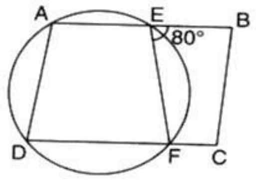
CBSE Test Paper 02

CH-10 Circles

1. Two circle are congruent if they have equal
    a. radius
    b. diameter
    c. secant
    d. chord
2. ABCD is a parallelogram. A circle passes through A and D and cuts AB at E and DC at F. If $\angle BEF = 80^o$, then $\angle ABC$ is equal to



    a. 80^o
    b. 75^o
    c. 120^o
    d. 100^o
3. Number of circles that can be drawn through three non-collinear points is
    a. 2
    b. 1
    c. 0
    d. 3
4. If P and Q are any two Points on a circle then PQ is called a

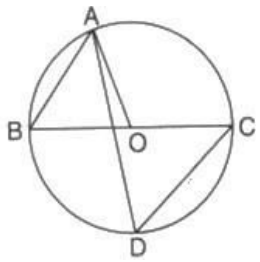
a. radius

b. diameter

c. secant

d. chord

5. BC is a diameter of the circle and $\angle BAO = 60^o$. Then $\angle ADC$ is equal to



a. 90^o

b. 45^o

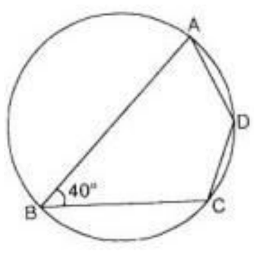
c. 60^o

d. 30^o

6. Fill in the blanks:

The line drawn through the centre of a circle to bisect a chord is_________to the chord.

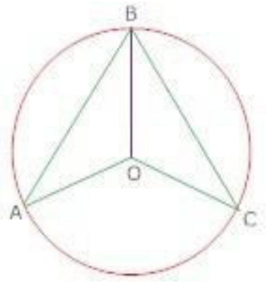
7. In the figure, $\angle ABC = 40^o$ then find $\angle ADC$



8. Prove that of all the chords of a circle through a given point within it, the least is one which is bisected at that point.

9. In figure, $\angle PQR = 100^\circ$, where P, Q, R are points on a circle with centre O. Find $\angle OPR$.

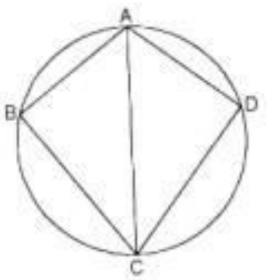
10. AB and CB are two chords of circle. Prove that BO bisects $\angle$ABC



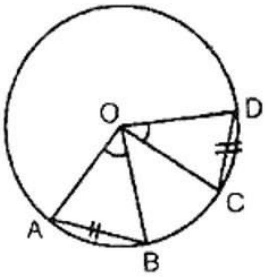
11. In the figure,

i. $\angle$BAC = 70^{o} and $\angle$DAC = 40^{o}, then find $\angle$BCD

ii. $\angle$BAC = 60^{o} and $\angle$BCA = 60^{o}, then find $\angle$ADC



12. Recall two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.



13. AB and BC are two chords of a circle whose centre is O such that $\angle ABO = \angle CBO$. Prove that AB = CB.

14. In figure, AB and AC are two equal chords of a circle whose centre of O. If $OD \perp AB$ and $OE \perp AC$, prove that ADE is an isosceles triangle.

15. If two equal chords of a circle intersect within a circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

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CH-10 Circles

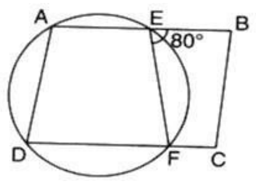
Solution

1. (a) radius

   Explanation: Equal radius would generate two same circles that are exact copy of each other, hence making them congruent.

2. (a) 80^o

   Explanation:



$\angle AEF + 80^0 = 180^0$ (Linear Pair)
$\angle AEF = 100^0$
$\angle ADF + \angle AEF = 180^0$ (Opposite angles of a cyclic quadrilateral)
$\angle ADF = 180^0 - 100^0 = 80^0$
$\angle ADF = \angle ABC = 80^0$(Opposite angles of a parallelogram)

3. (b) 1

   Explanation:

   Only 1 circle can be drawn from three non-collinear points.

4. (d) chord

   Explanation: A chord is a line formed by any two points on a circle.

5. (c) 60^o

   Explanation:

In triangle ABO,
OA = OB, $\angle A = \angle B = 60^0$
Now, $\angle ABO = \angle ADC = 60^0$

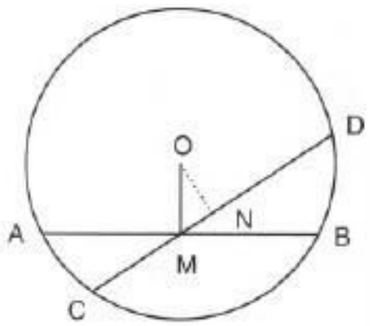
6. perpendicular

7. $\therefore$ $\angle$ABC + $\angle$ADC = 180° ($\because$ Opposite angles of a cyclic quadrilateral are supplementary)

$\angle$ADC = 180^o - $\angle$ABC

= 180^o - 40^o = 140^o

8. Let C(O, r) be a circle and let M be a point within it. Let AB be a chord whose mid-point is M. Let CD be another chord through M. We have to prove that AB < CD.



Join OM and draw ON $\perp$ CD.
In right triangle ONM, OM is the hypotenuse.
$\therefore$ OM > ON
$\Rightarrow$ Chord CD is nearer to O in comparison to AB
$\Rightarrow$ CD > AB [$\because$ Of any two chords of a circle, the one which is nearer to the centre is larger]
$\Rightarrow$ AB < CD

9. Since the angle subtented by an arc of a circle at its centre is twice the angle subtented by the same arc at a point on the circumference.
Therefore, $\angle$ROP = 2 $\angle$PQR
$\Rightarrow\angle$ ROP = $2 \times 100^\circ$ = 200°
Now $m\widehat{PR} + m\widehat{RP}$ = 360^o $\Rightarrow$ $\angle$ POR + $\angle$ ROP = 360° $\Rightarrow$ $\angle$ POR + 200° = 360° $\Rightarrow$ $\angle$ POR = $360^\circ - 200^\circ$ = 160° .....(i)
Now $\triangle$OPR is an isosceles triangle.
$\therefore$ OP = OR [Radii of the circle]

$\Rightarrow \angle OPR = \angle ORP$ [Angles opposite to equal sides are equal] .....(ii)

Now in isosceles triangle OPR,

$\angle OPR + \angle ORP + \angle POR = 180^\circ$ [The sum of the all angles of a traingle is 180°]

$\Rightarrow \angle OPR + \angle ORP + 160^\circ = 180^\circ$

$\Rightarrow 2\angle OPR = 180^\circ - 160^\circ$ [Using (i) & (ii)]

$\Rightarrow 2\angle OPR = 20^\circ$

$\Rightarrow \angle OPR = 10^\circ$

10. Join OA and OC

In $\triangle OAB$ And $\triangle OCB$

OA = OC (radii of circle)

OB = OB (common)

AB = BC (given)

$\triangle OAB \cong \triangle OCB$ (by SSS)

Therefore, $\angle ABO = \angle CBO$ [By CPCT]

Hence, BO bisects $\angle ABC$

11. i. $\angle BCD = 180^o - \angle BAD$ ($\because$ Opposite angles of a cyclic quadrilateral are supplementary)

$= 180^o - (\angle BAC + \angle DAC)$

$= 180^o - (70^o + 40^o) = 70^o$

ii. $\angle CBA = 180^o - (\angle BAC + \angle BCA)$ ($\because$Opposite angles of a cyclic quadrilateral are supplementary)

$= 180^o - (60^o + 20^o) = 100^o$

$\angle ADC = 180o - \angle CBA$

$= 180^o - 100^o = 80^o$

12. I Part: Two circles are said to be congruent if and only if one of them can be superposed on the other so as to cover it exactly.

Let C (O) and C (O') be two circles. Let us imagine that the circle C (O') is superposed on C (O) so that O' coincide with O. Then it can easily be seen that C (O') will cover C (O) completely.

Hence we can say that two circles are congruent, if and only if they have equal radii.

II Part: Given: In a circle (O), AB and CD are two equal chords, subtend $\angle$AOB and $\angle$COB at the centre.

To Prove: $\angle$AOB = $\angle$COD

Proof: In $\triangle$ AOB and $\triangle$ COD,

AB = CD [Given]

AO = CO [Radii of the same circle]

BO = DO [Radii of the same circle]

$\triangle$AOB $\cong$ $\triangle$COD [By SSS congruency]

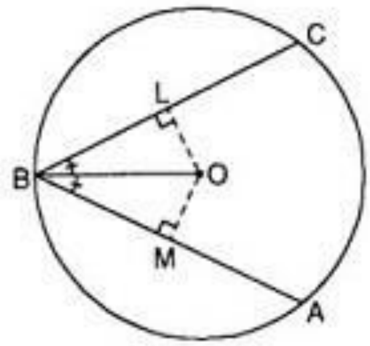
$\Rightarrow$ $\angle$AOB = $\angle$COD [By CPCT]

Hence Proved.

13. AB and BC are two chords of a circle whose centre is O such that $\angle ABO = \angle CBO$.

To prove : AB = CB

Construction : Draw $\mathrm{OL} \perp \mathrm{BC}$ and $OM \perp BA$.



Proof: In $\triangle OBL$ and $\triangle OBM$

$\angle OBL = \angle OBM$ |Given

$\angle OLB = \angle OMB$ |Each = 90^{o} (by const.)

OB = OB |Common

$\therefore \Delta OBL \cong \triangle OBM$ |AAS

BL = BM |c.p.c.t

$\Rightarrow$ 2BL = 2BM

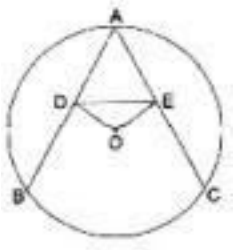
$\Rightarrow$ BC = BA

[L and M are the mid-points of BC and BA respectively since the perpendicular draw from the centre of a circle to a chord bisects the chord]

$\Rightarrow$ BA = BC

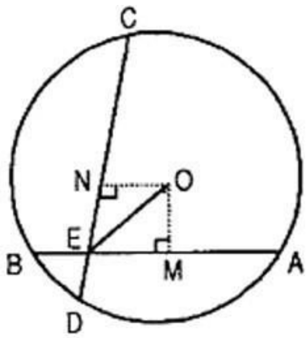
$\Rightarrow$ AB = CB

14. Given: In figure, AB and AC are two equal chords of a circle whose centre is O.
$OD \perp AB$ and $OE \perp AC$
To prove: ADE is an isosceles triangle



Proof: AB = AC
OD = OE | $\because$ Equal chords are equidistant from the centre
$\therefore$ In $\triangle ODE$
$\angle ODE = \angle OED$ |Angle opposite to equal sides
$\Rightarrow 90^\circ - \angle ODE = 90^\circ - \angle OED$
$\Rightarrow \angle ODA - \angle ODE = \angle OEA - \angle OED$
$\Rightarrow \angle ADE = \angle AED$
$\therefore AD = AE$ |Sides opposite to equal angles
$\therefore \triangle ADE$ is an isosceles triangle.

15. Given: Let AB and CD are two equal chords of a circle of centers O intersecting each other at point E within the circle.
To prove: (a) AE = CE (b) BE = DE
Construction: Draw OM $\perp$ AB, ON $\perp$ CD. Also join OE.



Proof: In right triangles OME and ONE,
$\angle$OME = $\angle$ONE = 90°
OM = ON
[Equal chords are equidistance from the centre]
OE = OE [Common]
$\therefore \triangle$OME $\cong \triangle$ONE [RHS rule of congruency]

$\therefore$ ME = NE [By CPCT] ...(i)

Now, O is the centre of circle and OM $\perp$ AB

$\therefore$ AM = $\frac{1}{2}$ AB [Perpendicular from the centre bisects the chord] ...(ii)

Similarly, NC = $\frac{1}{2}$ CD ...(iii)

But AB = CD [Given]

From eq. (ii) and (iii), AM = NC ...(iv)

Also MB = DN ...(v)

Adding (i) and (iv), we get,

AM + ME = NC + NE

$\Rightarrow$ AE = CE [Proved part (a)]

Now AB = CD [Given] ...(v)

AE = CE [Proved] ...(vi)

Subtracting eq. (vi) from eq. (v), we have

$\Rightarrow$ AB - AE = CD - CE

$\Rightarrow$ BE = DE [Proved part (b)]