

CBSE Test Paper 03
Chapter 8 Introduction to Trigonometry

1. The value of $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$ is **(1)**
- 0
 - 1
 - 2
 - 2
2. Choose the correct option and justify your choice: $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$ **(1)**
- $\tan 90^\circ$
 - 1
 - $\sin 45^\circ$
 - 0
3. The value of $\operatorname{cosec}^4 A - 2 \operatorname{cosec}^2 A + 1$ is **(1)**
- $\tan^4 A$
 - $\sec^4 A$
 - $\operatorname{cosec}^4 A$
 - $\cot^4 A$
4. Choose the correct option. Justify your choice. $(\sec A + \tan A)(1 - \sin A)$ **(1)**
- $\cos A$
 - $\sec A$
 - $\sin A$
 - $\operatorname{cosec} A$
5. $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) =$ **(1)**
- $\tan^2 \theta + \cos^2 \theta$
 - $\tan^2 \theta - \cos^2 \theta$
 - $\tan^2 \theta + \sin^2 \theta$

d. $\tan^2 \theta - \sin^2 \theta$

6. If $\sin (20^\circ + \theta) = \cos 30^\circ$, then find the value of θ . **(1)**
7. Write the value of $4\tan^2 \theta - \frac{4}{\cos^2 \theta}$. **(1)**
8. Find the value of x, if $2 \sin 3x = \sqrt{3}$. **(1)**
9. Prove that: $\frac{1}{(\operatorname{cosec} \theta - \cot \theta)} = (\operatorname{cosec} \theta + \cot \theta)$ **(1)**
10. If $\cos \theta = \frac{2}{3}$, then find the value of $(4 + 4 \tan^2 \theta)$. **(1)**
11. Find the value of θ if $\sqrt{3} \tan 2\theta - 3 = 0$. **(2)**
12. Prove that $\tan 1^\circ \tan 11^\circ \tan 21^\circ \tan 69^\circ \tan 79^\circ \tan 89^\circ = 1$ **(2)**
13. Prove that: $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ = 2$ **(2)**
14. If $\tan \theta + \frac{1}{\tan \theta} = 2$, find the value of $\tan^2 \theta + \frac{1}{\tan^2 \theta}$ **(3)**
15. Prove the trigonometric identity:

$$\left(\tan \theta + \frac{1}{\cos \theta} \right)^2 + \left(\tan \theta - \frac{1}{\cos \theta} \right)^2 = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right)$$
 (3)
16. In $\triangle ABC$ right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$ find value of: $\sin A \cos C + \cos A \sin C$ **(3)**
17. If $\sin \theta = \frac{3}{5}$, evaluate $\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$. **(3)**
18. Prove that: $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$. **(4)**
19. Prove that: $\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$. **(4)**
20. Prove that: $\frac{1 + \cos^2 A}{\sin^2 A} = 2 \operatorname{cosec}^2 A - 1$ **(4)**

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Solution

1. c. 2

Explanation: Given: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$
 $= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

2. d. 0

Explanation: $= \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$

Hence the correct answer is 0.

3. d. $\cot^4 A$

Explanation: Given: $\cos^4 A - 2\cos^2 A + 1$
 $= (\cos^2 A - 1)^2$
 $= (\cot^2 A)^2$
 $= \cot^4 A$

4. a. $\cos A$

Explanation: $(\sec A + \tan A)(1 - \sin A)$
 $= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$
 $= \left[\frac{1 + \sin A}{\cos A}\right] \times (1 - \sin A) = \frac{1 - \sin^2 A}{\cos A}$
 $= \frac{\cos^2 A}{\cos A} = \cos A$

Hence, the correct choice is $\cos A$

5. c. $\tan^2 \theta + \sin^2 \theta$

Explanation: Given: $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta)$
 $= (\sec^2 \theta - \cos^2 \theta)$
 $= (1 + \tan^2 \theta - 1 + \sin^2 \theta)$
 $= (\tan^2 \theta + \sin^2 \theta)$

6. $\sin (20^\circ + \theta) = \cos 30^\circ$

$\cos (90^\circ - (20^\circ + \theta)) = \cos 30^\circ$ [$\because \cos (90^\circ - \theta) = \sin \theta$]

$\cos (90^\circ - 20^\circ - \theta) = \cos 30^\circ$

$$\cos (70^\circ - \theta) = \cos 30^\circ$$

$$70^\circ - \theta = 30^\circ$$

$$70^\circ - 30^\circ = \theta$$

$$40^\circ = \theta$$

Hence, the value of θ is 40° .

$$\begin{aligned} 7. \quad & 4\tan^2\theta - \frac{4}{\cos^2\theta} \\ &= 4\frac{\sin^2\theta}{\cos^2\theta} - \frac{4}{\cos^2\theta} \\ &= \frac{4\sin^2\theta - 4}{\cos^2\theta} \\ &= \frac{4(\sin^2\theta - 1)}{\cos^2\theta} = \frac{-4(1 - \sin^2\theta)}{\cos^2\theta} \\ &= \frac{-4 \times (\cos^2\theta)}{\cos^2\theta} \\ &= -4 \end{aligned}$$

8. Given,

$$2\sin 3x = \sqrt{3}$$

$$\Rightarrow \sin 3x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 3x = \sin 60^\circ \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right]$$

$$\Rightarrow 3x = 60^\circ$$

$$\Rightarrow x = \frac{60^\circ}{3} = 20^\circ$$

9. We have,

$$\begin{aligned} \text{L.H.S.} &= \frac{1}{(\operatorname{cosec}\theta - \cot\theta)} = \frac{1}{(\operatorname{cosec}\theta - \cot\theta)} \times \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}\theta + \cot\theta)} \\ &= \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}^2\theta - \cot^2\theta)} = \operatorname{cosec}\theta + \cot\theta = \text{R.H.S.} \left[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1 \right] \\ \therefore \text{L.H.S.} &= \text{R.H.S.} \end{aligned}$$

$$10. \quad \cos \theta = \frac{2}{3} \therefore \cos^2 \theta = \frac{4}{9} .$$

$$\text{Now, } 4 + 4\tan^2\theta = 4(1 + \tan^2\theta) = 4\sec^2\theta = 4(1/\cos^2\theta)$$

$$= 4 \times \frac{1}{4/9}$$

$$= 9$$

$$11. \quad \sqrt{3} \tan 2\theta - 3 = 0$$

$$\Rightarrow \sqrt{3} \tan 2\theta = 3$$

$$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \tan 2\theta = \sqrt{3} \dots (i)$$

$$\text{Also, } \tan 60^\circ = \sqrt{3} \dots (ii)$$

$\therefore$ on equating (i) and (ii), we get

$$2\theta = 60^\circ$$

$$\Rightarrow \theta = 30^\circ$$

$$\begin{aligned} 12. \text{ LHS} &= \tan 1^\circ \tan 11^\circ \tan 21^\circ \tan 69^\circ \tan 79^\circ \tan 89^\circ \\ &= \tan(90^\circ - 89^\circ) \tan(90^\circ - 79^\circ) \tan(90^\circ - 69^\circ) \tan 69^\circ \tan 79^\circ \tan 89^\circ \\ &= \cot 89^\circ \cot 79^\circ \cot 69^\circ \tan 69^\circ \tan 79^\circ \tan 89^\circ \\ &= (\cot 89^\circ \tan 89^\circ) (\cot 79^\circ \tan 79^\circ) (\cot 69^\circ \tan 69^\circ) \\ &= 1 \times 1 \times 1 \\ &= 1 = \text{RHS.} \end{aligned}$$

13. We have,

$$\begin{aligned} &\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \\ &= \sec(90^\circ - 40^\circ) \sin 40^\circ + \cos 40^\circ \operatorname{cosec}(90^\circ - 40^\circ) \\ &= \operatorname{cosec} 40^\circ \sin 40^\circ + \cos 40^\circ \sec 40^\circ \left[\begin{array}{l} \because \sec(90^\circ - \theta) = \operatorname{cosec} \theta \\ \operatorname{cosec}(90^\circ - \theta) = \sec \theta \end{array} \right] \\ &= \frac{\sin 40^\circ}{\sin 40^\circ} + \frac{\cos 40^\circ}{\cos 40^\circ} = 1 + 1 = 2. \end{aligned}$$

14. We have,

$$\tan \theta + \frac{1}{\tan \theta} = 2$$

Squaring both sides, we get

$$\begin{aligned} &\Rightarrow \left(\tan \theta + \frac{1}{\tan \theta} \right)^2 = 2^2 \\ &\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 \times \tan \theta \times \frac{1}{\tan \theta} = 4 \\ &\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} + 2 = 4 \\ &\Rightarrow \tan^2 \theta + \frac{1}{\tan^2 \theta} = 2 \end{aligned}$$

Alternate method, We have

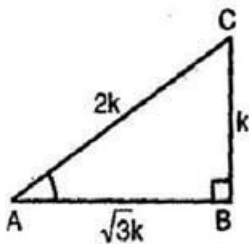
$$\begin{aligned} &\tan \theta + \frac{1}{\tan \theta} = 2 \\ &\Rightarrow \tan^2 \theta + 1 = 2 \tan \theta \\ &\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0 \\ &\Rightarrow (\tan \theta - 1)^2 = 0 \\ &\Rightarrow \tan \theta = 1 \end{aligned}$$

$$\therefore \tan^2 \theta + \frac{1}{\tan^2 \theta} = 1 + 1 = 2$$

$$\begin{aligned}
 15. \text{ LHS} &= \left(\tan \theta + \frac{1}{\cos \theta} \right)^2 + \left(\tan \theta - \frac{1}{\cos \theta} \right)^2 \\
 &= \left(\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right)^2 + \left(\frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} \right)^2 \left(\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right) \\
 &= \frac{(\sin \theta + 1)^2}{\cos^2 \theta} + \frac{(\sin \theta - 1)^2}{\cos^2 \theta} \\
 &= \frac{(\sin \theta + 1)^2 + (\sin \theta - 1)^2}{\cos^2 \theta} \\
 &= \frac{\sin^2 \theta + 1 + 2 \sin \theta + \sin^2 \theta + 1 - 2 \sin \theta}{\cos^2 \theta} \\
 &= \frac{2 \sin^2 \theta + 2}{\cos^2 \theta} \\
 &= \frac{2(\sin^2 \theta + 1)}{1 - \sin^2 \theta} \left[\because \cos^2 \theta = 1 - \sin^2 \theta \right] \\
 &= 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right) \\
 &= \text{RHS.}
 \end{aligned}$$

Hence proved

16. Consider a triangle ABC in which $\angle B = 90^\circ$



Let $BC = k$ and $AB = \sqrt{3}k$

Then, using Pythagoras theorem,

$$\begin{aligned}
 AC &= \sqrt{(BC)^2 + (AB)^2} = \sqrt{(k)^2 + (\sqrt{3}k)^2} \\
 &= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k
 \end{aligned}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

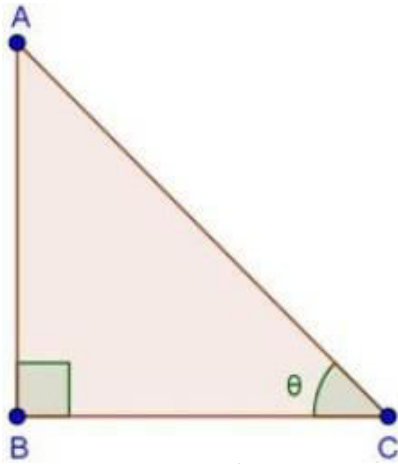
For $\angle C$, Base = BC, Perpendicular = AB and Hypotenuse = AC

$$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

17.



$$\text{Given } \sin \theta = \frac{3}{5} = \frac{AB}{AC}$$

$$\text{Let } AB = 3K$$

$$\text{and, } AC = 5K$$

In $\triangle ABC$, by Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$(3K)^2 + BC^2 = 25K^2$$

$$BC^2 = 25K^2 - 9K^2$$

$$BC^2 = 16K^2$$

$$BC = \sqrt{16K^2} = 4K$$

$$\therefore \cos \theta = \frac{BC}{AC} = \frac{4K}{5K} = \frac{4}{5}$$

$$\tan \theta = \frac{AB}{BC} = \frac{3K}{4K} = \frac{3}{4}$$

$$\cot \theta = \frac{BC}{AB} = \frac{4K}{3K} = \frac{4}{3}$$

$$\text{Now } \frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$$

$$= \frac{\frac{4}{5} - \frac{4}{3}}{2 \times \frac{4}{3}}$$

$$= \frac{\frac{12-20}{15}}{\frac{8}{3}}$$

$$= \frac{-\frac{8}{15}}{\frac{8}{3}}$$

$$= \frac{-8}{15} \times \frac{3}{8}$$

$$= \frac{-1}{5}$$

$$18. = (\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$$

$$= \sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta + \cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta$$

$$= \sin^2 \theta + \cos^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \times \frac{1}{\sin \theta} + \sec^2 \theta + 2 \cos \theta \times \frac{1}{\cos \theta}$$

$$\begin{aligned}
 &= 1 + 1 + \cot^2 \theta + 2 + 1 + \tan^2 \theta + 2 \\
 &= 1 + 1 + 2 + 1 + 2 + \tan^2 \theta + \cot^2 \theta \\
 &= 7 + \tan^2 \theta + \cot^2 \theta
 \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$19. \text{ LHS} = \sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}}$$

Rationalise the denominator and we get,

$$\begin{aligned}
 &= \sqrt{\frac{(\sec \theta - 1)^2}{\sec^2 \theta - 1}} + \sqrt{\frac{(\sec \theta + 1)^2}{\sec^2 \theta - 1}} \\
 &= \frac{(\sec \theta - 1) + (\sec \theta + 1)}{\sqrt{(\sec \theta + 1)(\sec \theta - 1)}} \\
 &= \frac{2 \sec \theta}{\sqrt{\sec^2 \theta - 1}} = \frac{2 \sec \theta}{\sqrt{\tan^2 \theta}} = \frac{2 \sec \theta}{\tan \theta} \\
 &= 2 \times \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} \\
 &= 2 \times \frac{1}{\sin \theta} \\
 &= 2 \operatorname{cosec} \theta = \text{RHS (Hence Proved)}
 \end{aligned}$$

$$20. \frac{1 + \cos^2 A}{\sin^2 A} = 2 \operatorname{cosec}^2 A - 1$$

L.H.S.

$$\begin{aligned}
 &= \frac{1 + \cos^2 A}{\sin^2 A} = \frac{1 + (1 - \sin^2 A)}{\sin^2 A} \left[\because 1 - \sin^2 A = \cos^2 A \right] \\
 &= \frac{2 - \sin^2 A}{\sin^2 A} = \frac{2}{\sin^2 A} - 1 \\
 &= 2 \operatorname{cosec}^2 A - 1 \left[\because \frac{1}{\sin^2 A} = \operatorname{cosec}^2 A \right] \\
 &= \text{R.H.S. (Hence Proved)}
 \end{aligned}$$