CBSE Test Paper 02

Chapter 05 Arithmetic Progression

1. Find the sum of first'n' terms of an A.P.? **(1)**
   a. 7n – 8
   b. S = $\frac{n}{2}$[2a + (n - 1)d]
   c. 2n + 3
   d. $n^2 + 2$
2. The sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$ is **(1)**
   a. 680
   b. 672
   c. 640
   d. 600
3. The 17th term of an AP exceeds its 10th term by 7, then the common difference is **(1)**
   a. -1
   b. 1
   c. 2
   d. 0
4. The first term of an AP is 5, the last term is 45 and the sum is 400. The number of terms is **(1)**
   a. 16
   b. 20
   c. 17
   d. 18
5. If a, b and c are in A. P., then the value of $\frac{a-b}{b-c}$ is **(1)**
   a. $\frac{a}{b}$
   b. 1
   c. $\frac{c}{a}$
   d. $\frac{b}{c}$
6. Find the common difference of the A.P. and write the next two terms of A.P. 119,136,153,170,..... **(1)**

7. If 2x, x + 10, 3x + 2 are in A.P., find the value of x. **(1)**

8. Find 7^{th} term from the end of the AP : 7, 10 , 13, ...,184. **(1)**

9. Find k, if the given value of x is the k^{th} term of the given AP 25,50, 75,100, .., x = 1000. **(1)**

10. Find the 11th term from the end of the AP 10,7,4, ... ,-62. **(1)**

11. Find the common difference d and write three more terms. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ **(2)**

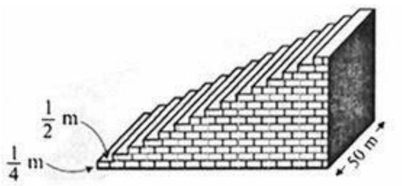
12. The house of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the following it. Find this value of x. **(2)**

13. Divide 24 in three parts such that they are in AP and their product is 440. **(2)**

14. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8th term, we get 6. **(3)**

15. Find the second term and n^{th} term of an A.P. whose 6^{th} term is 12 and the 8^{th} term is 22. **(3)**

16. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace. **(3)**



17. If 9^{th} term of an A.P. is zero, prove that its 29^{th} term is double the 19^{th} term. **(3)**

18. If the sum of Rs 1890 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 50 less than its preceding prize. Then find the value of each of the prizes. **(4)**

19. Let a sequence be defined by $a_1 = 1$, $a_2 = 1$ and, $a_n = a_{n-1} + a_{n-2}$ for all n > 2. Find $\frac{a_{n+1}}{a_n}$ for n = 1,2,3,4. **(4)**

20. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find. n and S_n, if a = 5, d = 3 and $a_n = 50$. **(4)**

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Solution

1. b. S = $\frac{n}{2}$ [2a + (n - 1)d]

   Explanation: let a = 1^{st} term, d = common difference,

   S_n = sum of 1st n terms of an AP

   then S_n = (a) + (a + d) + (a + 2d) + ... {a + (n - 3) d} + { a + (n - 2)d} + {a + (n - 1)d}

   .... (i)

   Now Rewrite S_n as follows

   S_n = {a +(n-1)d} + {a +(n-2)d }+ {a + (n - 3)d} .... (a + 3d) + (a + 2d) + (a + d) +

   a .... (ii)

   adding the terms i and ii vertically

   adding 1st term of both we get (a) + {a + (n-1)d} = 2a + (n-1)d

   adding 2nd term of both (a + d) + {a + (n - 2)d} = 2a + (n-1)d

   adding 3rd terms of both (a + 2d) + {a + (n-3)d} = 2a + (n-1)d

   since there are n terms in each of the equations i and ii , adding both the

   equations we get

   $2S_n$ = n{2a + (n-1)d}

   S_n = $\frac{n}{2}$ {2a + (n-1)d}.

2. b. 672

   Explanation: Given: $a_n = 3 + 2n$

   $\therefore a_1 = 3 + 2 \times 1 = 3 + 2 = 5$

   $a_2 = 3 + 2 \times 2 = 3 + 4 = 7$

   $\therefore d = a_2 - a_1 = 7 - 5 = 2$

   Now, $\mathrm{S}_n = \frac{n}{2}[2a + (n-1)d]$

   $\Rightarrow \mathrm{S}_{24} = \frac{24}{2}[2 \times 5 + (24-1)2]$

   $\Rightarrow \mathrm{S}_{24} = 12[10 + 23 \times 2]$ = 12 [10 + 46] = 672

3. b. 1

   Explanation: According to question,

   Given that the 17th term of an A.P exceeds its 10th term by 7 .

d = ?

$\Rightarrow$ a + 16d = a + 9d + 7

$\Rightarrow$16d - 9d = 7

$\Rightarrow$7d = 7

$\Rightarrow$ d = $\frac{7}{7}$ = 1

Therefore , common difference = 1.

4. a. 16

Explanation: Given: $a = 5, l = 45, \mathrm{S}_n = 400$

$\therefore \mathrm{S}_n = \frac{n}{2}(a + l)$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow 800 = n \times 50$

$\Rightarrow n = 16$

5. b. 1

Explanation: If a, b and c are in A.P.,

$b - a = c - b$

$-(a - b) = -(b - c)$

$a - b = b - c$

dividing both sides by $b - c$

$\frac{a-b}{b-c}$ = $\frac{b-c}{b-c}$

$\frac{a-b}{b-c}$ =1

6. Given A.P is

119, 136, 153, 170........

We know that common differnce is difference between any consecutive terms of an A.P.

So, common difference = 136 - 119 = 17

5th term = 170 + 17 = 187 (a_5 = a + 4d)

6th term = 187 + 17 = 204. (a_6 = a + 5d)

7. If 2x, x + 10, 3x + 2 are in A.P.,we have to find the value of x.

Since, 2x, x + 10, 3x + 2 are in A.P.therefore 2 (x + 10) = 2x + 3x + 2

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18 $\Rightarrow$ x = 6

8. Given, AP is 7, 10 , 13,... ,184.

   we have to find 7th term from the end

   reversing the AP , 184,.....,13,10,7.

   now, d = common difference = 7-10 = -3

   ∴ 7^{th} term from the beginning of AP $= a + (7-1)d = a + 6d$

   $= 184 + (6 \times (-3))$

   = 184 - 18 = 166

9. $a = 25,\ d = 50 - 25 = 25,\ \ x = 1000$

   A.T.Q. , $a_k = x$

   $\Rightarrow$ a$+(k-1)d = 1000$

   $\Rightarrow$ 25 + (k - 1)25 = 1000

   $\Rightarrow$ (k - 1)25 = 975 $\Rightarrow$ k - 1 = $\frac{975}{25}$

   $\Rightarrow$ k - 1 = 39 $\Rightarrow$ k = 40

10. We have

   a = 10, d = (7-10) = -3, l = -62 and n = 11.

   ∴11th term from the end = [l - (n -1) $\times$ d]

   = {-62 - (11 -1) $\times$ (-3)}

   = (-62 + 30) = -32.

   Hence, the 11th term from the end of the given AP is -32.

11. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

   $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$
   $a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$
   $a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

   i.e. a_{k+1} - a_k is the same every time.

   So, the given list of numbers forms an AP with the common difference d = $\sqrt{2}$.

   The next three terms are:

   $\sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$
   $5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$
   and $6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

12. The consecutive numbers on the houses of a row are 1, 2, 3, ..., 49

   Clearly this list of number forming an AP.

Here, a = 1

d = 2 - 1 = 1

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x-1}{2}[2a+(x-1-1)d] - \frac{x}{2}[2a+(x-1)d]$

$\because S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \frac{x-1}{2}[2(1)+(x-2)(1)] = \frac{49}{2}[2(1)+(48)(1)] - \frac{x}{2}[2(1)+(x-1)(1)]$

$\Rightarrow \frac{x-1}{2}[x] = 1225 - \frac{x(x+1)}{2}$

$\Rightarrow \frac{(x-1)(x)}{2} + \frac{x(x+1)}{2} = 1225$

$\Rightarrow \frac{x}{2}(x-1+x+1) = 1225$

$\Rightarrow x^2 = 1225$

$\Rightarrow x = \sqrt{1225} \Rightarrow x = 35$

Hence, the required value of x is 35.

13. Let the required numbers in A.P.are (a - d), a and (a + d).

Sum of these numbers = (a - d) + a + (a + d) = 3a

Product of these numbers = $(a - d) \times a \times (a + d) = a(a^2 - d^2)$

But given, sum = 24 and product = 440

$\therefore 3a = 24 \Rightarrow a = 8$

and $a(a^2 - d^2) = 8(64 - d^2) = 440$ [$\because a = 8$]

Or, $64 - d^2 = 55$

Or, $d^2 = 64 - 55$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

When a = 8 and d = 3

The required numbers are (5, 8, 11).

When a = 8 and d = -3

The required numbers are (11, 8, 5).

14. Let the first term be a and the common difference be d.

$a_n = a + (n - 1)d$

Here given, $a_3 = 9$

or, a + 2d = 9.....(i)

a_8 - a_5 = 6

or, (a + 7d) - (a + 4d) = 6

a + 7d - a - 4d = 6

or, 3d = 6

or, d = 2 .... (ii)

Substituting this value of d from (ii) in (i), we get

or, a + 2(2) = 9

or, a + 4 = 9

or a = 9 - 4

or, a = 5

a = 5 and d = 2

So, A.P. is 5,7,9,11,....

15. Given a_6 = 12

$\Rightarrow$ a + (6 - 1)d = 12

$\Rightarrow$ a + 5d = 12.............(i)

and, a_8 = 22

$\Rightarrow$ a + (8 - 1)d = 22

$\Rightarrow$ a + 7d = 22.............(ii)

Subtracting equation (i) from (ii), we get

(a + 7d) - (a + 5d) = 22 - 12

$\Rightarrow$ a + 7d - a - 5d = 10

$\Rightarrow$ 2d - 10

$\Rightarrow d = \frac{10}{2} = 5$

Using value of d in equation (i), we get

a + 5 $\times$ 5 = 12

$\Rightarrow$ a = 12 - 25 = -13

Second term(a_2) = a + (2 - 1)d

= -13 + 1(5)

= -13 + 5 = -8

nth term(a_n) = a + (n - 1)a

= -13 + (n - 1)(5)

= 5n - 18

16. Volume of concrete required to build the first step, second step, third step, ... (in m^2) are

$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50$

$\Rightarrow \frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots\ldots$

$\therefore$ Total volume of concrete required = $\frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \ldots.$

$= \frac{50}{8}[1 + 2 + 3 + \ldots\ldots.]$

$S_n = \frac{n}{2}[(2a + (n - 1)d]$

$S_{15} = \frac{50}{8} \times \frac{15}{2}[2 \times 1 + (15 - 1) \times 1]\ [\because n = 15]$

$= \frac{50}{8} \times \frac{15}{2} \times 16 = 750\ m^3$

17. We have,

$a_9 = 0$

$\Rightarrow a + (9 - 1)d = 0$

$\Rightarrow a + 8d = 0$

$\Rightarrow a = -8d$

To prove: $a_{29} = 2a_{19}$

Proof: LHS = a_{29}

$= a + (29 - 1)d$

$= a + 28d$

$= -8d + 28d = 20d$

RHS = $2a_{19}$

$= 2\ a + (19 - 1)d]$

$= 2[-8d + 18d]$

$= 2 \times 10d$

$= 20d$

$\therefore$ LHS = RHS

Hence, 29^{th} term is double the 19^{th} term.

18. Let 1^{st} prize be Rs x.

The series in A.P. is x, x - 50, x - 100, x - 150,........

Where a = x, d = - 50, S_n = 1890, n = 7.

As we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow \frac{7}{2}[2x + (6)(-50)] = 1890$
$\Rightarrow \frac{7}{2}[2x - 300] = 1890$
$\Rightarrow 2x - 300 = 1890(2/7)$
$\Rightarrow 2x = 540 + 300$
$\Rightarrow x = \frac{840}{2} = 420$
The prizes are: Rs 420, Rs 370, Rs 320, Rs 270, Rs 220, Rs 170, Rs 120.

19. Given $a_1 = 1$ and $a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

So $a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

Now putting n = 1,2,3 and 4 in a_{n+1}/a_n we get
$\frac{a_2}{a_1} = \frac{1}{1} = 1$
$\frac{a_3}{a_2} = \frac{2}{1} = 2$
$\frac{a_4}{a_3} = \frac{3}{2} = 1.5$
$\frac{a_5}{a_4} = \frac{5}{3} = 1.67$

20. Given,

First term(a) = 5

Common difference(d) = 3

and, nth term (a_n) = 50

$\Rightarrow a + (n - 1)d = 50$

$\Rightarrow 5 + (n - 1)(3) = 50$

$\Rightarrow 5 + 3n - 3 = 50$

$\Rightarrow 3n = 50 - 5 + 3$

$\Rightarrow 3n = 48$

$\Rightarrow \quad n = \frac{48}{3} = 16$
Therefore, $S_n = \frac{n}{2}[a + a_n]$
$= \frac{16}{2}[5 + 50]$
$= 8 \times 55 = 440$