CBSE Test Paper 04

Chapter 9 Some Applications of Trigonometry

1. If the angle of depression of a car from a 100 m high tower is 45^{o}, then the distance of the car from the tower is **(1)**

a. 100 m

b. 200 m

c. $100\sqrt{3}$ m

d. $200\sqrt{3}$ m

2. A kite is flying at a height of 200 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 45^{o}. The length of the string, assuming that there is no slack in the string is **(1)**

a. 100 m

b. 200 m

c. $200\sqrt{2}$ m

d. $100\sqrt{2}$ m

3. The angles of elevation of the top of a tower from two points on the ground at distances 8 m and 18 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is **(1)**

a. 12 m

b. 18 m

c. 8 m

d. 16 m

4. The______________is the line drawn from the eye of an observer to the point in the object viewed by the observer. **(1)**

a. Horizontal line

b. line of sight

c. None of these

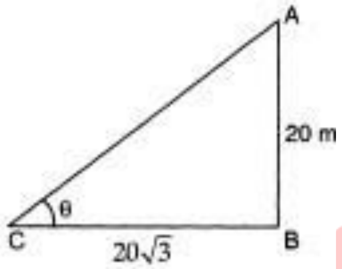
d. Vertical line

5. A pole 10 m high cast a shadow 10 m long on the ground, then the sun’s elevation is **(1)**

a. 60°
b. 15°
c. 45°
d. 30°

6. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^o with the wall, then find the height of the wall. **(1)**

7. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long Find the Sun's altitude. **(1)**



8. If the angles of elevation of the top of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are respectively 30^o and 60^p, then find the height of the tower. **(1)**

9. If the height and length of shadow of a man are the same, then find the angle of elevation of the Sun. **(1)**

10. An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of the tower from her eyes is 45°. What is the height of the tower? **(1)**

11. A vertical tower of height 90 m stands on the ground. The angle of elevation of the top of the tower as observed from a point on the ground is 60^o. Find the distance of the point from the foot of the tower. **(2)**

12. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of pole observed from a point A on the ground is 60^o and the angle of depression of the point A from the top of the tower is 45^o. Find the height of the tower. (Take $\sqrt{3} = 1.732$) **(2)**

13. A pole casts a shadow of length $2\sqrt{3}\ m$ on the ground, When the Sun's elevation is

60^o. Find the height of the pole. **(2)**

14. The pilot of an aircraft flying horizontally at a speed of 1200 km/hr. observes that the angle of depression of a point on the ground changes from 30° to 45° in 15 seconds. Find the height at which the aircraft is flying. **(3)**

15. A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β. Prove that the height of the top from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$. **(3)**

16. At the foot of a mountain the elevation of its summit is 45°. After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain. **(3)**

17. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships. **(3)**

18. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects. **(4)**

19. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it. **(4)**

20. A man standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. **(4)**

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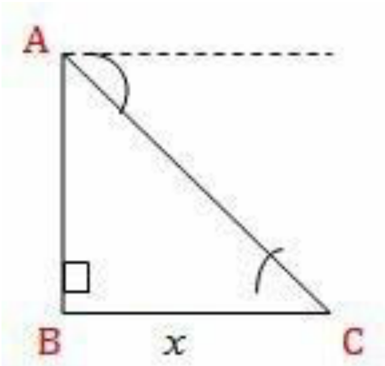
Solution

1. a. 100 m

Explanation:



Let the distance of the car from the tower be x meters.

$\therefore \tan 45^\circ = \frac{AC}{BC}$

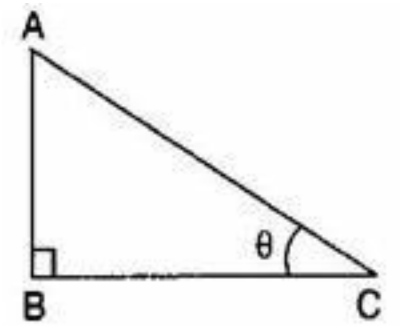
$\Rightarrow 1 = \frac{100}{x}$ m

$\Rightarrow x = 100$ m

Therefore, the distance of the car from the tower is 100 m.

2. c. $200\sqrt{2}$ m

Explanation:



Here, in triangle ABC, Height of the slide = AB = 200 m

Angle of elevation = $\theta = 45^\circ$

To find: Length of the string = AC

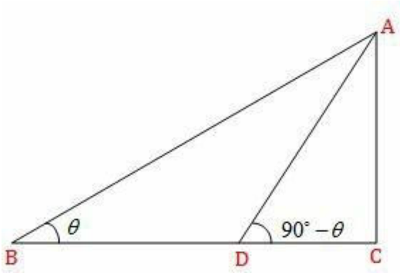
$\therefore \sin 45^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{200}{AC}$

$\Rightarrow AC = 200\sqrt{2}$ m

3. a. 12 m

Explanation:



In triangle ABC, $\tan\theta = \frac{h}{18}$ ....(i)

And in triangle ADC, $\tan(90^\circ - \theta) = \frac{h}{8}h$

$\Rightarrow \cot\theta = \frac{h}{8}$ ....(ii)

Multiplying eq. (i) and (ii), we get

$\tan\theta . cot\theta = \frac{h}{18} \times \frac{h}{8}$

$\Rightarrow 1 = \frac{h^2}{144}$

$\Rightarrow h^2 = 144$

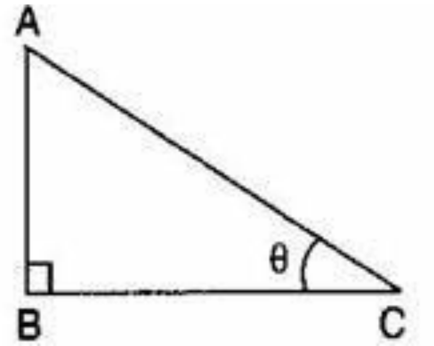
$\Rightarrow h = 12$ m

4. b. line of sight

Explanation: The line of sight is the imaginary line drawn from the eye of an observer to the point in the object viewed by the observer. The angle between the line of sight and the ground is called angle of elevation

5. c. 45°

Explanation:



Let the length of the shadow BC be 10 meters.

Then the height of the pole AB is 10meter.

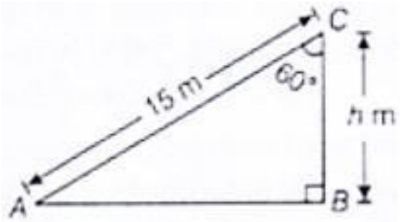
$\therefore \tan\theta = \frac{\text{AB}}{\text{BC}}$

$\Rightarrow \tan\theta = \frac{10}{10}$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45^\circ$

$\Rightarrow \theta = 45^\circ$

6. 

Let AC be the ladder and BC be the wall . Then, we have

AC = 15 m and $\angle ACB = 60^o$

Now , let BC = h m

Clearly, In $\triangle ABC$, we have

$\cos 60^o = \frac{B}{H} = \frac{BC}{AC}$

$\cos 60^o = \frac{h}{15}$

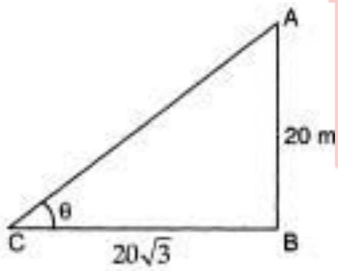
$\Rightarrow \frac{1}{2} = \frac{h}{15} [\because\ Cos\ 60^o = \ \frac{1}{2}]$

$\Rightarrow h = \frac{15}{2}\ m$

$\Rightarrow h = 7.5\ m$

Therefore, height of wall = 7.5 m

7. Let the tower AB = 20m and shadow BC $= 20\sqrt{3}m$



$\angle ACB = \theta$

$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$

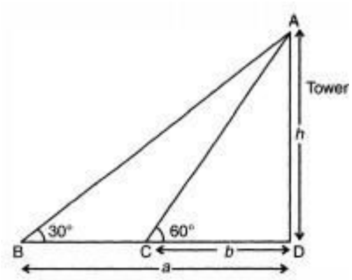
$\Rightarrow \quad \theta = 30^\circ$

Therefore ,the Sun's altitude is 30°.

8. Let the height of tower be h.

From $\triangle ABD\ \frac{h}{a} = \tan 30^\circ$

$\therefore h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad \ldots(\text{i})$



From $\triangle ACD$, $\frac{h}{b} = \tan 60^\circ$

$h = b \times \sqrt{3} = b\sqrt{3} \quad \ldots(\text{ii})$

From (i) $a = \sqrt{3}h$

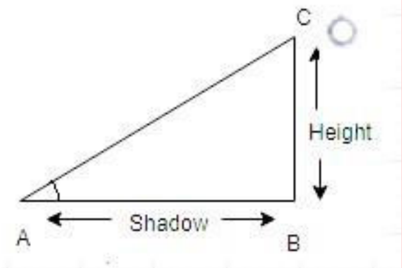
From (ii) $b = \frac{h}{\sqrt{3}}$

$\therefore \quad a \times b = \sqrt{3}h \times \frac{h}{\sqrt{3}}$

$\Rightarrow \quad h^2 = ab$

$\Rightarrow \quad h = \sqrt{ab}$

Hence, the height of the tower = $\sqrt{ab}$

9. 

Let BC be the height of man and AB be the shadow of the man.

According to the question, $AB = BC$

Again, let the angle of elevation of the Sun be θ.

In right-angled triangle,

$\Rightarrow \tan\theta = \frac{P}{B} = \frac{BC}{AB}$

$\Rightarrow \tan\theta = \frac{BC}{BC}$ [height of pole = length of shadow or $AB = BC$]

$\Rightarrow \tan\theta = 1$

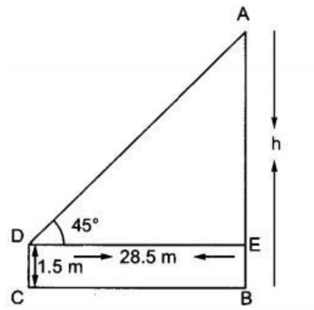
$\Rightarrow tan\theta = tan45^o (\because \tan 45^o = 1)$

$\Rightarrow \theta = 45^o$

$\because$ Angle of elevation of Sun is 45^o.

10. Let AB be the tower of height h and CD be the observer of height 1.5 m at a distance of 28.5 m from the tower AB.

In $\triangle AED$, we have



$\tan 45^\circ = \frac{AE}{DE}$

$\Rightarrow 1 = \frac{AE}{28.5}$

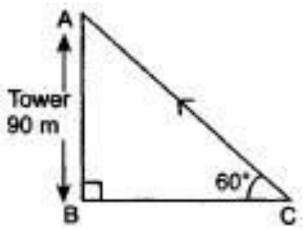
$\Rightarrow AE = 28.5\text{m}$

$\therefore$ h = AE + BE

= AE + DC

= (28.5 + 1.5) m = 30m

Hence, the height of the tower is 30 m.

11. 

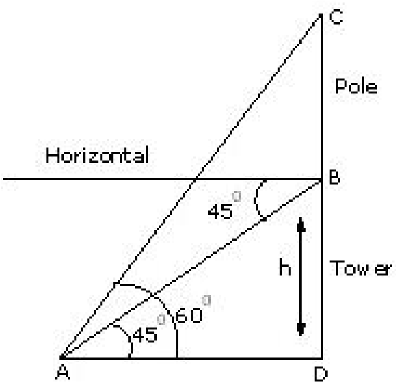
$AB = 90m$

In right $\triangle$ABC,

$\frac{\text{AB}}{\text{BC}} = \tan 60^\circ$

$\Rightarrow \frac{90}{BC} = \sqrt{3}$

$BC = \frac{90}{\sqrt{3}} = 30\sqrt{3}\text{m}$

12. 

given: the height of the pole (CB) = 5m

let 'h' be the height of the tower, then;

In $\triangle ABD$,

$\frac{AD}{h} = \cot 45^\circ$

$\Rightarrow \frac{AD}{h} = 1$

$\Rightarrow$ AD = h

In $\triangle ADC$,

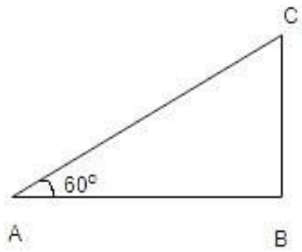
$\frac{AD}{h+5} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h}{h+5} = \frac{1}{\sqrt{3}} [\because AD = h]$

$\Rightarrow h + 5 = \sqrt{3}h$

$\Rightarrow 5 = (\sqrt{3} - 1)h$

$\Rightarrow h = \left(\frac{5}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{5(1.732+1)}{3-1}$ = 6.83 m

13. 

AB = $2\sqrt{3}\ m$ BC is the height of pole.

In right-angle $\triangle ABC, \angle B = 90^o$

$\tan 60^o = \frac{P}{B} = \frac{BC}{AB}$

$\Rightarrow \sqrt{3} = \frac{BC}{AB}$

$\Rightarrow \sqrt{3} = \frac{BC}{2\sqrt{3}}$

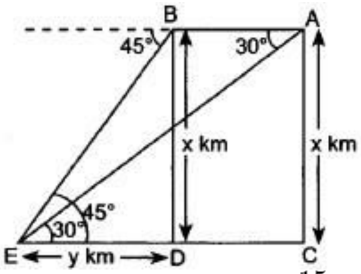
$\Rightarrow BC = \sqrt{3} \times 2\sqrt{3}$

$\Rightarrow BC = 6$

Therefore, height of the pole = 6 m.

14. Distance covered in 15 seconds $= AB$

Speed $= 1200\ km/hr$.



$\therefore \text{AB} = 1200 \times \frac{15}{3600} = 5\text{km}$

$AB = DC = 5km$

Let height = x km

In rt. $\triangle$ BDE,

$\frac{BD}{ED} = \tan 45^\circ \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$

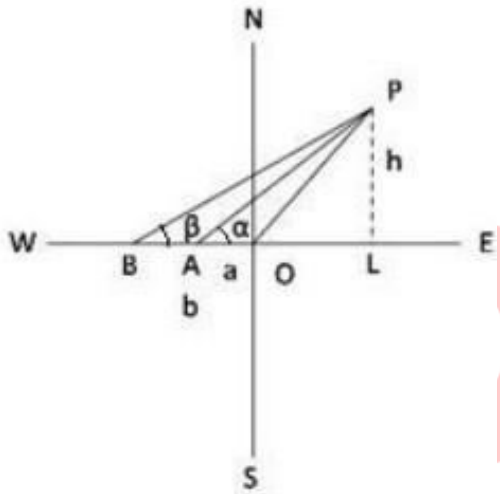
In rt. $\triangle$ ACE,

$\frac{AC}{EC} = \tan 30^\circ \Rightarrow \frac{x}{y+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x}{x+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}x = x + 5 \Rightarrow (\sqrt{3} - 1)x = 5$

$\therefore \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2} = 6.83\text{km}$

15. 

Let OP be the tree and A, B be two points such that $OA = a$ and $OB = b$. In $\Delta's\ ALP$ and BLP, we have

$\tan\alpha = \frac{h}{OL+a}$ and $\tan\beta = \frac{h}{OL+b}$

$\Rightarrow OL + a = h\cot\alpha$ and $OL + b = h\cot\beta$

$\Rightarrow b - a = h\cot\beta - h\cot\alpha$

$\Rightarrow h = \frac{(b-a)}{\cot\beta - \cot\alpha} = \frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$

16. Let AB be the mountain of height h m and C be its foot.

CE = 1000 m, $\angle ACB = 45^o$, $\angle ECB = 30^o$ and $\angle AEF = 60^o$

If $\triangle ACB$, we have

$\tan 45^\circ = \frac{h}{CB} \Rightarrow 1 = \frac{h}{CB}$

$\Rightarrow h = CB$..........(i)

In $\triangle CGE$, we have $\sin 30^\circ = \frac{EG}{1000}$

$\frac{1}{2} = \frac{EG}{1000} \Rightarrow EG = 500$............(ii)

In $\triangle CGE$, we have $\cos 30^\circ = \frac{CG}{1000}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\mathrm{CG}}{1000} \Rightarrow \mathrm{CG} = 500\sqrt{3}$ ....(iii)

Now, BG = BC - GC

$\Rightarrow BG = h - 500\sqrt{3}$ [from (i)]

In $\triangle AEF$, we have, $\tan 60^\circ = \frac{AF}{EF}$

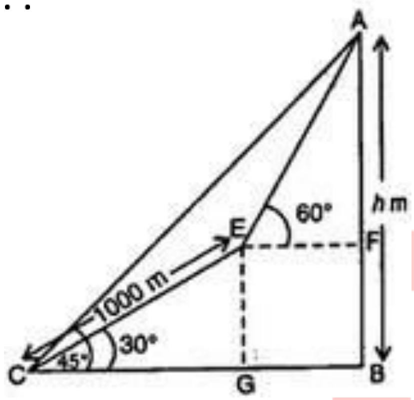
$\Rightarrow \sqrt{3} = \frac{\mathrm{h-BF}}{\mathrm{BG}} [\because \mathrm{BG} = \mathrm{EF} \text{ and } \mathrm{BF} = \mathrm{EG} \quad 500]$

$\Rightarrow \sqrt{3} = \frac{h-500}{h-500\sqrt{3}} \Rightarrow h - 500 = \sqrt{3}h - 500 \times 3$

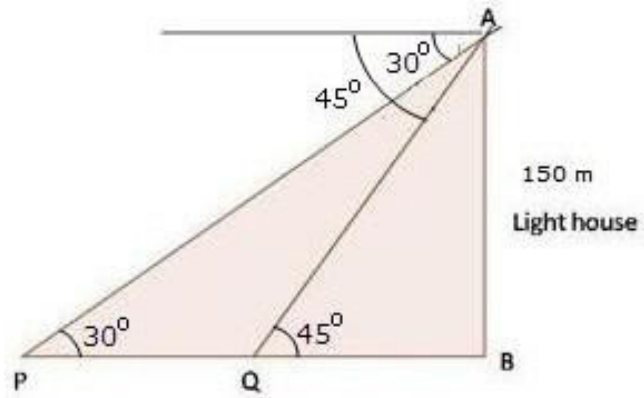
$\Rightarrow -500 + 1500 = h(\sqrt{3} - 1) \Rightarrow 1000 = h \times 0.73$

$\Rightarrow h = \frac{1000}{0.73} = 1369.86\mathrm{m}$

$\therefore$ the height of the mountain is 1369.86 m.



17.



Height of light house $AB = 150m$

In ΔABQ

$\tan 45^\circ = \frac{AB}{BQ}$

$\Rightarrow 1 = \frac{150}{BQ}$

$BQ = 150m$

In ΔABP

$\tan 30^\circ = \frac{AB}{PB}$

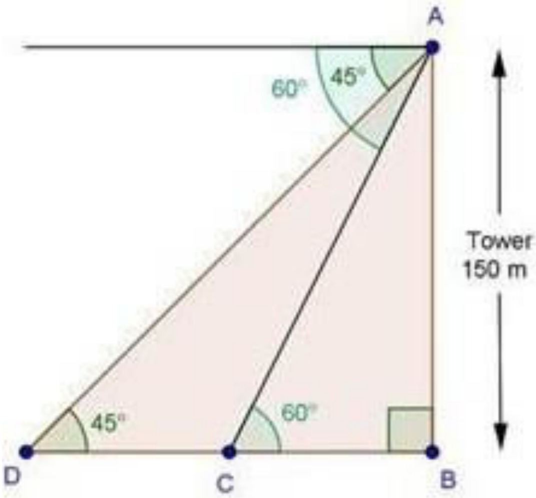
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{PB}$

$\Rightarrow PB = 150\sqrt{3} = 150 \times 1.73 = 259.5m$

∴ Distance between two ships PQ = PB - BQ

$= 259.5 - 150$

$= 109.5m$

18. 

Let AB be the tower of height 150 m and two objects are located when top of tower are observed, makes an

angle of depression from the top and bottom of tower are 45^0 and 60^0

In ΔABD

$\tan 45^0 = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{150}{BD}$

$\Rightarrow BD = 150m$

In ΔABC

$\tan 60^o = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{150}{BC}$

$\Rightarrow BC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow BC = \frac{150\sqrt{3}}{3}$

$\Rightarrow BC = 50\sqrt{3} = 50 \times 1.732 = 86.6m$

∴ Distance between two objects = DC

$= BD - BC$

= 150 - 86.6

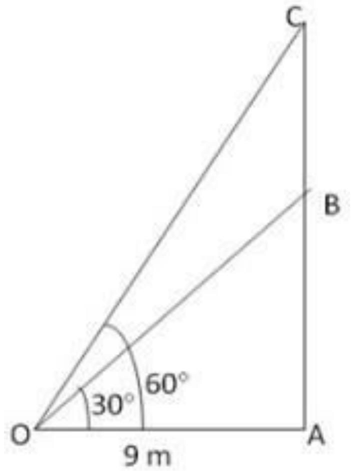
= 63.4 m

19. Let us suppose that AB be the tower and BC be flagpole

Let us suppose that O be the point of observation. Then, OA = 9m

According to question it is given that

$\angle AOB = 30^\circ$ and $\angle AOC = 60^\circ$



From right angled $\triangle \mathrm{BOA}$

$\frac{AB}{OA} = \tan 30^\circ$

$\Rightarrow \frac{AB}{9} = \frac{1}{\sqrt{3}} \Rightarrow AB = 3\sqrt{3}$

From right angled ΔOAC

$\frac{AC}{OA} = \tan 60^\circ$

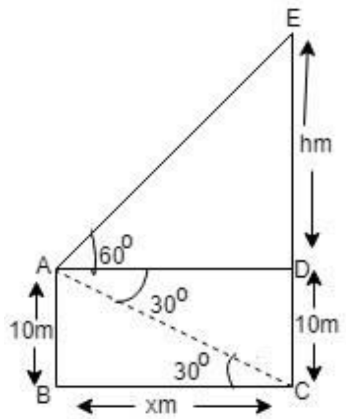
$\frac{AC}{9} = \sqrt{3} \Rightarrow AC = 9\sqrt{3}\mathrm{m}$

$\therefore$ BC = (AC - AB) $= 6\sqrt{3}\mathrm{m}$

Thus $AB = 3\sqrt{3}m = 5.196m$ and $BC = 6\sqrt{3}m = 10.392m$

Hence, height of the tower is 5.196m and the height of the flagpole is 10.392 m

20. Suppose the man is standing on the deck of a ship at point A and let CD be the hill. It is given that the angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°



Then, $\angle EAD = 60^\circ, \angle BCA = 30^\circ.$

Also, AB = 10m

In ΔAED, we have

$\tan 60^\circ = \frac{DE}{EA}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ .. ...(i)

In ΔABC, we have

$\tan 30^\circ = \frac{AB}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x} \Rightarrow x = 10\sqrt{3}$ .....(ii)

Putting $x = 10\sqrt{3}$ in equation (i), we get

$h = \sqrt{3} \times 10\sqrt{3} = 30$

$\Rightarrow \quad DE = 30\text{m}$

∴ CD = CE + ED = 10 + 30 = 40 metres

Hence, the distance of the hill from the ship is $10\sqrt{3}$ metres and the height of the hill is 40 metres.