

CBSE Test Paper 01
Chapter 8 Introduction to Trigonometry

1. Given that $\sin \alpha = \frac{1}{\sqrt{2}}$ and $\cos \beta = \frac{1}{\sqrt{2}}$, then the value of $(\alpha + \beta)$ is **(1)**
 - a. 90°
 - b. 45°
 - c. 60°
 - d. 30°
2. $\cot A \tan A$ **(1)**
 - a. $\tan A$
 - b. $\sec A$
 - c. 1
 - d. $\cot A$
3. If A and B are acute angles and $\sin A = \cos B$, then the value of $(A + B)$ is **(1)**
 - a. 0°
 - b. 90°
 - c. 30°
 - d. 60°
4. Which of the following statement is true: **(1)**
 - a. $\frac{\cos ec A}{\sin A} = \cos A$
 - b. $\frac{\cos A}{\sin A} = \sec A$
 - c. $\frac{\cos ec A}{\sin A} = \cot A$
 - d. $\frac{\cos ec A}{\cos A} = \tan A$
5. Choose the correct option and justify your choice: $\sin 2A = 2 \sin A$ is true when A = **(1)**
 - a. 45°
 - b. 0°
 - c. 30°
 - d. 60°
6. Express $\cos 83^\circ - \sec 76^\circ$ in terms of trigonometric ratios of angles between 0° and 45° . **(1)**
7. Prove the trigonometric identity:

$$\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A \quad (1)$$

8. Prove that $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ = 0$. **(1)**
9. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles, find the value of θ . **(1)**
10. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$ where $5A$ is an acute angle, then find the value of A . **(1)**
11. Prove that $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$. **(2)**
12. Prove the following identity : $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$ **(2)**
13. Verify that, $\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ = \frac{1}{2}$. **(2)**
14. If $\sin 3\theta = \cos(\theta - 6^\circ)$, where 3θ and $\theta - 6^\circ$ are both acute angles, find the value of θ . **(3)**
15. Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$, using identity $\sec^2 \theta = 1 + \tan^2 \theta$. **(3)**
16. Prove the identity:
 $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$ **(3)**
17. Find the value of the following without using trigonometric tables:
 $\frac{\cos 50^\circ}{2 \sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3 \tan^2 45^\circ} - \frac{2}{3} \tan 12^\circ \tan 78^\circ \cdot \sin 90^\circ$ **(3)**
18. Evaluate : $\frac{\cos 65^\circ}{\sin 25^\circ} - \frac{\tan 20^\circ}{\cot 70^\circ} - \sin 90^\circ + \tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$. **(4)**
19. If $\tan \theta = \frac{1}{\sqrt{5}}$
 i. Evaluate : $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$
 ii. Verify the identity : $\sin^2 \theta + \cos^2 \theta = 1$. **(4)**
20. Given the value of $\sec \theta = \frac{13}{12}$, Calculate all other trigonometric ratios. **(4)**

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Solution

1. a. 90°

Explanation: Given: $\sin \alpha = \frac{1}{\sqrt{2}}$

$$\Rightarrow \sin \alpha = \sin 45^\circ$$

$$\Rightarrow \alpha = 45^\circ$$

And $\cos \beta = \frac{1}{\sqrt{2}}$

$$\Rightarrow \cos \beta = \cos 45^\circ$$

$$\Rightarrow \beta = 45^\circ$$

$$\therefore \alpha + \beta = 45^\circ + 45^\circ = 90^\circ$$

2. c. 1

Explanation: $\cot A \tan A = \cot A \times \frac{1}{\cot A} = 1 \times 1 = 1$

3. b. 90°

Explanation: Given: $\sin A = \cos B$

Since A and B are acute angles, then

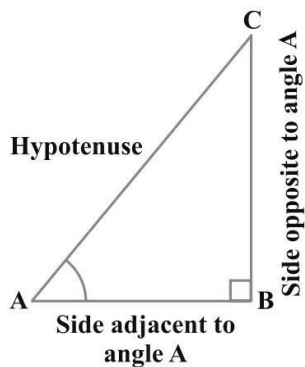
$$\sin A = \sin(90^\circ - B)$$

$$\Rightarrow A = 90^\circ - B$$

$$\Rightarrow A + B = 90^\circ$$

4. d. $\frac{\sin A}{\cos A} = \tan A$

Explanation: In right triangle ABC right angle at B



$$\sin A = \frac{BC}{AC}$$

$$\cos A = \frac{AB}{AC}$$

$$\begin{aligned}\text{Then } \frac{\sin A}{\cos A} &= \frac{\frac{BC}{AC}}{\frac{AB}{AC}} \\ \frac{BC}{AC} \times \frac{AC}{AB} &= \frac{BC}{AB} \\ L.H.S &= \frac{\sin A}{\cos A} = \frac{BC}{AB} \\ R.H.S &= \tan A \\ &= \frac{\sin A}{\cos A} = \frac{BC}{AB} = \frac{BC}{AB}\end{aligned}$$

L.H.S = R.H.S

Therefore $\frac{\sin A}{\cos A} = \tan A$ is true.

5. b. 0°

Explanation: When $A = 0^\circ$

$$\sin 2A = \sin 2(0^\circ) = 0$$

$$2 \sin A = 2 \sin(0^\circ) = 2(0) = 0$$

$$\therefore \sin 2A = 2 \sin A$$

Hence, the correct option is (A) = 0°

$$\begin{aligned}6. \cos 83^\circ - \sec 76^\circ &= \cos (90^\circ - 7^\circ) - \sec (90^\circ - 14^\circ) \\ &= (\sin 7^\circ - \operatorname{cosec} 14^\circ).\end{aligned}$$

7. We have,

$$\text{LHS} = \cos^4 A - \cos^2 A$$

$$\Rightarrow \text{LHS} = \cos^2 A (\cos^2 A - 1)$$

$$\Rightarrow \text{LHS} = -\cos^2 A (1 - \cos^2 A)$$

$$\Rightarrow \text{LHS} = -\cos^2 A \sin^2 A = -(1 - \sin^2 A) \sin^2 A = -\sin^2 A + \sin^4 A$$

$$\Rightarrow \text{LHS} = \sin^4 A - \sin^2 A = \text{RHS}$$

8. We have,

$$\text{LHS} = \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 89^\circ \cos 90^\circ \cos 91^\circ \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \times \cos 89^\circ \times 0 \times \cos 91^\circ \times \dots \cos 180^\circ = 0 = \text{RHS} [\because \cos 90^\circ = 0]$$

9. We have,

$$\sin 5\theta = \cos 4\theta$$

$$\Rightarrow \sin 5\theta = \sin(90^\circ - 4\theta) [\because \sin(90^\circ - \theta) = \cos \theta]$$

$$\Rightarrow 5\theta = 90^\circ - 4\theta$$

$$\Rightarrow 9\theta = 90^\circ$$

$$\Rightarrow \theta = 10^\circ$$

10. Given, $\sec 5A = \operatorname{cosec}(A + 30^\circ)$

$$\operatorname{cosec}(90^\circ - 5A) = \operatorname{cosec}(A + 30^\circ) [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta]$$

$$90^\circ - 5A = A + 30^\circ$$

$$90^\circ - 30^\circ = A + 5A$$

$$\Rightarrow 6A = 60^\circ$$

$$\text{Therefore, } A = 10^\circ$$

11. **LHS** = $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$

$$= [\sin(90^\circ - (40^\circ - \theta))] - \cos(40^\circ - \theta) + \tan(90^\circ - 89^\circ) \tan(90^\circ - 80^\circ) \tan(90^\circ - 70^\circ) \times \tan 70^\circ \tan 80^\circ \tan 89^\circ$$

$$= \cos(40^\circ - \theta) - \cos(40^\circ - \theta) + \cot 89^\circ \times \cot 80^\circ \times \cot 70^\circ \times \frac{1}{\cot 70^\circ} \times \frac{1}{\cot 80^\circ} \times \frac{1}{\cot 89^\circ}$$

$$= 0 + 1$$

$$= 1 = \text{RHS}$$

12. To prove : $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$

$$\text{LHS} = \sin^6 A + \cos^6 A$$

$$= (\sin^2 A)^3 + (\cos^2 A)^3$$

$$= (\sin^2 A + \cos^2 A)^3 - 3\sin^2 A \cdot \cos^2 A (\sin^2 A + \cos^2 A) [\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)]$$

$$= 1^3 - 3\sin^2 A \cdot \cos^2 A \times 1$$

$$= 1 - 3\sin^2 A \cdot \cos^2 A$$

$$= \text{RHS.}$$

Hence proved.

13. We have,

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \sin 30^\circ = \frac{1}{2}$$

therefore,

$$\text{R.H.S} = \cos^2 30^\circ - \sin^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\text{and, L.H.S} = \cos 60^\circ = \frac{1}{2}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

$$\text{i.e, } \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ = \frac{1}{2}$$

14. Given,

$$\sin 3\theta = \cos (\theta - 6^\circ)$$

$$\cos (90^\circ - 3\theta) = \cos (\theta - 6^\circ)$$

$$90^\circ - 3\theta = \theta - 6^\circ$$

$$4\theta = 90^\circ + 6^\circ = 96^\circ$$

$$\therefore \theta = \frac{96^\circ}{4} = 24^\circ$$

15. We have to prove that, $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ using identity $\sec^2 \theta = 1 + \tan^2 \theta$

$$\text{LHS} = \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} \quad [\text{dividing the numerator and denominator by } \cos \theta.]$$

$$= \frac{(\tan \theta + \sec \theta) - 1}{(\tan \theta - \sec \theta) + 1} = \frac{\{(\tan \theta + \sec \theta) - 1\}(\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \quad [\text{Multiplying and dividing by } (\tan \theta - \sec \theta)]$$

$$= \frac{(\tan^2 \theta - \sec^2 \theta) - (\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= \frac{-1 - \tan \theta + \sec \theta}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \quad [\because \tan^2 \theta - \sec^2 \theta = -1]$$

$$= \frac{-(\tan \theta - \sec \theta + 1)}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} = \frac{-1}{\tan \theta - \sec \theta}$$

$$= \frac{1}{\sec \theta - \tan \theta} = \text{RHS}$$

Hence Proved.

16. Take ,

$$\text{LHS} = 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1$$

$$\Rightarrow \text{LHS} = 2\{(\sin^2 \theta)^3 + (\cos^2 \theta)^3\} - 3\{(\sin^2 \theta)^2 + (\cos^2 \theta)^2\} + 1$$

Using $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ and $a^2 + b^2 = (a + b)^2 - 2ab$ in above expression,

where $a = \sin^2 \theta$ & $b = \cos^2 \theta$; we get :-

$$\text{LHS} = 2\{(\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)\} - 3\{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta\} + 1$$

$$\Rightarrow \text{LHS} = 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta) + 1 \quad [\text{Since, } \sin^2 A + \cos^2 A = 1]$$

$$\Rightarrow \text{LHS} = 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta + 1$$

Hence, L.H.S. = 0 = R.H.S.

Hence, proved.

17. We know that,

$$\cos 50^\circ = \cos (90^\circ - 40^\circ) = \sin 40^\circ$$

$$\operatorname{cosec}^2 59^\circ = \operatorname{cosec}^2 (90^\circ - 31^\circ) = \sec^2 31^\circ$$

$$\text{and } \tan 78^\circ = \tan (90^\circ - 12^\circ) = \cot 12^\circ$$

Now,

$$\begin{aligned} &= \frac{\cos 50^\circ}{2 \sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3 \tan^2 45^\circ} - \frac{2}{3} \tan 12^\circ \tan 78^\circ \sin 90^\circ \\ &= \frac{\sin 40^\circ}{2 \sin 40^\circ} + \frac{4(\sec^2 31^\circ - \tan^2 31^\circ)}{3 \times (1)^2} - \frac{2}{3} \tan 12^\circ \cot 12^\circ \times 1 \\ &= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6} \end{aligned}$$

18. First, we need to solve given equation in parts

$$\Rightarrow \frac{\cos 65^\circ}{\sin 25^\circ} = \frac{\cos 65^\circ}{\sin(90^\circ - 65^\circ)} = \frac{\cos 65^\circ}{\cos 65^\circ} = 1$$

$$\Rightarrow \frac{\tan 20^\circ}{\cot 70^\circ} = \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} = \frac{\cot 70^\circ}{\cot 70^\circ} = 1$$

$$\Rightarrow \sin 90^\circ = 1$$

$$\Rightarrow \tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$$

$$= \tan (90^\circ - 85^\circ) \tan (90^\circ - 55^\circ) \tan 55^\circ \tan 60^\circ \tan 85^\circ$$

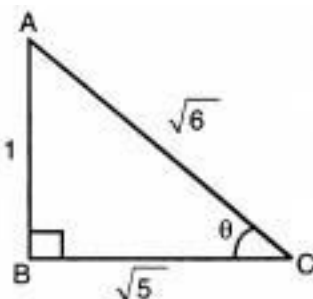
$$= \cot 85^\circ \tan 85^\circ \cot 55^\circ \tan 55^\circ \cdot \sqrt{3}$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

$$\therefore \text{Given Expression} = 1 - 1 - 1 + \sqrt{3}$$

Therefore, $\sqrt{3} - 1$ is the answer.

19. Given, $\tan \theta = \frac{P}{B} = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$



$$\text{In } \triangle ABC, AC^2 = AB^2 + BC^2 = 1 + 5 = 6$$

$$\therefore AC = \sqrt{6}$$

i. $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$

$$\begin{aligned}
&= \frac{(1+\cot^2 \theta) - (1+\tan^2 \theta)}{(1+\cot^2 \theta) + (1+\tan^2 \theta)} \\
&= \frac{\cot^2 \theta - \tan^2 \theta}{2 + \cot^2 \theta + \tan^2 \theta} \\
&= \frac{(\sqrt{5})^2 - \left(\frac{1}{\sqrt{5}}\right)^2}{2 + (\sqrt{5})^2 + \left(\frac{1}{\sqrt{5}}\right)^2} \\
&= \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} \\
&= \frac{25 - 1}{35 + 1} \\
&= \frac{24}{36} = \frac{2}{3}
\end{aligned}$$

ii. LHS = $\sin^2 \theta + \cos^2 \theta$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2 \\
&= \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1 = \text{R.H.S}
\end{aligned}$$

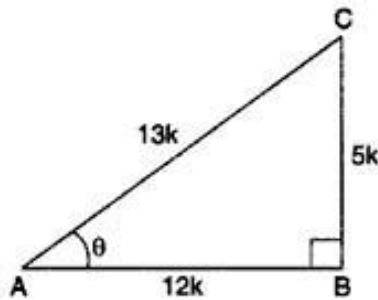
20. Let us draw a right triangle ABC in which $\angle BAC = \theta$

$\sec \theta = \frac{13}{12}, \dots$, Given

$$\Rightarrow \frac{AC}{AB} = \frac{13}{12} \Rightarrow \frac{AC}{13} = \frac{AB}{12} = k(\text{say})$$

where k is a positive number.

$$\Rightarrow AC = 13k, AB = 12k$$



By using the Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (13k)^2 = (12k)^2 + BC^2$$

$$\Rightarrow 169k^2 = 144k^2 + BC^2$$

$$\Rightarrow BC^2 = 169k^2 - 144k^2$$

$$\Rightarrow BC^2 = 25k^2 \Rightarrow BC = \sqrt{25k^2}$$

$$\Rightarrow BC = 5k$$

$$\text{Therefore, } \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\begin{aligned}
 &\left. \begin{aligned} \cos \theta &= \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13} \\ \cos \theta &= \frac{1}{\sec \theta} = \frac{12}{13} \end{aligned} \right\} \\
 &\left. \begin{aligned} \tan \theta &= \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12} \end{aligned} \right\} \\
 &\left. \begin{aligned} \operatorname{cosec} \theta &= \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5} \\ \operatorname{cosec} \theta &= \frac{1}{\sin \theta} = \frac{13}{5} \end{aligned} \right\} \\
 &\cot \theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5} \text{ or } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}
 \end{aligned}$$

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