CBSE Test Paper 03

Chapter 9 Some Applications of Trigonometry

1. If the height of the tower is $\sqrt{3}$ times of the length of its shadow, then the angle of elevation of the sun is **(1)**
   a. 15°
   b. 30°
   c. 60°
   d. 45°
2. A ramp for disabled people in a hospital must slope at not more than 30°. If the height of the ramp has to be 1 m, then the length of the ramp be **(1)**
   a. 3 m
   b. 1 m
   c. 2 m
   d. $\sqrt{3}$ m
3. In a right triangle ABC, $\angle C = 90^{\circ}$. If AC = $\sqrt{3}$ BC and $\angle B = \phi$, then find its value **(1)**
   a. 45°
   b. 30°
   c. None of these
   d. 60°
4. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50m high, then the height of the hill is **(1)**
   a. $50\sqrt{3}$ m
   b. 150m
   c. $150\sqrt{3}$ m
   d. $100\sqrt{3}$ m
5. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. The length of the string, assuming that there is no slack in the string is **(1)**

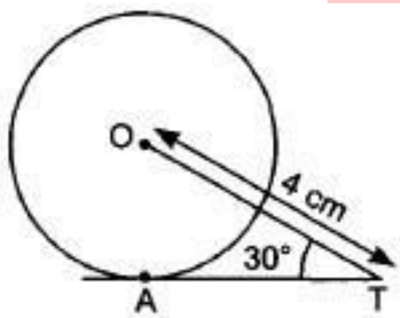
a. $90\sqrt{3}$ m
b. $60\sqrt{3}$ m
c. 90 m
d. 45 m

6. At some time of the day the length of the shadow of a tower is equal to its height. Find the sun's altitude at that time. **(1)**

7. If the elevation of the sun at a given time is 30°, then find the length of the shadow cast by a tower of 150 feet height at that time. **(1)**

8. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30^{o}. Calculate the distance covered by the artist in climbing to the top of the pole. **(1)**

9. In figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Find AT. **(1)**



10. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60^{o}. Find the height of the tower. **(1)**

11. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30^{o} with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. **(2)**

12. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^{o}. Find the length of the string assuming that there is no slack in the

string. $\sqrt{3}=17.32]$ **(2)**

13. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30^o and 45^o. If the distance between two ships is 100 m. Find the height of light-house. **(2)**

14. A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30^o with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 metres. Find the height of the tree. **(3)**

15. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr. **(3)**

16. At a point on level ground, the angle of an elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$ on walking 192 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of the tower. **(3)**

17. A flagstaff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flagstaff is 60^o and from the same point, the angle of elevation of the top of the tower is 45^o. Find the height of the flagstaff. **(3)**

18. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at an angle of ϕ, it is found that the angle of elevation is α. Show that the height of the cliff is $\frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$ metres. **(4)**

19. The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower, when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, find the height of the first tower. **(4)**

20. The angle of elevation of a cloud from a point 120 m above a lake is 30^o and the angle of depression of its reflection in the lake is 60^o. Find the height of the cloud. **(4)**

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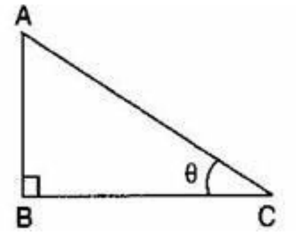
Solution

1. c. 60°

Explanation: 

Let the length of the shadow be x meters.

Then the height of the tower be $\sqrt{3}x$ meter.

$\therefore \tan\theta = \frac{\mathrm{AB}}{\mathrm{BC}} \Rightarrow \tan\theta = \frac{\sqrt{3}x}{x}$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60^{\circ}$

$\Rightarrow \theta = 60^{\circ}$

2. c. 2 m

Explanation: Let height of the ramp be AB = 1 m, the slope of the ramp AC and angle of elevation = $\theta = 30^{\circ}$

In triangle ABC,

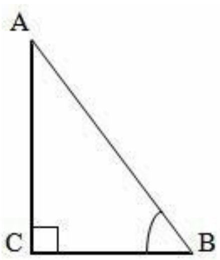
$\sin 30^{\circ} = \frac{\mathrm{AB}}{\mathrm{AC}}$

$\Rightarrow \frac{1}{2} = \frac{1}{\mathrm{AC}}$

$\Rightarrow$ AC = 2 meters

Therefore, the length of the ramp is 2 m.

3. d. 60°

Explanation: 

Given: $\angle C = 90^{\circ}$. If AC = $\sqrt{3}$ BC and $\angle B = \phi$,

$\therefore \tan\phi = \frac{\mathrm{AC}}{\mathrm{BC}}$

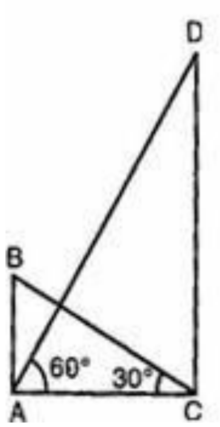
$\Rightarrow \tan\phi = \frac{\sqrt{3}\mathrm{BC}}{\mathrm{BC}}$ = $\sqrt{3}$

$\Rightarrow \tan\phi = \tan 60^{\circ}\phi$

$\Rightarrow \phi = 60^\circ$

4. b. 150m

Explanation:



Let the height of the hill be h m.

$\tan 60^\circ = \frac{\text{DC}}{\text{AC}}$

$\Rightarrow \sqrt{3} = \frac{h}{\text{AC}}$

$\Rightarrow$ AC = $\frac{h}{\sqrt{3}}$ m .... (i)

In right triangle ABC,

$\tan 30^\circ = \frac{50}{\text{AC}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{\text{AC}}$

$\Rightarrow$AC = $50\sqrt{3}$ m .... (ii)

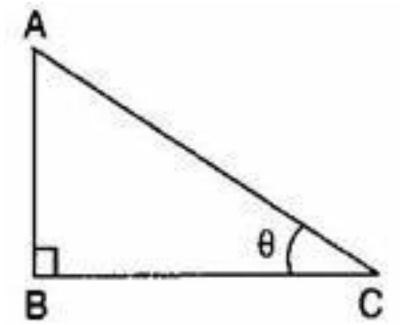
From eq. (i) and (ii),

$50\sqrt{3} = \frac{h}{\sqrt{3}}$

$\Rightarrow h = 50 \times \sqrt{3} \times \sqrt{3} = 150$ m

5. b. $60\sqrt{3}$ m

Explanation:



Let Height of the flying kite = AB = 90 m

And the angle of elevation = $\theta = 60^\circ$

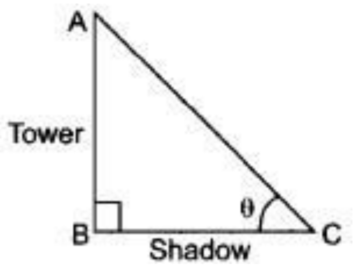
And the length of the string = AC

$\therefore \sin 60^\circ = \frac{\text{AB}}{\text{AC}}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{90}{\text{AC}}$$

$$\Rightarrow \text{AC} = \frac{90 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 60\sqrt{3} \text{ m}$$

6.



given, the length of the shadow of a tower is equal to its height i.e. AB=BC

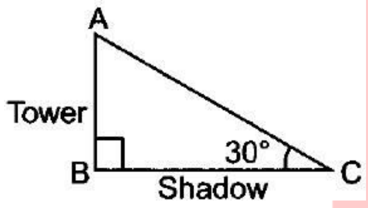
In right $\triangle$ABC

$\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

7.



height of the tower = 150 feets

In right $\triangle ABC$

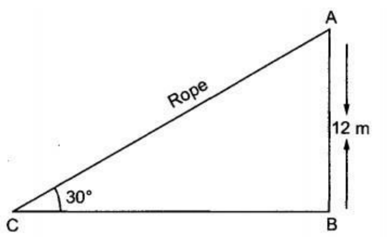
$\tan\theta = \frac{AB}{BC}$

$\frac{150}{\text{BC}} = \frac{1}{\sqrt{3}}$

$\text{BC} = 150\sqrt{3}$ feet

8. Clearly, distance covered by the artist is equal to the length of the rope AC. Let AB be the vertical pole of height 12 m. It is given that $\angle ACB = 30^\circ$.

Thus, in right-angled triangle ABC, we have Perpendicular AB = 12 m, $\angle ACB = 30^\circ$ and we wish to find hypotenuse AC.

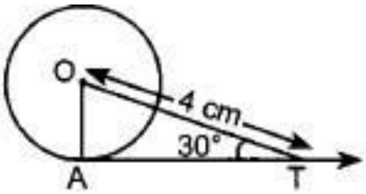


$\therefore \quad \sin 30^\circ = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{12}{AC}$

$\Rightarrow \quad AC = 24\text{m}$

Hence, the distance covered by the circus artist is 24 m.

9. 

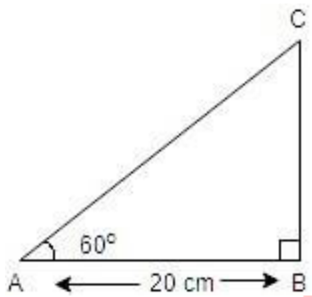
$\angle$OAT = 90°

In right angled $\triangle$ OAT,

$\frac{\text{AT}}{\text{OT}} = \cos 30^\circ \Rightarrow \frac{\text{AT}}{4} = \frac{\sqrt{3}}{2}$

$\Rightarrow \text{AT} = 2\sqrt{3}\text{cm}.$

10. $\angle A = 60^0$ and AB = 20 cm



In $\triangle ABC$,

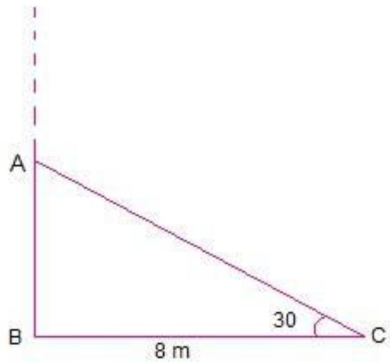
$\tan\theta = \frac{P}{B} = \frac{CB}{AB} = \frac{CB}{20}$

$\Rightarrow \tan 60^o = \frac{CB}{20}$

$\Rightarrow \sqrt{3} = \frac{CB}{20} [\because \tan 60^o = \sqrt{3}]$

$\Rightarrow CB = 20\sqrt{3}$

Therefore, Height of tower = $20\sqrt{3}\ m$

11. 

Let AC be the broken part of the tree.

$\therefore$ Total height of the tree = AB + AC

In right $\triangle$ABC,

$\cos 30^o = \frac{BC}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} = \frac{8}{AC}$

$\Rightarrow$ AC = $\frac{16}{\sqrt{3}}$

Also,

$\tan 30^o = \frac{AB}{BC}$

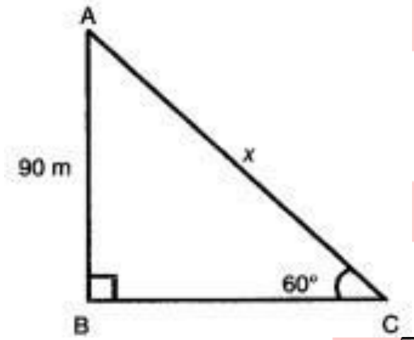
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$

$\Rightarrow$ AB = $\frac{8}{\sqrt{3}}$

Total height of the tree = AB + AC = $\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

12. In right $\triangle ABC$, $\frac{AB}{AC} = \sin 60^\circ$

$$\Rightarrow \quad \frac{90}{x} = \frac{\sqrt{3}}{2}$$



$$\Rightarrow \quad x = \frac{90\times 2}{\sqrt{3}} = \frac{180\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

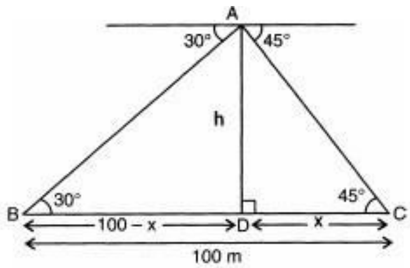
$$\Rightarrow \quad x = \frac{180\sqrt{3}}{3}$$

$$\Rightarrow \quad x = 60\sqrt{3}$$

$$= 60 \times 1.732$$

Hence length of string = 103.92 m.

13. Let AD be the height (h) of the light house and BC is the distance between two ships



Given, BC = 100 m, BD = 100 - x, DC = x (let)

In $\triangle$ADC, $\tan 45^\circ = \frac{h}{DC}$

$\Rightarrow$ DC = h.

$\Rightarrow$ x = h... (i)

In $\triangle$ABD, $\tan 30^\circ = \frac{h}{100 - DC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100-x}$

$\Rightarrow 100 - x = h\sqrt{3}$

$\Rightarrow 100 - h = h\sqrt{3}$ [By (i)]

$\Rightarrow \quad 100 = h + h\sqrt{3}$

$\Rightarrow \quad 100 = h(1+\sqrt{3})$

$\Rightarrow h = \frac{100}{1+\sqrt{3}}$

Rationalising numerator and denominator

$\Rightarrow h = \frac{100}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$\Rightarrow \quad h = \frac{100(\sqrt{3}-1)}{3-1}$

$= 50(\sqrt{3}-1)$

= 50(1.732 - 1)

$= 50 \times 0.732$

= 36.6 m

Height of light house = 36.6 m.

14. Let AB be the tree broken at point D, so that AD takes the position DC.

Let DB = x m , AD = DC = y m

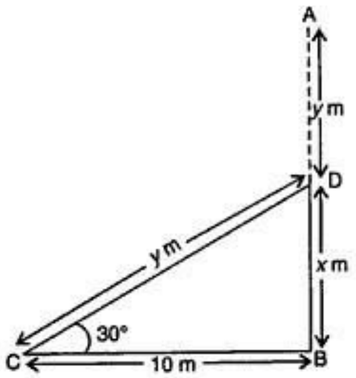
In $\triangle DBC$, we have $\cos 30^\circ = \frac{10}{\text{y}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{y} \Rightarrow y = \frac{20}{\sqrt{3}}$ ....(i)

Again in $\triangle$ DBC, we have $\sin 30^\circ = \frac{\text{x}}{\text{y}}$

$\Rightarrow \frac{1}{2} = \frac{x}{\frac{20}{\sqrt{3}}}$ [From (i)]

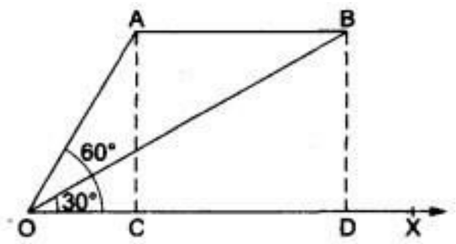
$\Rightarrow 2\sqrt{3}\text{x} = 20 \Rightarrow \text{x} = \frac{20}{2\sqrt{3}} = \frac{10}{\sqrt{3}}$



$\therefore$ height of the tree = x + y = $\left(\frac{20}{\sqrt{3}}+\frac{10}{\sqrt{3}}\right)\mathrm{m}=\frac{30}{\sqrt{3}}\mathrm{m}=10\sqrt{3\mathrm{m}}$

$\therefore$ height of the tree = $10\sqrt{3}\ m$

15. Let A and B be the two positions of the aeroplane.



Let $AC\perp OX$ and $BD\perp OX$. Then,

$\angle COA=60^{\circ},\angle DOB=30^{\circ}$

and AC = BD = $1500\sqrt{3}\mathrm{m}.$

From right $\Delta OCA,$ we have

$\frac{OC}{AC}=\cot 60^{\circ}=\frac{1}{\sqrt{3}}$

$\Rightarrow\frac{OC}{1500\sqrt{3}}=\frac{1}{\sqrt{3}}\Rightarrow OC=1500\mathrm{m}$

From right $\Delta ODB,$we have

$\frac{OD}{BD}=\cot 30^{\circ}=\sqrt{3}\Rightarrow\frac{OD}{1500\sqrt{3}\mathrm{m}}=\sqrt{3}$

$\Rightarrow OD=(1500\times 3)\mathrm{m}=4500\mathrm{m}.$

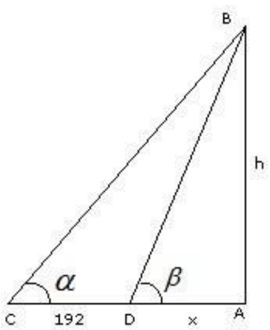
$\therefore CD=(OD-OC)=(4500-1500)m=3000m.$

Thus, the aeroplane covers 300m in 15 seconds.

$\therefore$ speed of the aeroplance $=\left(\frac{3000}{15}\times\frac{60\times 60}{1000}\right)\mathrm{km/hr}$

= 720 km /hr.

16.



Suppose height of tower is h meter

In $\triangle ABD$

$\tan\beta=\frac{h}{x}=\frac{3}{4}$ ......(i)

In $\triangle ABC$

$\tan\alpha = \frac{h}{192+x}$

$\Rightarrow \frac{h}{192+x} = \frac{5}{12}$.........(ii)

(i) + (ii)

$\frac{\frac{h}{x}}{\frac{h}{192+x}} = \frac{\frac{3}{4}}{\frac{5}{12}}$

$\Rightarrow x = 240$

Putting the value of x in equation 1 to find the value of h(height of tower)

h = 180 m

Hence, the height of the tower = 180 m

17. Let the height of flagstaff = h m = CB

height of tower = 5 m = AB

Height of top of flagstaff from ground = (h + 5) m = AC

Let distance of point P from tower = x

Using $\triangle PAB$, $\frac{x}{5} = cot45^0 \rightarrow \frac{x}{5} = 1 \implies x = 5m$

Using $\triangle PAC$, $\frac{\mathrm{x}}{\mathrm{h+5}} = \cot 60^\circ$

$\Rightarrow \frac{x}{h+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{h+5}{\sqrt{3}}$........(ii)

From (i) and (ii), we get

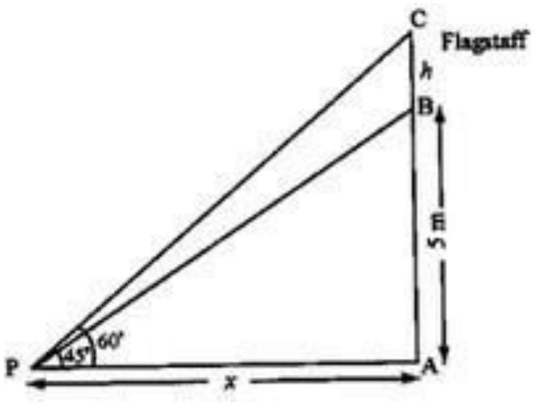
$\frac{h+5}{\sqrt{3}} = 5$

$\Rightarrow h+5 = 5\sqrt{3}$

$\therefore h = 5\sqrt{3} - 5$

= 5(1.73 - 1) = $5 \times .73 = 3.65$ m

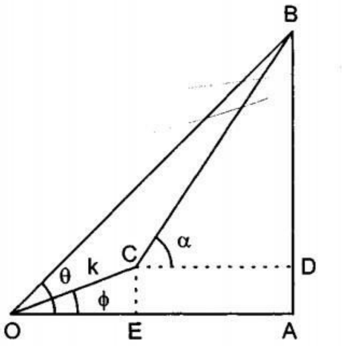
∴ the height of the flagstaff is 3.65 m approx.



18. Let AB be the cliff and O be the fixed point such that the angle of elevation of the cliff

from O is θi.e., $\angle AOB = \theta$. Let $\angle AOC = \phi$ and OC = k metres. From C draw CD and CE perpendiculars on AB and OA respectively. Then, $\angle DCB = \alpha$. Let h be the height of the cliff AB.

In $\triangle OCE$, we have



$\sin\phi = \frac{CE}{OC}$

$\Rightarrow \sin\phi = \frac{CE}{k}$

$\Rightarrow CE = k\sin\phi$

$\Rightarrow AD = k\sin\phi$ .....(i) [∵CE = AD]

and, $\cos\phi = \frac{OE}{OC}$

$\Rightarrow \cos\phi = \frac{OE}{k}$

$\Rightarrow OE = k\cos\phi$ ....(ii)

In $\triangle OAB$, we have

$\tan\theta = \frac{AB}{OA}$

$\Rightarrow \tan\theta = \frac{h}{OA}$

$\Rightarrow OA = h\cot\theta$ ....(iii)

∴ CD = EA = OA - OE = h cot ϕ...(iv)[Using (ii) and (iii)]

and, BD = AB - AD = AB - CE = h - k sin ϕ ...(v) [Using (i)]

In $\triangle$BCD, we have

$\tan\alpha = \frac{BD}{CD}$

$\Rightarrow \tan\alpha = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$

$\Rightarrow \frac{1}{\cot\alpha} = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$

$\Rightarrow h\cot\alpha - k\sin\phi\cot\alpha = h\cot\theta - k\cos\phi$

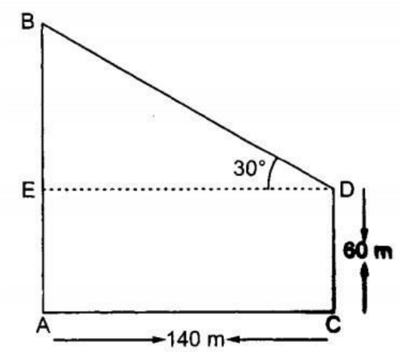
$\Rightarrow h(\cot\theta - \cot\phi) = k(\cos\phi - \sin\phi\cot\alpha)$

$\Rightarrow h = \frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$

19. Let AB and CD be two towers of height h metres and 60 metres respectively such that

the distance AC between them is 140 m. The angle of elevation of top B of tower AB as seen from D (top of tower CD) is 30°.

$\triangle DEB$, we have



$\tan 30^{\circ} = \frac{BE}{DE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{140}$ [∵DE = AC = 140 m]

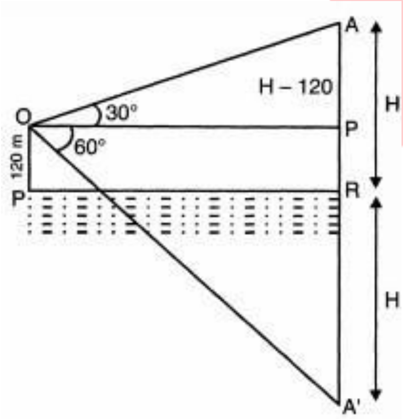
$\Rightarrow \quad BE = \frac{140}{\sqrt{3}}\text{m} = \frac{140}{1.732}\text{m} = 80.83\text{m}$

∴ AB = AE + BE = CD + BE = 60 + 80.83 m = 140.83 m

Hence, the height of the second tower is 140.83 m.

20. In $\triangle AOP$, $\tan 30^{\circ} = \frac{H-120}{OP}$

$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$



$\Rightarrow \quad OP = (H-120)\sqrt{3}$ ...(i)

In $\triangle$OPA'

$\frac{PA'}{OP} = \frac{H+120}{(H-120)\sqrt{3}} = tan60 = \sqrt{3}$

H + 120 = 3H - 360

H = 240 meter

Hence height of cloud is 240 meter