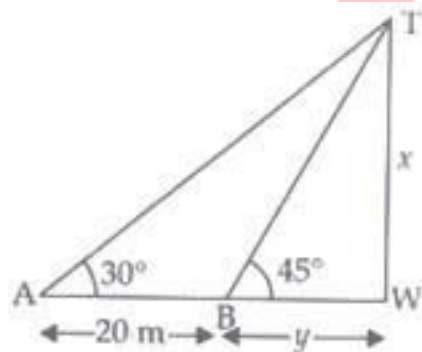
$$\begin{aligned} & \frac{\sec 41^\circ \sin 49^\circ + \cos 29^\circ \operatorname{cosec} 61^\circ - \frac{2}{\sqrt{3}} (\tan 20^\circ \tan 60^\circ \tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)} \\ &= \frac{\sec(90^\circ - 41^\circ) \sin 49^\circ + \cos 29^\circ \operatorname{cosec}(90^\circ - 29^\circ) - \frac{2}{\sqrt{3}} [\tan 20^\circ \sqrt{3} \tan(90^\circ - 20^\circ)]}{3[\sin^2 31^\circ + \sin^2(90^\circ - 31^\circ)]} \\ &= \frac{\operatorname{cosec} 49^\circ \sin 49^\circ + \cos 29^\circ \sec 29^\circ - \frac{2}{\sqrt{3}} [\tan 20^\circ \sqrt{3} \cot 20^\circ]}{3(\sin^2 31^\circ + \cos^2 31^\circ)} \\ &= \frac{1+1-2}{3} = \frac{2-2}{3} \\ &= 0 \end{aligned}$$

18. Let the height of the vertical tower (TW) = x m

When an observer at A, makes angle of elevation at the top of tower is 30° .

Now, angle of elevation of the top of tower is increased by 15° when observer moves 20m towards the tower.

i.e., it becomes $30^\circ + 15^\circ = 45^\circ$.



In $\triangle TWB$,

$$\tan 45^\circ = \frac{P}{B} = \frac{x}{y}$$

$$\Rightarrow 1 = \frac{x}{y}$$

$$\Rightarrow x = y \dots\dots (i)$$

Now, $\triangle TWA$, we have

$$\tan 30^\circ = \frac{P}{B} = \frac{x}{20+y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{20+x} \text{ [From (i)]}$$

$$\begin{aligned}
&\Rightarrow \sqrt{3}x = 20 + x \\
&\Rightarrow \sqrt{3}x - x = 20 \\
&\Rightarrow x(\sqrt{3} - 1) = 20 \\
&\Rightarrow x = \frac{20}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{(\sqrt{3}+1)} \\
&\Rightarrow x = \frac{20(\sqrt{3}+1)}{3-1} = \frac{20(\sqrt{3}+1)}{2} \\
&\Rightarrow x = 10(1.732 + 1) \\
&\Rightarrow x = 10 \times 2.732 = 27.32m
\end{aligned}$$

Hence, the height of the tower (TW) is 27.32 m.

$$\begin{aligned}
19. \text{ LHS} &= \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\cos^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\frac{1}{\sin^2 \theta} - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{1 - \cos^4 \theta} + \frac{\sin^2 \theta}{1 - \sin^4 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta (1 - \sin^4 \theta) + \sin^2 \theta (1 - \cos^4 \theta)}{(1 - \cos^4 \theta)(1 - \sin^4 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta (1 - \sin^2 \theta)(1 + \sin^2 \theta) + \sin^2 \theta (1 - \cos^2 \theta)(1 + \cos^2 \theta)}{(1 - \cos^2 \theta)(1 + \cos^2 \theta)(1 - \sin^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta \cdot \cos^2 \theta (1 + \sin^2 \theta) + \sin^2 \theta \sin^2 \theta (1 + \cos^2 \theta)}{\sin^2 \theta (1 + \cos^2 \theta) \cos^2 \theta (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\
&= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{\sin^2 \theta \cos^2 \theta (1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \\
&= \left(\frac{\cos^4 \theta + \cos^4 \theta \sin^2 \theta + \sin^4 \theta + \sin^4 \theta \cos^2 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \\
&= \left(\frac{\cos^4 \theta + \sin^4 \theta + \cos^4 \theta \sin^2 \theta + \sin^4 \theta \cos^2 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \\
&= \frac{(\cos^2 \theta + \sin^2 \theta)^2 - 2\cos^2 \theta \sin^2 \theta + \cos^2 \theta \sin^2 \theta (\cos^2 \theta + \sin^2 \theta)}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \\
&= \frac{1 - 2\cos^2 \theta \sin^2 \theta + \cos^2 \theta \sin^2 \theta \times 1}{1 + \sin^2 \theta + \cos^2 \theta + \cos^2 \theta \sin^2 \theta} \quad [\sin^2 \theta + \cos^2 \theta = 1] \\
&= \frac{1 - \cos^2 \theta \sin^2 \theta}{1 + 1 + \cos^2 \theta \sin^2 \theta}
\end{aligned}$$

$$= \frac{1 - \cos^2 \theta \sin^2 \theta}{2 + \cos^2 \theta \sin^2 \theta}$$

$$= \text{RHS}$$

Hence proved

$$20. \frac{\sec^2(90^\circ - \theta) - \tan^2 \theta}{4(\cos^2 48^\circ + \cos^2 42^\circ)} - \frac{2 \tan^2 30^\circ \sec^2 52^\circ \sin^2 38^\circ}{\sec^2 70^\circ - \tan^2 20^\circ}$$

$$= \frac{\sec^2 \theta - \tan^2 \theta}{4[\cos^2(90^\circ - 42^\circ) + \cos^2 42^\circ]} - \frac{2\left(\frac{1}{\sqrt{3}}\right)^2 \sec^2(90^\circ - 38^\circ) \sin^2 38^\circ}{\sec^2(90^\circ - 20^\circ) - \tan^2 20^\circ}$$

$$= \frac{1}{4(\sin^2 42^\circ + \cos^2 42^\circ)} - \frac{\frac{2}{3} \sec^2 38^\circ \sin^2 38^\circ}{\sec^2 20^\circ - \tan^2 20^\circ}$$

$$\therefore \sec^2 \theta - \tan^2 \theta = 1$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\sec(90^\circ - \theta) = \sec \theta$$

$$\sec(90^\circ - \theta) = \sec \theta$$

$$= \frac{1}{4(1)} - \frac{\frac{2}{3} \cdot \frac{1}{\sin^2 38^\circ} \cdot \sin^2 38^\circ}{1}$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\sec \theta = \frac{1}{\sin \theta}$$

$$\frac{1}{4} - \frac{2}{3} = \frac{3-8}{12} = \frac{-5}{12}$$