CBSE Test Paper 01

Chapter 9 Some Applications of Trigonometry

1. The____________of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level. **(1)**
   a. angle of projection
   b. angle of depression
   c. angle of elevation
   d. none of these

2. From a point on the ground which is 15m away from the foot of a tower, the angle of elevation is found to be 60° . The height of the tower is **(1)**
   a. $15\sqrt{3}$ m
   b. $20\sqrt{3}$ m
   c. $10\sqrt{3}$ m
   d. 10 m

3. From a point P on the level ground, the angle of elevation of the top of a tower is 30° . If the tower is 100m high, the distance between P and the foot of the tower is **(1)**
   a. $300\sqrt{3}$ m
   b. $150\sqrt{3}$ m
   c. $200\sqrt{3}$ m
   d. $100\sqrt{3}$ m

4. An electric pole is $10\sqrt{3}$ m high and its shadow is 10 m in length, then the angle of elevation of the sun is **(1)**
   a. 45°
   b. 15°
   c. 30°
   d. 60°

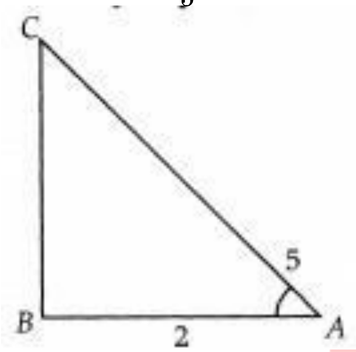
5. If the shadow of a boy ‘x’ metres high is 1.6m and the angle of elevation of the sun is

45°, then the value of 'x' is **(1)**

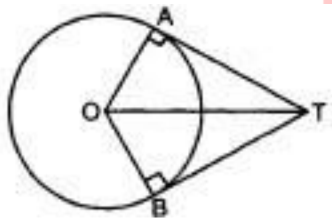
a. 0.8 m
b. 1.6 m
c. 3.2 m
d. 2 m

6. The angle of depression of car parked on the road from the the top of a 150 m hightower is 30^o. Find the distance of the car from the tower. **(1)**

7. If cos $A = \frac{2}{5}$, find the value of 4 + 4tan^{2}A. **(1)**



8. In figure if ∠ATO = 40°, find ∠AOB. **(1)**



9. A ladder 15 m long leans against a wall making an angle of 60^o with the wall. Find the height of the wall from the point the ladder touches the wall. **(1)**

10. A pole 6 m high casts a shadow $2\sqrt{3}$ long on the ground, then find the Sun's elevation. **(1)**

11. A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30^o. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45^o. Find the distance of the bird from the girl. **(1)**

12. Find the angle of elevation of the sun when the shadow of a pole h m high is $\sqrt{3}\ h$ m long. **(2)**

13. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60^o and 45^o respectively. Find the height of the tower correct to one place of decimal. **(2)**

14. The tops of two towers of height x and y, standing on level ground, subtend angles of 30^o and 60^o respectively at the centre of the line joining their feet, then find x : y. **(3)**

15. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that tan $\theta = 15/8$ then find the height of the kite from the ground. Assume that there is no slack in the string. **(3)**

16. A man standing on the deck of a ship which is 10 m above the water level observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. **(3)**

17. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60^o. At a point R, 40 m vertically above X, the angle of elevation of the top Q of tower is 45^o. Find the height of the tower PQ and the distance PX. **(3)**

18. The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30^o and 45^o respectively. Find the height of the tower and also the horizontal distance between the building and the tower. **(4)**

19. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break? **(4)**

20. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is r sin β cosec$\frac{\alpha}{2}$ **(4)**

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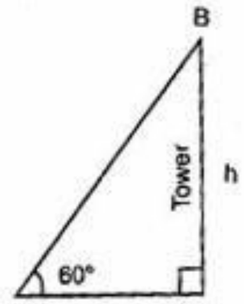
Solution

1. c. angle of elevation

   Explanation: The angle of elevation of an object is the angle formed by the line of sight with the horizontal when the object is above the horizontal level.

2. a. $15\sqrt{3}$ m

   Explanation: Let the height of the tower be h metres.



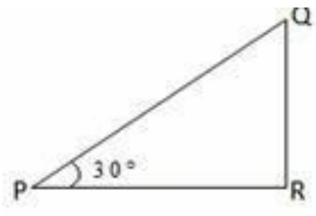
In triangle AOB, $\tan 60^\circ = \frac{\text{AB}}{\text{OA}}$

$\Rightarrow \tan 60^\circ = \frac{h}{15}$
$\Rightarrow \sqrt{3} = \frac{h}{15}$
$\Rightarrow h = 15\sqrt{3}$ m

Therefore, the height of the tower is $15\sqrt{3}$ meters.

3. d. $100\sqrt{3}$ m

   Explanation:



Let QR be the height of the tower, then QR = 100 m

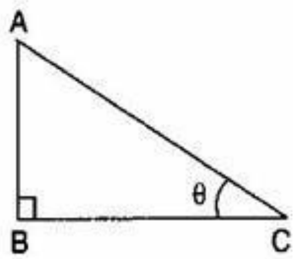
And the angle of elevation of the top of the tower be $\angle\text{QPR} = 30^\circ$

$\therefore \tan 30^\circ = \frac{\text{QR}}{\text{PR}}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{\text{PR}}$ m
$\Rightarrow$PR = $100\sqrt{3}$ meters

Therefore, the distance between P and the foot of the tower is $100\sqrt{3}$ meters.

4. d. 60°

Explanation:



Let AB be the electric pole of height $10\sqrt{3}$ m and its shadow be BC of length 10 m. And the angle of elevation of the sun be θ.

$\therefore \tan\theta = \frac{\text{AB}}{\text{BC}}$

$\Rightarrow \tan\theta = \frac{10\sqrt{3}}{10}$

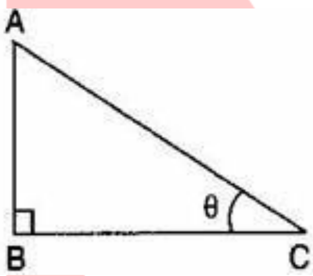
$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60^{\circ}$

$\Rightarrow \theta = 60^{\circ}$

5. b. 1.6 m

Explanation:



Given: Height of the boy = AB = x meters

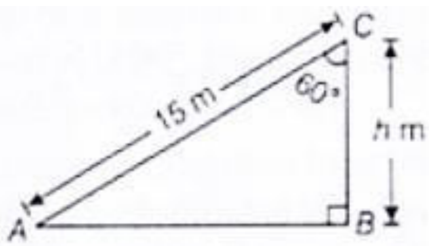
And the length of the shadow of the boy = BC = 1.6 m

And angled of elevation θ $=45^{\circ}$

$\therefore \tan 45^{\circ} = \frac{\text{AB}}{\text{BC}} \Rightarrow 1 = \frac{x}{1.6}$

$\Rightarrow x = 1.6$ m

6. The angle of depression of car parked on the road from the the top of a 150 m hightower is 30^{o}.



Let $AB =$ 150 m be the height of the tower

and angle of depression is $\angle DAC = 30^{o}$.

Therefore, $\angle ACB = \angle DAC = 30^o$[$\because$alternate angles]

Now, in right-angled $\triangle ABC$, $\angle B = 90^o$

$\tan 30^o = \frac{P}{B} = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{BC}$ [$\because \tan 30^o = \frac{1}{\sqrt{3}}$]

$\Rightarrow BC = 150\sqrt{3}$ m

Therefore, Distance of car from tower = $150\sqrt{3}\ m$

7. Given, $cos A = \frac{5}{2}$

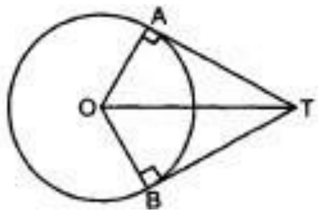
$= 4 + 4\tan^2 A$

$= 4(1 + tan^2 A)$

= $4\sec^2 A = \frac{4}{\cos^2 A} = 4 \times \frac{25}{4} = 25$

8. According to the question,

$\angle$ATO = 40°



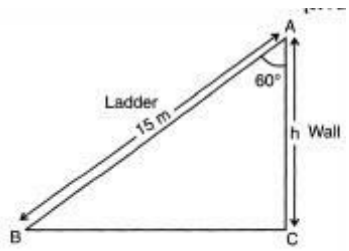
In $\triangle$OAT, $\angle$OAT = 90°

$\angle$AOT = 50° [Angle sum property]

Now $\angle$BTO = 40° as OT bisects $\angle$ATB

Similarly, $\angle$BOT= 50°

$\angle$AOB = $\angle AOT + \angle$BOT = 50° + 50° = 100°

9. 

Let ABC be a right angled triangle where AB isis ladder = 15m and angle a = 60°

Let AC be the height of the wall

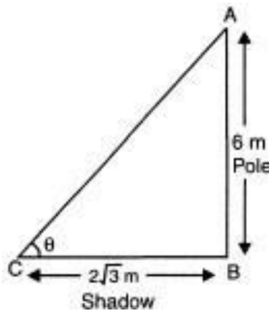
Therefore by Pythagoras theorem

$\frac{h}{15} = \cos 60^\circ$

$\Rightarrow \quad h = 15 \times \cos 60^\circ$

$= 15 \times \frac{1}{2}$

= 7.5 m

10. 

Let the Sun's elevation be θ

Length of pole = 6 m, length of shadow = $2\sqrt{3}\text{m}$

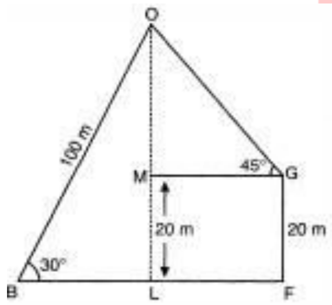
From $\triangle ABC$, $\frac{AB}{BC} = \tan\theta$(using Pythagoras theorem)

$\Rightarrow \quad \frac{6}{2\sqrt{3}} = \tan\theta$

$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$

$\Rightarrow \quad \theta = 60^\circ$

Hence sun's elevation is 60^{o}

11. 

Let O be the position of the bird and B be the position of the boy. Let FG be the building and G be the position of the girl.

In $\triangle$OLB,

$\frac{OL}{BO} = \sin 30^\circ$

$\Rightarrow \quad \frac{OL}{100} = \frac{1}{2}$

$\Rightarrow$ OL = 50 m

OM = OL - ML

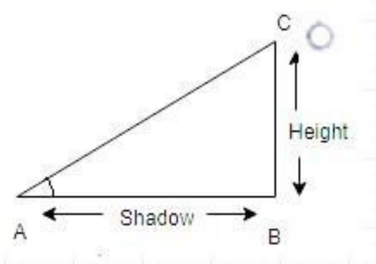
= OL - FG

= 50 - 20 = 30 m

In $\triangle$OMG

$\frac{OM}{OG}$ = sin 45^{o} = $\frac{1}{\sqrt{2}}$

OG = OM$\sqrt{2}$ = 30 $\sqrt{2}$ = 42.3 meter

12. 

Let BC be the height and BA be the shadow of a man.

According to the question, AB = BC

The shadow of a pole AB = h m high BC = $\sqrt{3}\ h$ m long.

Again, let the angle of elevation of the Sun be θ.

In right-angled $\triangle ABC$

$\tan\theta = \frac{P}{B} = \frac{BC}{AB}$

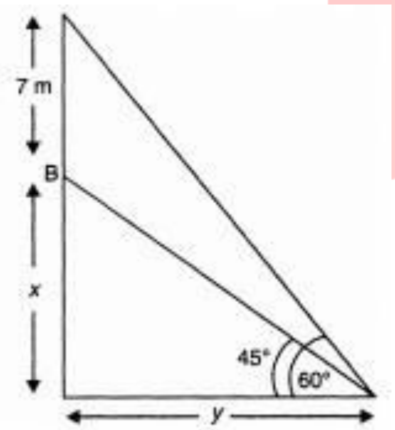
$\Rightarrow \tan\theta = \frac{h}{\sqrt{3}h}$ {∵AB = h m and BC = $\sqrt{3}\ h$}

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow tan\theta = tan30^o (\because \tan 30^o = \frac{1}{\sqrt{3}})$

$\Rightarrow \theta = 30^o$

Therefore, Angle of elevation of Sun is 30^o

13. 

$\frac{x}{y} = \tan 45^\circ = 1$

$\Rightarrow$ x = y

Now in big triangle

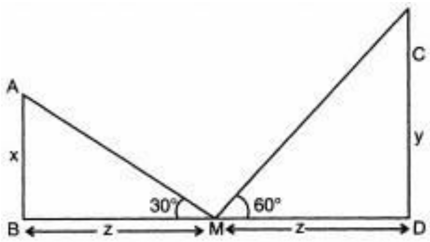
tan 60° = $\frac{x+7}{x}$

√3 = $\frac{x+7}{x}$

x($\sqrt{3}$ - 1) = 7

So height of the tower

$x = \frac{\gamma}{1.73-1} = 9.58$

14. 

Let M be the centre of the line joining their feet.

Let BM = MD = z

$\therefore \tan\theta = \frac{\text{perpendicular}}{\text{base}}$

In $\triangle ABM, \therefore \frac{x}{z} = \tan 30^\circ$

$\Rightarrow \quad x = z \times \frac{1}{\sqrt{3}}$ ....(i)

In $\triangle$MCD we have

$\frac{y}{z} = tan60^o = \sqrt{3}$

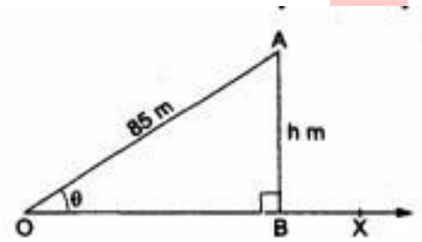
$y = z\sqrt{3}......(ii)$

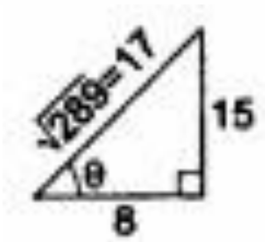
From (i) and (ii) we get

$\frac{x}{y} = \frac{z}{\sqrt{3}} \times \frac{1}{\sqrt{3}z} = \frac{1}{3}$

Hence x : y = 1 : 3

15. Let OX be the horizontal ground and let A be the position of the kite. Let O be the position of the observer and OA be the string. Draw $AB \perp OX$.





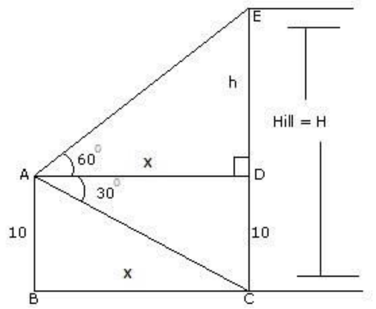
Then, $\angle BOA = \theta$ such that $\tan\theta = \frac{15}{8}$, $OA = 85m$ and $\angle OBA = 90^\circ$.

Let AB = h m.

From right ΔOBA, we have

$\frac{AB}{OA} = \sin\theta = \frac{15}{17} \left[\because \tan\theta = \frac{15}{8} \Rightarrow \sin\theta = \frac{15}{17}\right]$

$\Rightarrow \frac{h}{85} = \frac{15}{17} \Rightarrow h = \frac{15}{17} \times 85 = 75.$

16. 

Let H = Height of hill

Let AD = BC = x meters

CE = CD + DE = 10 + h

In right $\triangle$ADE, tan 30^o = $\frac{AD}{DE}$

$\frac{x}{h} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ x = $\frac{h}{\sqrt{3}}$

In right $\triangle ADC, \frac{x}{10} = \cot 30^\circ = \sqrt{3}$

$\Rightarrow x = 10\sqrt{3}$

Equating the values of x, we get

$\frac{h}{\sqrt{3}} = 10\sqrt{3} \Rightarrow h = 30\,cm$

$\therefore$ From H = 10 + h = 10 + 30 = 40 m

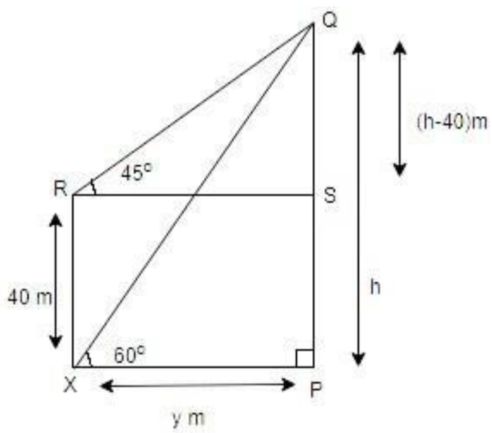
And x = distance of hill from ship = $10\sqrt{3}$ m

17. Let h be the height of the tower.

i.e, PQ = h m and let PX = y m

Now, draw $RS \parallel XP$,

Then, we have RX =SP = 40 m, $\angle QXP = 60^o$ and $\angle QRS = 45^o$



In right angled $\triangle XPQ$,

$\tan 60^o = \frac{P}{B} = \frac{PQ}{XP}$

$\Rightarrow \frac{\sqrt{3}}{1} = \frac{h}{y}$ [$\because \tan 60^o = \sqrt{3}$]

$\Rightarrow y = \frac{h}{\sqrt{3}}$ ..(i)

In right angled $\triangle RSQ$,

$\tan 45^o = \frac{P}{B} = \frac{QS}{RS}$

$\Rightarrow \tan 45^o = \frac{PQ-SP}{XP}$

$\Rightarrow 1 = \frac{h-40}{y}$

$\Rightarrow y = h - 40$....(ii)

Now,solve Eq(i) and Eq(ii) , to find h and y.

$\frac{h}{\sqrt{3}} = h - 40$

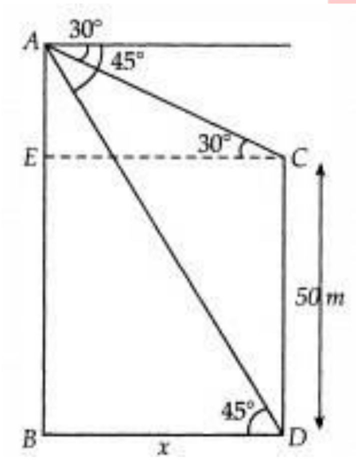
$(\sqrt{3} - 1)$ h = $40\sqrt{3}$

h = $\frac{40\sqrt{3}}{\sqrt{3}-1} = \frac{40(1.732)}{1.732 - 1} = \frac{68.28}{0.732} = 94.64$

$\Rightarrow y = 94.64 - 40$

$\Rightarrow y = 54.64$

$\Rightarrow PQ = 94.64\ m\ and\ PX = 54.64\ m$

18. 

Let the height of the tower be AB = hm

Let the building be CD = 50 m

and let distance between BD = x

Now, In $\triangle ABD$

$\frac{AB}{BD} = \tan 45^\circ$

$\Rightarrow \frac{h}{x} = 10$

h = x ..(i)

In $\triangle AEC$, $\frac{AE}{EC} = \tan 30^\circ$

$\Rightarrow \frac{h-50}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = h\sqrt{3} - 50\sqrt{3}$ ..(ii)

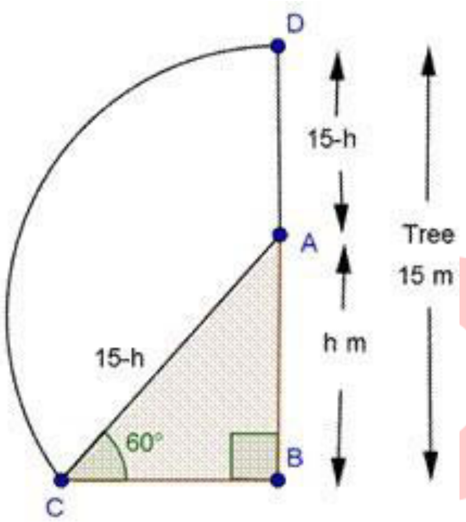
From (i) and (ii) we get

$h = \sqrt{3}(h - 50)$
$h(\sqrt{3} - 1) = 50$
$h = \frac{50}{\sqrt{3}-1} = \frac{50(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{50(\sqrt{3}+1)}{3-1} = 25(1.73 + 1)$
$= 25 \times 2.73 = 68.25$meter

Hence the height of tower = 68.25 meter

and distance between the building and tower x= h = 68.25 meter

19.



The height of the tree (DB) = 15 m

Suppose it broke at A and its top D touches the ground at C.

Suppose AB = h Then AD = AC = (15 - h) m

In ΔABC

$\sin 60^o = \frac{AB}{AC}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{15-h}$
$\Rightarrow 2h = 15\sqrt{3} - \sqrt{3}h$
$\Rightarrow 2h + \sqrt{3}h = 15\sqrt{3}$
$\Rightarrow h(2 + \sqrt{3}) = 15\sqrt{3}$
$\Rightarrow h = \frac{5\sqrt{3}}{2+\sqrt{3}}$
$\Rightarrow h = \frac{5\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$
$\Rightarrow h = \frac{30\sqrt{3}-45}{4-3}$
$\Rightarrow h = 15\,(2\sqrt{3} - 3)$
$\Rightarrow h = 15\,[2 \times 1.73 - 3]$

$\Rightarrow h = 15\,[3.46 - 3]$

$\Rightarrow h = 15 \times 0.46$

$\Rightarrow h = 6.9m$

$\therefore$ Height above the ground from where the tree broke is 6.9 meter.

20. Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA, PB be tangents from P to the balloon. Then, $\angle APB = \alpha$.

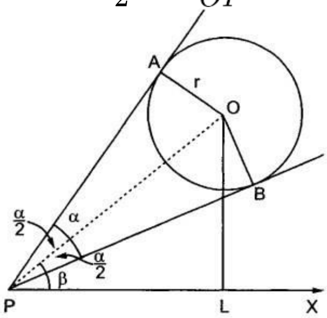
$\therefore \angle APO = \angle BPO = \frac{\alpha}{2}$

Let OL be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the balloon is β i.e, $\angle OPL = \beta$.

In $\triangle OAP$, we have

$\sin\frac{\alpha}{2} = \frac{OA}{OP}$

$\Rightarrow \sin\frac{\alpha}{2} = \frac{r}{OP}$



$\Rightarrow OP = r\ cosec\frac{\alpha}{2}$

In $\triangle OPL$, we have

$\sin\beta = \frac{OL}{OP}$

$\Rightarrow OL = OP\sin\beta$ = r cosec $\frac{\alpha}{2}$ sin β [Using equation (i)]

Hence, the height of the centre of the balloon is r sin β cosec $\frac{\alpha}{2}$