CBSE Test Paper 04

Chapter 7 Coordinate Geometry

1. The centroid of a triangle whose vertices are (3, -7), (-8, 6) and (5, 10) is **(1)**
    a. (0, 3)
    b. (1, 3)
    c. (3, 3)
    d. (0, 9)
2. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then AP is equal to **(1)**
    a. $AP = \frac{1}{4}AB$
    b. $AP = \frac{1}{2}AB$
    c. $AP = \frac{1}{3}AB$
    d. AP = PB
3. If the points (x, y), (1, 2) and (7, 0) are collinear, then the relation between ‘x’ and ‘y’ is given by **(1)**
    a. 3x – y – 7 = 0
    b. 3x + y + 7 = 0
    c. x + 3y – 7 = 0
    d. x – 3y + 7 = 0
4. The points A(1, 2), B(5, 4), C(3, 8) and D(– 1, 6) are the vertices of a **(1)**
    a. Rectangle
    b. Rhombus
    c. Square
    d. Parallelogram
5. The triangle whose vertices are (– 3, 0), (1, – 3) and (4, 1) is____________triangle. **(1)**
    a. Obtuse triangle
    b. equilateral
    c. right angled isosceles
    d. scalene
6. If 18, a ,b ,- 3 are in A.P., then find a + b. **(1)**

7. Find the radius of the circle whose end points of diameter are (24,1) and (2,23)] **(1)**

8. Find the distance between the following pairs of points: (2, 3), (4,1) **(1)**

9. Find the coordinates of the point on y-axis which is nearest to the point (- 2, 5). **(1)**

10. What is the distance between the points A(c,0) and B(0, - c)? **(1)**

11. Use distance formula to show that the points A (- 2,3), B (1, 2) and C (7,0) are collinear. **(2)**

12. If the mid-point of the line joining (3,4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0 find the value of k. **(2)**

13. Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10). **(2)**

14. If (5,2), (- 3,4) and (x, y) are collinear, show that x + 4y - 13 = 0. **(3)**

15. If the point C(-1, 2) divides the line segment AB in the ratio 3 : 4, where the coordinates of A are (2, 5), find the coordinates of B. **(3)**

16. The three vertices of a parallelogram ABCD taken in order are A (-1, 0), B(3, 1) and C(2, 2). Find the height of a parallelogram with AD as its base. **(3)**

17. Find the ratio in which the line segment joining the points A(3, - 3) and B(- 2,7) is divided by the x-axis. Also, find the coordinates of the point of division. **(3)**

18. Find the point on the x-axis which is equidistant from (2,-5) and (-2,9) **(4)**

19. The points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of $\triangle$ABC.

i. The median from A meets BC at D. Find the coordinates of the point D.

ii. Find the coordinates of the point P on AD such that AP : PD = 2:1.

iii. Find the points of coordinates Q and R on medians BE and CP respectively such that BQ : QE = 2 :1 and CR : RP = 2 :1.

iv. What are the coordinates of the centroid of the triangle ABC? **(4)**

20. Find the coordinates of the points Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A(–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B. **(4)**

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Solution

1. a. (0, 3)

   Explanation: Given: $(x_1, y_1) = (3, -7), (x_2, y_2) = (-8, 6)$ and $(x_3, y_3) = (5, 10)$

   Coordinates of Centroid of triangle = $x = \frac{x_1+x_2+x_3}{3}$ and $y = \frac{y_1+y_2+y_3}{3}$

   $\therefore x = \frac{3-8+5}{3} = \frac{8-8}{3} = 0$

   and $y = \frac{-7+6+10}{3} = \frac{9}{3} = 3$

   Therefore, the coordinates of centroid of triangle are (0, 3).

2. b. $AP = \frac{1}{2}AB$

   Explanation: AP = $\sqrt{(2-4)^2 + (1-2)^2}$

   = $\sqrt{4+1}$ = $\sqrt{5}$ = units

   AB = $\sqrt{(8-4)^2 + (4-2)^2}$

   = $\sqrt{16+4}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

   Here AB = 2 $\times$ AP

   $\therefore$ AP = $\frac{1}{2}$ AB

3. c. x + 3y – 7 = 0

   Explanation: Given: $(x_1, y_1) = (x, y), (x_2, y_2) = (1, 2)$ and $(x_3, y_3) = (7, 0)$ and these are collinear

   $\therefore \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

   $\Rightarrow \frac{1}{2}|x(2-0) + 1(0-y) + 7(y-2)| = 0$

   $\Rightarrow \frac{1}{2}|2x - y + 7y - 14| = 0$

   $\Rightarrow 2x + 6y - 14 = 0 \Rightarrow x + 3y - 7 = 0$

4. c. Square

   Explanation: Given: The points A(1, 2), B(5, 4), C(3, 8) and D(– 1, 6)

   $\therefore$ AB = $\sqrt{(5-1)^2 + (4-2)^2}$ = $\sqrt{16+4}$ = $2\sqrt{5}$ units

   BC = $\sqrt{(3-5)^2 + (8-4)^2}$ = $\sqrt{4+16}$ = $2\sqrt{5}$ units

   CD = $\sqrt{(-1-3)^2 + (6-8)^2}$ = $\sqrt{16+4}$ = $2\sqrt{5}$ units

AD = $\sqrt{(-1-1)^2+(6-2)^2}$ = $\sqrt{4+16}$ = $2\sqrt{5}$ units

Therefore the 4 sides AB , BC, CD and DA are equal

and the diagonal AC = $\sqrt{(3-1)^2+(8-2)^2}$ = $\sqrt{4+36}$ = $2\sqrt{10}$ units

and BD = $\sqrt{(-1-5)^2+(6-4)^2}$ = $\sqrt{36+4}$ = $2\sqrt{10}$ units

Therefore diagonals AC and BD are equal

Since, all 4 sides are equal and both diagonals are also equal.

Therefore, the given quadrilateral is a square.

5. c. right angled isosceles

Explanation: Let A (-3, 0), B(1, -3) and C (4, 1) are the vertices of a triangle ABC.

$\therefore$ AB = $\sqrt{(1+3)^2+(-3-0)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units

BC = $\sqrt{(4-1)^2+(1+3)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ = 5 units

CA = $\sqrt{(-3-4)^2+(0-1)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ = $5\sqrt{2}$ units

Now, check if $AC^2 = AB^2 + BC^2$

$\Rightarrow (5\sqrt{2})^2 = (5)^2 + (5)^2$

$\Rightarrow 50 = 50$

Therefore, ΔABC is a right-angled triangle.and also AB = BC = 5 units

Therefore triangle ABC is a right-angled isosceles triangle

6. Since 18, a, b, and - 3 are in A.P., Then

a - 18 = - 3 - b

or, a + b = - 3 + 18

or, a + b = 15

7. (x_1, y_1) = (24,1) and (x_2, y_2) = (2,23)

Diameter of Circle = $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-24)^2+(23-1)^2}$

$= \sqrt{(-22)^2+(22)^2} = \sqrt{(22)^2(1+1)}$

$= 22\sqrt{2}$ units

Therefore , Radius of circle, $r = \frac{d}{2} = \frac{22\sqrt{2}}{2} = 11\sqrt{2}$ units

8. Applying Distance Formula to find distance between points (2, 3) and (4,1), we get

d = $\sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$

9. The point on y-axis that is nearest to the point(-2,5) is (0,5).

10. AB = $\sqrt{(0-c)^2+(-c-0)^2}$

$= \sqrt{c^2+c^2}$

$= \sqrt{2c^2}$

$= \sqrt{2}c$

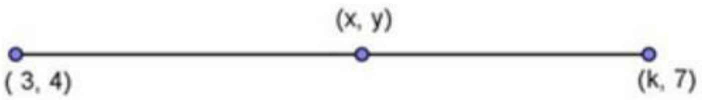
11. AB = $\sqrt{(1+2)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(7-1)^2+(0-2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

AC = $\sqrt{(7+2)^2+(0-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

Since AB + AC = $= \sqrt{10}+2\sqrt{10}$ = (1 + 2)$\sqrt{10} = 3\sqrt{10}$ = AC

Hence, the points A, B and C are colinear.

12. 

Since, (x, y) is the mid-point

$x = \frac{3+k}{2}, y = \frac{4+7}{2} = \frac{11}{2}$

Again,

2x + 2y + 1 =0

$\Rightarrow \quad 2 \times \frac{(3+k)}{2} + 2 \times \frac{11}{2} + 1 = 0$

$\Rightarrow$ 3 + k + 11 + 1 = 0

$\Rightarrow$ 3 + k + 12 = 0

$\Rightarrow$ k + 15 = 0

$\Rightarrow$ k = -15

13. Let A(5, 7), B(3, 9), C(8, 6) and D(0, 10) be the given points. Therefore,by mid-point formula,we have,

Coordinates of the mid-point of AB are $\left(\frac{5+3}{2}, \frac{7+9}{2}\right) = (4, 8)$

Coordinates of the mid-point of CD are $\left(\frac{8+0}{2}, \frac{6+10}{2}\right) = (4, 8)$

Therefore, the mid-point of AB = mid point of CD.

14. Since the points are collinear

The area of triangle = 0

$\therefore$Area of triangle =0

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$ =0

$\frac{1}{2}[5(4-y)+(-3)(y-2)+x(2-4)]=0$
= $\frac{1}{2}[20-5y-3y+6+(-2x)]=0$
$\frac{1}{2}[-2x-8y+26]=0$
x + 4y - 13 = 0

Hence Proved.

15. Given: A (2,5) and C(-1,2)

Let the coordinate of the point B be (a,b).

it is given that AC : BC = 3:4

Then, by section formula , coordinates of C are given by

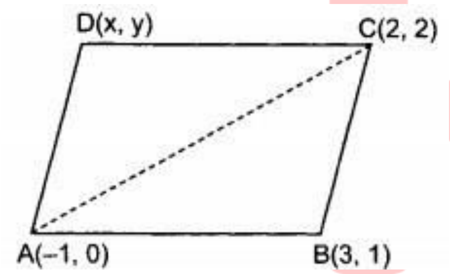
$-1=\frac{3\times a+4\times 2}{3+4}$ and $2=\frac{3\times b+4\times 5}{3+4}$

∴ - 7= 3a+8 and 14 = 3b+20

∴ 3a = -15 and 3b = -6

∴ a = - 5 and b = - 2

Hence, coordinates of B are (-5,-2).

16. 

Area of $\triangle$ ABC

$=\frac{1}{2}$ [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

$=\frac{1}{2}$ [-1(1 - 2) + 3(2 - 0) + 2(0 - 1)]

$=\frac{1}{2}$ [1 + 6 - 2] = $\frac{5}{2}$ sq. units

Area of $\|\text{gm}$ = 2 $\times$ area of $\triangle$ABC

$\Rightarrow$ Area of $\|\text{gm}$ = 2 $\times$ $\frac{5}{2}$ = 5 sq. units

Let coordinates of D are (x, y)

Mid point of AC $=\left(\frac{-1+2}{2}, \frac{0+2}{2}\right)=\left(\frac{1}{2}, 1\right)$

Mid-point of BD $=\left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

∴ Diagonals of a $\|\text{gm}$ bisect each other

∴ Mid-point of BD = Mid-point of AC

$\Rightarrow\left(\frac{3+x}{2}, \frac{1+y}{2}\right)=\left(\frac{1}{2}, 1\right)$

$\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{1+y}{2} = 1$

$\Rightarrow$ x = - 2

$\Rightarrow$ y = 1

Now AD = $\sqrt{(-1+2)^2+(0+1)^2} = \sqrt{2}$

Also area of $||\text{gm}$ = base $\times$ height

$\Rightarrow$ AD $\times$ height = 5

$\Rightarrow \sqrt{2} \times$ height = 5

$\Rightarrow$ height$= \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}$ units.

17. According to the question,

A (3,-3) and B (- 2, 7)

On the x-axis, the y-coordinate is zero

So, let the point be (x, 0)

Let the ratio be k : 1

$(x,0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$

$\Rightarrow \frac{7k-3}{k+1} = 0$

$\Rightarrow$7k - 3 = 0

$\Rightarrow k = \frac{3}{7}$

$\therefore$ The line is divided in the ratio of 3:7

$\Rightarrow \frac{-2k+3}{k+1} = x$

$\Rightarrow \frac{-2\times\frac{3}{7}+3}{\frac{3}{7}+1} = x$

$\Rightarrow \frac{-\frac{6}{7}+3}{\frac{10}{7}} = x$

$\Rightarrow \frac{\frac{-6+21}{7}}{\frac{10}{7}} = x$

$\Rightarrow \frac{\frac{15}{7}}{\frac{10}{7}} = x$

$\Rightarrow x = \frac{3}{2}$

Coordinate of y is 0 at x-axis,

$\therefore$ The coordinates of the point at which x axis divides AB is $\left(\frac{3}{2}, 0\right)$ in ratio of 3:7.

18. Let the point of x-axis be P(x, 0)

Given A(2, -5) and B(-2, 9) are equidistant from P

That is PA = PB

Hence $PA^2 = PB^2$ → (1)

Distance between two points is $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

PA = $\sqrt{[(2-x)^2+(-5-0)^2]}$

$PA^2 = 4 - 4x + x^2 + 25$

$= x^2 - 4x + 29$

Similarly, $PB^2 = x^2 + 4x + 85$

Equation (1) becomes

$x^2 - 4x + 29 = x^2 + 4x + 85$

$-8x = 56$

$x = -7$

Hence the point on x-axis is (-7, 0)

19. $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the three vertices of ΔABC.

i. Median from A meets BC at D.

∴ D is the mid-point of BC.

∴ Coordinates of $D = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

ii. P divides AD in the ratio 2 : 1.

∴ Coordinates of P = $\left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$

$= \left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{3}\right)$

iii. Median from B meet AC at E and median from C meets AB at F.

∴ E is the mid-point of AC and F is the mid-point of AB.

∴ Coordinates of $E = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$ and

Coordinates of $F = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Q divides BE in the ratio 2 : 1.

∴ Coordinates of Q = $\left(\frac{2\times\frac{x_1+x_3}{2}+1\times x_2}{2+1}, \frac{2\times\frac{y_1+y_3}{2}+1\times y_2}{2+1}\right)$

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

R divides CF in the ratio 2 : 1.

$\therefore$ Coordinates of R = $\left(\frac{2\times\frac{x_1+x_2}{2}+1\times x_3}{2+1}, \frac{2\times\frac{y_1+y_2}{2}+1\times y_3}{2+1}\right)$

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

iv. Coordinates of centroid of

$\Delta ABC = \left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{3}\right)$

20. Let Q(x, 0) be a point on x–axis which lies on the perpendicular bisector of AB.

Therefore, QA = QB

$\Rightarrow QA^2 = QB^2$

$\Rightarrow (-5 - x)^2 + (-2 - 0)^2 = (4 - x)^2 + (-2 - 0)^2$

$\Rightarrow (x + 5)^2 + (-2)^2 = (4 - x)^2 + (-2)^2$

$\Rightarrow x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$

$\Rightarrow 10x + 8x = 16 - 25$

$\Rightarrow 18x = -9$

$\Rightarrow x = \frac{-9}{18} = \frac{-1}{2}$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now, $QA^2 = \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2$

$= \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$

$\Rightarrow QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow QA = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

Now, $QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$

$\Rightarrow QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

and $AB = \sqrt{(4+5)^2 + [-2-(-2)]^2} = \sqrt{(9)^2}$ = 9 units

$\Rightarrow$ AB = 9 units

As QA = QB

So, Δ QAB is an isosceles Δ.