ANSWERS WITH EXPLANATION

Physics

1. (2) $M \propto [E]^a[V]^b[F]^c$ ...(1)

$M^1 = K[M^1L^2T^{-2}]^a\,[L^1T^{-1}]^b\,[M^1L^1T^{-2}]^c$

$M^1L^0T^0 = KM^{a+c}\,L^{2a+b+c}\,T^{-2a-b-2c}$

$a + c = 1$ ...(2)

$2a + b + c = 0$ ...(3)

$2a + b + 2c = 0$ ...(4)

On solving equation (2), (3) and (4), we get

$a = 1, b = -2, c = 0$

Putting values of a, b and c in equation (1)

$[M = EV^{-2}]$

2. (1) We know that, $s = ut + \left(a\frac{(t^2)}{2}\right)$

Here, object starts from $u = 0$ and time$(t) = n$

$s_n = \frac{1}{2}an^2$, $s_{n-2} = \frac{1}{2}a(n-2)^2$

Displacement in last 2 seconds

$$s = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= \frac{a}{2}(2n-2)2 = 2a(n-1)$$

But $v = u + at$

$\Rightarrow$ $v = 0 + an$ or $a = \frac{v}{n}$

$$s = \frac{2v}{n}(n-1)$$

3. (1) $\Delta Q = \left[1\times\frac{1}{2}\times 10 + 1\times 80 + 1\times 1\times 100 + 1\times 536\right]$

$= 721$ cal $= 3045$ J

Option (1) is correct.

4. (2) Average kinetic energy

$$= \frac{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_2^3 + ...}{N}$$

$$= \frac{1}{2}\frac{m}{N}\left(v_1^2 + v_2^2 + v_3^2 + ...\right) \neq 0$$

Average momentum $= m\vec{v}_{av} = m(0) = 0$

$$\text{Average density} = \frac{\text{Mass}}{\text{Volume}} = \frac{nm}{V} \neq 0$$

$$\text{Average Speed} = v_{av} = \sqrt{\frac{8kT}{\pi m}} \neq 0$$

5. (1) $\Delta U = \Delta Q - \Delta W$

Now $\Delta W = P\Delta V$

$= 50\,[4 - 10] = -300$ J

$\Delta Q = 100$ J

So $\Delta U = 400$ J increased

6. (1) $R_A = \frac{L}{K_1 a}$

$R_B = \frac{L}{K_2 a}$

Where a is the cross sectional area of the rod and L is the length of the rod and K_1 and K_2 are the thermal conductivity of each rod.

$$T_J = \frac{R_B}{R_A + R_B}(100 - 0)$$

$$= \frac{1}{\frac{R_A}{R_B} + 1}\times 100$$

$$= \frac{100}{3+1}$$

$= 25°C$

7. (1) $V = xyz$

$V' = (x + \Delta x)\,(y + \Delta y)\,(z + \Delta z)$

Neglecting terms

$V' = xyz + xy\,\Delta z + (\Delta x)\,yz + x\Delta yz$

$$V'_2 = V + \frac{V\Delta z}{z} + \frac{V\Delta x}{x} + \frac{V\Delta y}{y}$$

$V'_2 = V[1 + (\alpha_1 + \alpha_2 + \alpha_3)\Delta T]$

(We know that $\alpha \times \Delta T = \frac{\Delta \ell}{\ell}$)

$= V(1 + \alpha_{eq}\Delta T)$

$\Rightarrow$ $\alpha_{eq} = (\alpha_1 + \alpha_2 + \alpha_3)$

8. (1) Given $r_A = 2r_B$; $\ell_A = \ell_B$

$v = \sqrt{\frac{T}{\mu}}$; T = same for both

$v \propto \sqrt{\frac{T}{\mu}}$; $\mu = \frac{m}{\ell}$

$m = \delta\,A\,\ell$

$$\ell = \text{same}, \rho = \text{same}$$
$$m_1 : m_2 = A_1 : A_2 = \pi(2r)^2 : \pi(r)^2$$
$$m_1 : m_2 = 4 : 1$$
$$\mu_1 : \mu_2 = m_1 : m_2 = 4 : 1$$
$$v \propto \frac{1}{\sqrt{\mu}}$$
$$\Rightarrow v_A : v_B = 1 : 2 \text{ or } \frac{1}{2}$$

9. (4)

$$\tan\theta = \frac{v^2}{rg}$$
$$\Rightarrow \quad v = \sqrt{rg\tan\theta}$$
$$\Rightarrow \quad = \sqrt{(10)\,(9.8)\tan 10'} = 4.2 \text{ m/s}$$

10. (4)

According to keplers law.

$T^2 \propto R^3$

$$\Rightarrow \quad \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3 \text{ or } \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{4R}{R}\right)^3$$
$$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{64} = 8$$
$$\Rightarrow \quad T_2 = 8T_1 \text{ or } \boxed{T_2 = 8T}$$

11. (3) For series limit of Paschen series, transition from $n_1 = 3$ to $n_2 = \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$= R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$$
$$\frac{1}{\lambda} = \frac{R}{9}$$
$$\Rightarrow \quad \lambda = \frac{9}{R} = 9 \times 912 \text{ Å}$$
$$= 82.8 \times 10^{-9}$$
$$= 822 \text{ nm}$$

12. (1)

Orbital velocity of satellite near the Earth's surface is :

$$V_0 = \sqrt{gRe}$$

$\delta = 9.8$ m/s^2, Re $= 6.4 \times 10^6$ m

$$\Rightarrow \quad V_0 = 7\text{-}92 \times 10^3 \text{ m/s}$$
$$\Rightarrow \quad 7.22 \text{ km/s}$$
$$\simeq 8 \text{ km/s}$$

13. (3) $\frac{1}{2}mv_{max}^2 = eV_0 = (hv - hv_0)$

$$eV_0 = (hv - \phi_0)$$

when frequency doubled

$$eV_0' = (2hv - \phi_0)$$
$$\frac{V_0'}{V_0} = \frac{(2hv - \phi_0 - \phi_0 + \phi_0)}{(hv - \phi_0)}$$
$$\frac{V_0'}{V_0} = \frac{[2(hv - \phi_0) + \phi_0]}{(hv - \phi_0)}$$
$$\frac{V_0'}{V_0} = 2 + \left[\frac{(\phi_0)}{(hv - \phi_0)}\right]$$

$\Rightarrow$ V_0' will become more than $2V_0$

14. (2)

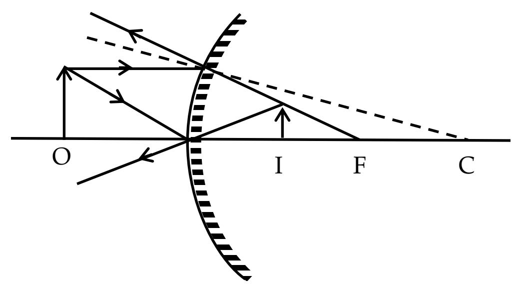
Given -

$B = 5 \times 10^{-6}$ T, $r = 20$ cm $= 0.2$ m

Magnetic Field $\left[B = \frac{u_0 m}{4\pi r^3}\right]$

$$\Rightarrow \quad 5 \times 10^{-6} = \frac{10^{-7} \times M}{(0.2)^B}$$
$$\Rightarrow \quad \mathbf{M} = \frac{5 \times 10^{-6} \times 0.2 \times 0.2 \times 0.2}{10^{-7}}$$
$$= 0.4 \text{ JT}$$

15. (2)



$$u = -30 \text{ cm}$$
$$f = 30 \text{ cm}$$
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$\frac{—}{v} = \frac{1}{30} + \frac{1}{30}$$
$$\Rightarrow \quad v = 15 \text{ cm.}$$

Hence, the formation of image will be at 15 cm behind convex mirror, which is virtual, erect and diminished Hence, the separation between the object and the image is 30 cm + 15 cm = 45 cm.

16. (1) Given, Thickness of glass slab = 4 cm

Let the number of waves be x

Thickness of water = 5 cm

The number of waves = x

(∵ it is given in the question)

We know, wave number $y_g = \frac{x}{4}$ and $y_w = \frac{x}{5}$

But, we know that refractive index $(\mu) \propto \frac{1}{\lambda}$ and $\frac{1}{\lambda}$ = wave number

So, $\mu \propto y$

Now, $\frac{\mu_g}{\mu_w} = \frac{y_g}{y_w}$

$\Rightarrow \frac{\mu_g}{\mu_w} = \frac{\frac{x}{4}}{\frac{x}{5}}$

$\Rightarrow \frac{\mu_g}{\mu_w} = \frac{5}{4}$

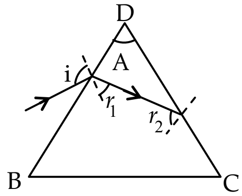
$\mu_g = \frac{5}{4} \times \mu_w$

Now, substituting the values

$\mu_g = \frac{5}{4} \times \frac{4}{3}$

$\Rightarrow \mu_g = \frac{5}{3}$

17. (4)



A ray of light will not emerge out from prism for all values of angle of incidence provided, if for maximum angle of incidence (i_1 = 90° = maximum) at face DB, angle of incidence on face DC, i.e. r_2 is greater than critical angle θ_C

At DB, $1 \times \sin 90° = \mu \sin r_1$

$\Rightarrow r_1 = \sin^{-1}\left(\frac{1}{\mu}\right)$

or $r_1 = \theta_C$ ...(1)

For no emergence

$r_2 > \theta_C$ ...(2)

Adding equation (1) and (2)

$r_1 + r_2 > 2\theta_C$ ...(3)

Also $r_1 + r_2 = A$ ...(4)

We know that for any triangle, sum of all angle = 180

$90 - r_1 + 90 - r_2 + A = 180$

$A = r_1 + r_2$

Now, solving equation (3) and (4)

$A > 2\theta_C$

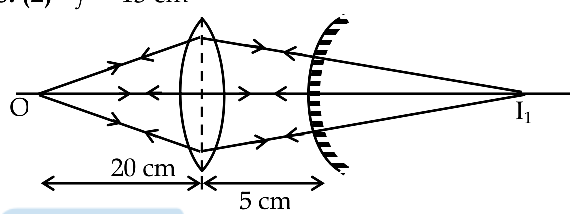
$\Rightarrow \sin\frac{A}{2} > \sin\theta_C$

$\mu > \operatorname{cosec}\frac{A}{2}$

So for emergence, maximum value of

$\mu = \operatorname{cosec}\frac{A}{2} = \sqrt{1+\cot^2\frac{A}{2}}$

18. (2) f = 15 cm



In the absence of convex mirror, I_1 be the image formed by convex lens. Since mirror is at distance 5 cm from the lens, So, I_1 is at distance 60 – 5 = 55 cm. From convex mirror. Now final image I_2 is formed at O itself, means rays after reflection from mirror, incident on mirror normally. Therefore I_1 is centre of curvature.

Hence, R_1 = 55 cm.

19. (1) Separation between the slit $d = 3\lambda$

So, fringe width $\beta = \frac{\lambda D}{d} = \frac{\lambda D}{3\lambda} = \frac{D}{3}$

When A thin film of thickness 3λ and refractive index 2 has been placed in front of the upper slit then distance of the central maxima on the screen from O is

$$x = \frac{\beta}{\lambda}(\mu - 1)t$$

here $\beta = \frac{D}{3}$, $t = 3\lambda$ and $\mu = 2$

So, distance $x = \frac{D}{3\lambda}(2-1)\times 3\lambda$

$x = D$

20. (2) Diameter of bigger piston = 35 cm

Diameter of smaller piston = 10 cm

$P_1 - P_2 = h\rho g$

$$\frac{20\times9.8}{\pi(0.05)^2}-\frac{F}{\pi(0.175)^2} = 1.5 \times 750 \times 9.8$$

$$249680-\frac{F}{0.096} = 11025$$

$$F = (24968 - 11025) \times 0.096$$
$$F = 1338 \text{ N}$$
$$F = 1.3 \times 10^3 \text{ N}$$

21. [0.829] Angular speed $= \omega = \frac{21}{44}\text{rps} = \frac{21}{44}\times 2\pi$ rad/s

$$\therefore v = \omega R = \frac{21}{22}\pi\times\frac{1}{2} = \frac{21\pi}{44}$$

$$\left[\because \text{diameter} = 1, \text{radius} = \frac{1}{2}\right]$$

For next part of motion, using $s_y = \frac{1}{2}gt^2$

$$\Rightarrow \quad 1.5 = \frac{9.8t^2}{2}$$

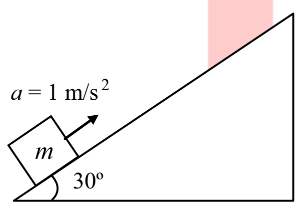
$$t = \sqrt{\frac{3}{9.8}}$$

Now using $s_x = u_x t = vt$

$$s_x = \frac{21}{44}\times\sqrt{\frac{3}{9.8}} = 0.829 \text{ m}$$

Drops hit the floor at a horizontal distance of 0.829 m from umbrella

22. [48.00]



at $t = 4$ sec
if $u = 0$
then $v = u + at$

$v = 0 + 1 \times 4$
$v = 4$ m/s
$m = 2$ kg
$a = 1$ m/s^2

So, power delivered at $t = 4$ sec

F – mg sin 30° = 2 × 1

$$F = \frac{2\times10}{2}+2$$
$$F = 12 \text{ N}$$

$$P = \vec{F}\cdot\vec{v}$$
$$= m\vec{a}_r\cdot\vec{v}$$
$$= 2 \times 6 \times 4$$
$$= 48 \text{ J/s}$$

23. [4.66] $m_2 = 6 \times 10^{24}$ kg mass of earth

v_1 when ball reaches earths surface $= \sqrt{2gh}$

$$= \sqrt{2\times9.8\times10}$$
$$v_1 = \sqrt{196} = 14 \text{ m/s}$$

Since, there is no external force, so velocity of center of mass must remain zero.

$$\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2} = 0$$
$$m_2\vec{v}_2 = -m_1\vec{v}_1$$
$$v_2 = \frac{20\times14}{6\times10^{24}}$$
$$= 4.66 \times 10^{-23} \text{ m/s}$$

24. [1.15]

$$\frac{mv^2}{r} = f_{s(\max)}$$

(To prevent Car from slipping)

$$\Rightarrow \quad = \frac{mv^2}{r} = \mu_s mg$$

$$\Rightarrow \mu_s = \frac{v^2}{rg} = \frac{(15)^2}{20\times9.8} = 1.15$$

$(\because 54 \text{ km/h} = 15 \text{ ms}^{-1})$

25. [7.92] Let mass of earth = M

Radius = R

Then escape velocity from earth's surface

$$V_e = \sqrt{2gR}$$

Velocity of satellite $V_s = \frac{V_e}{2} = \frac{\sqrt{2gR}}{2}$ ...(1)

$$V_s = \sqrt{\frac{GM}{r}} = \sqrt{\frac{R^2g}{R+h}}$$

$$V_s^2 = \frac{R^2g}{R+h} \quad \text{...(2)}$$

From equation (1) and (2)

We get $h = R = 6400$ km

(ii) Now total energy at height h

= total energy at earth's surface

$$\therefore 0-\frac{GMm}{R+h} = \frac{1}{2}mV^2-\frac{GMm}{R}$$

Solving we get $V = \sqrt{\frac{R}{g}}$

$$V = \sqrt{64000\times10^3\times9.8}$$
$$V = 7.92 \text{ km/sec}$$

26. [4.00] The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ then $T = 2\pi\sqrt{\frac{\ell'}{g'}}$

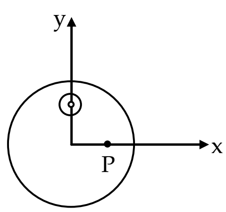
$$\ell' = \frac{\ell}{\sin\theta} = \sqrt{2}\ell$$

$$g' = g\cos\theta = \frac{g}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{2\ell}{g}} = \frac{2\pi}{5} = \frac{4\pi}{10}$$

$$x = 4.$$

27. [35.16]



Assume, given sphere is solid, potential V_1 at P is to be calculated. But in cavity there is no charge, therefore potential V_2 due to charge assumed in cavity must be subtracted from V_1.

Charge on solid sphere $= \frac{4}{3}\pi R^3 \times \rho$

$$= \frac{5}{3}\times 10^{-10}\ \text{C}$$

Potential at P can be calculated, say V_1

V_2 = Potential due to cavity sphere

$$= \frac{\frac{4}{3}\pi r^3\rho}{4\pi\varepsilon_0 a} = 0.24\ \text{V}$$

Potential at P $= V_1 - V_2 = 35.16$ Volt

28. [20.00] $\frac{q_1}{C_1} = \frac{q_2}{C_2}$

$$q_1 + q_2 = 2Q_0$$

$$C_1 = \frac{\varepsilon A}{d_0 + Vt},\ C_2 = \frac{\varepsilon A}{d_0 - Vt}$$

$$\frac{q_1}{q_2} = \frac{d_0 - Vt}{d_0 + Vt}$$

$$\Rightarrow q_1 + q_2\left(\frac{d_0 - Vt}{d_0 + Vt}\right) = 2Q_0$$

$$\Rightarrow q_2\left(\frac{2d_0}{d_0 + Vt}\right) = 2Q_0$$

$$\mu\ q_2 = \frac{2Q_0}{2d_0}(d_0 + Vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 V}{d_0} = 20\ \text{A}$$

29. [18.00] Current through one bulb $= \frac{100}{120}$

$$15 = N \times \frac{100}{120}$$

$$N = 18$$

Where, 'N' is number of bulbs.

30. [12.00] $R = \frac{mv}{qB}$

$$q \times 12 \times 10^3 = \frac{1}{2}m\times\left(10^6\right)^2$$

$$\frac{24\times 10^3}{10^{12}} = \frac{m}{q}$$

$$R = \frac{24\times 10^3\times 10^6}{10^{12}\times 0.2}$$

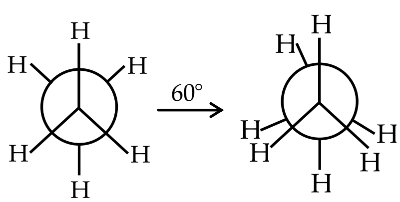
$$R = 12 \times 10^{-2}\ \text{m}$$

$$R = 12\ \text{cm}$$

Chemistry

31. (3) According to IUPAC, the correct name is 4-methyl hex-2-yne.

32. (2) There are two conformation of ethane which are staggered and eclipsed.



staggered eclipsed

33. (2)

Bond strength ∝ B. O.

Bond order $= 1/2(N_b - N_a)$

Molecule	Bond order
O^2	2.0
O_2^+	2.5

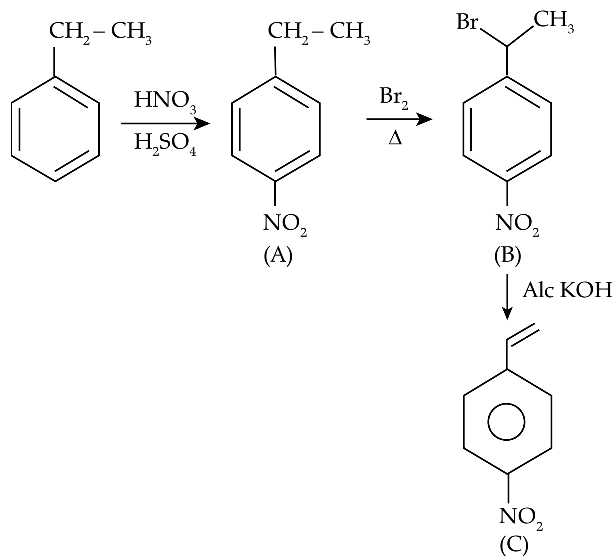
The bond order of O_2 increases from 2 to 2.5 when an electron is removed.

$$O_2 \rightarrow O_2^+$$

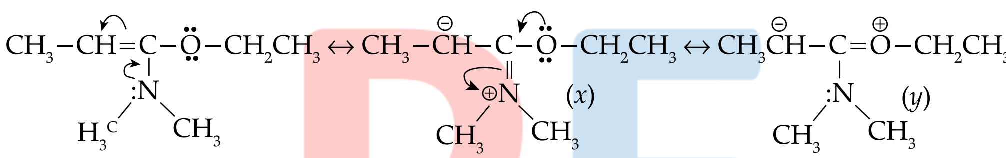
34. (2)

Electron gain enthalpies along a period increases and down the group decreases. Electron gain enthalpy of F is less than Cl due to small size of fluorine atom. Interelectronic repulsions experience seven valence electrons of F. The added one electron experiences much repulsion on smaller F. Hence, F electron gain enthalpy is less.

35. (2)

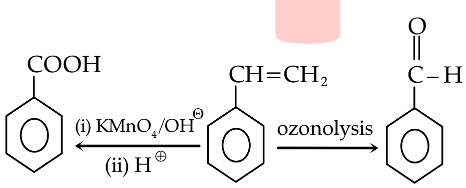


36. (1) In the resonating structures all the atome have complete octet. Thus, (*x*) and (*y*) are acceptable resonating structure.



37. (1) $CH_2 = CH_2 \xrightarrow[KMnO_4]{\text{cold dil}} \underset{OH}{CH_2}-\underset{OH}{CH_2}$ (diol)

38. (3)



Y = COOH *Meta*directing

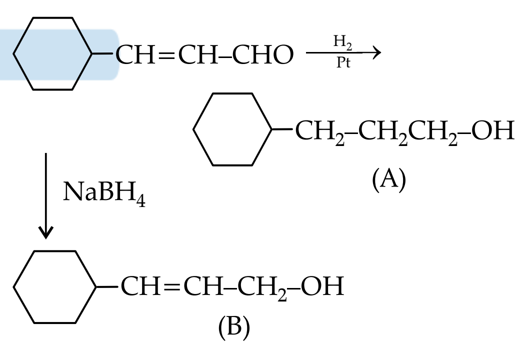
$-CH = CH_2$ or *Para*directing

x = CHO *Meta*directing

39. (3)

$$\mu_{calculated} = e \times d$$
$$= 4.8 \times 10^{-10} \times 1.275 \times 10^{-8}$$
$$d = 1.25 \times 10^{-8} \text{ cm}$$
$$1 \text{ D} = 1 \times 10^{-18} \text{ esu-cm}$$
$$\mu_{observed} = 1.03 \text{ D}$$
$$= 1.03 \times 10^{-18} \text{ esu-cm}$$
$$\% = \frac{\mu_{observed}}{\mu_{cal}} \times 100$$
$$= \frac{1.03 \times 10^{-18} \times 100}{4.8 \times 10^{-10} \times 1.275 \times 10^{-8}}$$

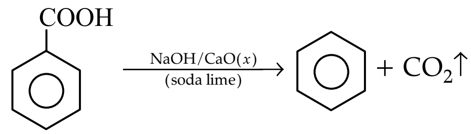
40. (2) $NaBH_4$ reduced only functional group (–CHO)



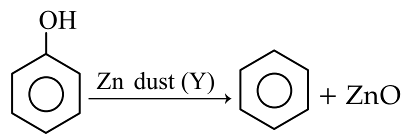
41. (2) $CH_3-\underset{OH}{CH}-CH_3 \xrightarrow{K_2Cr_2O_7/H^+} CH_3-\underset{O}{\overset{\|}{C}}-CH_3 \xrightarrow{I_2 + NaOH} CHI_3 + CH_3-\underset{O}{\overset{\|}{C}}-Na$

Yellow ppt.

42. (4)



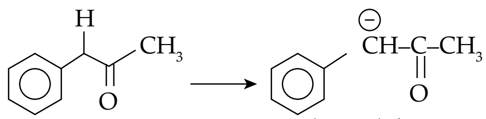
Benzoic acid



Phenol

Therefore X and Y are respectively sodalime and Zn-dust.

43. (3)



(Most acidic)

Resonance through benzene ring (most stable)

The given compound have maximum acidic hydrogen so it has lowest pKa value.

44. (4) X = 75.8% $\frac{75.8}{75} \simeq 1$

Y = 24.2% $\frac{24.2}{16} \simeq \frac{3}{2}$

X : Y = 2 : 3. X_2Y_3

45. (4) $nCH_3-CH=CH_2 \longrightarrow -\!\left[CH(CH_3)-CH_2\right]_n\!-$

Reagent : 1. Trimethyl aluminum and titanium tetrachloride

2. Zeigler natta eatalyst

46. (2) $2Cu(NO_3)_2 \xrightarrow{\Delta} 2Cu_2O + 4NO_2 + O_2$

47. (4) Δ_0 for $RhCl_6^{-3}$ is 234 kJ/mol

So, $E = \frac{\Delta}{6.023 \times 10^{23}} = \frac{234 \times 1000}{6.023 \times 10^{23}}$

$E = 38851.07 \times 10^{-23}$

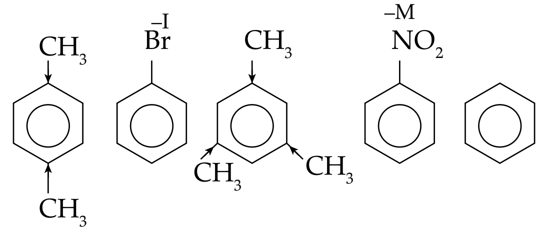
$E = \frac{hc}{\lambda}$

$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{38851.071 \times 10^{-23}}$

$= 511$ nm

48. (2)

D<B<E<A<C



A. p-xylene $-CH_3$ is activating group for nitration

B. bromobenzene –Br is deactivating group

C. mesitylene $-CH_3$ is activating group

D. nitrobenzene $-NO_2$ is a strong deactivating group

E. benzene –NO group present

49. (4) $2AgNO_3 \xrightarrow{\Delta} Ag_2O + 2NO_2\uparrow + 1/2O_2$

Brown gas

$AgNO_3 + KI \rightarrow AgI\downarrow + KNO_3$

yellow ppt.

$2AgNO_3 + BaCl_2 \rightarrow 2AgCl + Ba(NO_3)_2$

yellow ppt.

50. (4) In neutral or alkaline medium, thiosuphate is quantitavely oxidized by $KMnO_4$ into SO_4^{2-}

$$8\overset{+7}{Mn}O_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8\overset{+4}{Mn}O_2 + 6SO_4^{2-} + 2OH^-$$

Change in oxidation state of Mn is from +7 to +4 is 3.

51. [13.71] (A) Given, $V_0 = 22.8$ c.c.

t (min.)	V_t (c.c.)	$= \frac{2.303}{} \log \frac{V_0}{V}$
10	13.8	$k = \frac{2.303}{t} \log \frac{22.8}{13.8}$ $= \frac{2.303}{10} \log 1.652$ $= 0.2303 \times .2180$ $= 0.0502054$
20	8.25	$k_2 = \frac{2.303}{10} \log \frac{22.8}{8.25}$ $= 0.0508387$

$\because$ $k_1 = k_2$

$\therefore$ Reaction is of the first order.

(B) $t_{1/2} = \frac{0.693}{k}$

$= \frac{0.693}{0.0505221}$

$= 13.71$ min

52. [59.00] Radioactive decay reaction follows first order reaction

$$\lambda = \frac{2.303}{t}\log\frac{N_0}{N}$$

$$\frac{0.693}{t_{\frac{1}{2}}} = \frac{2.303}{t}\log\frac{N_0}{N} \qquad \left[\lambda = \frac{0.693}{t_{\frac{1}{2}}}\right]$$

$$\frac{0.693}{5.26} = \frac{2.303}{4}\log\frac{N_0}{N}$$

$$\log\frac{N_0}{N} = 0.2288$$

$$\frac{N_0}{N} = 1.693$$

$$\Rightarrow \qquad \frac{N}{N_0} = \frac{1}{1.693}$$

$\therefore$ % activity remaining = 59%

53. [13.842] According to Gibb's Helmholtz equation, heat of reaction ΔH, given as

$$\Delta H = nF\left[T\left(\frac{\delta E}{\delta T}\right)_P - E\right]$$

T = 273 + 25 = 298 K, n = 2, F = 96500 C,

$E_{cell} = 0.03$ V

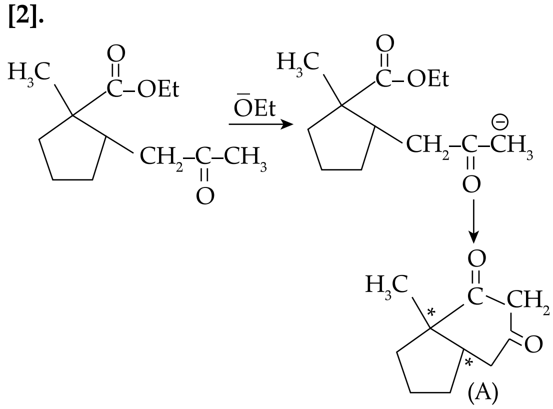
$$\left(\frac{\delta E}{\delta T}\right)_P = -1.4 \times 10^{-4} \text{ V/K}$$

$\Delta H = 2 \times 96500\,[298 \times (-1.4 \times 10^{-4})] - 0.03$

$= -13842$ J

$= -13.842$ kJ/mol

54. [2].



In this compound, there are two chiral carbon.

55. [0].

Half-life period is directly proportional to the pressure. This is applicable for a zero-order reaction.

Given:

$T_1 = 340$

$T_2 = 170$

$P_1 = 55.5$

$P_2 = 27.8$

$t_{1/2} \times 1/[P_0]^{n-1}$

$t_1/t_2 = (P_2)^{n-1}/(P_1)^{n-1}$

$340/170 = (27.8/55.8)^{n-1}$

$n = 0$

At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340s. When the pressure was 27.8 kPa, the half life was found to be 170 s. The order of the reaction is 0.

56. [23.44] Relative lowering of vapour pressure

$$= \frac{P^0 - P_s}{P_s} = \frac{W_2 \times M_1}{W_1 \times M_2}$$

$$= \frac{W_2 \times 1000}{W_1 \times M_2} \times \frac{M}{1000}$$

$$= \text{molality} \times \frac{M}{1000}$$

$$\Rightarrow \quad \frac{P^0 - P_s}{P_s} = \frac{\Delta T}{K_f} \times \frac{M}{100}$$

(As $\Delta T_f = K_f \times m$)

$$\Rightarrow \quad \frac{23.51 - P_s}{P_s} = \frac{0.3}{1.86} \times \frac{18}{1000}$$

So, $P_s = 23.44$ mm Hg

57. [3].

Manganese (VI) disproportionates in acidic medium as

$3MnO_4^{2-} + 4H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Oxidation state of Mn of MnO_4^- = +7

Oxidation state of Mn of MnO_2^- = +4

Difference in oxidation states of Mn in the products formed = 7 – 4 = 3

58. [2.324] Molar heats of formation

$NH_4NO_3(s) = -367.54$ kJ

$N_2O(g) = 81.46$ kJ

$H_2O\,(l) = -285.8$ kJ

$NH_4NO_3\,(s) \rightarrow N_2O(g) + 2H_2O(\ell)$

$\Delta H_{reaction} = \Sigma H_P - \Sigma H_R$

$= [81.46 + 571.6] - [-367.54]$

$$= -490.14 + 367.54$$
$$= -122.6 \text{ kJ}$$

and $\Delta H = \Delta E + \Delta n_g RT$

$$\Delta H = \Delta E - 1 \times \frac{8.314 \times 298}{1000}$$

$$\Delta E = \Delta H - (1) \times \frac{8.314 \times 298}{1000}$$

$$= -122.6 - 2.477$$

$$= -125.076 \text{ kJ}$$

$$\Delta H - \Delta E = -122.6 - (-125.076)$$
$$= 2.324 \text{ kJ}$$

59. [92.40] Let the wt. of $CuFeS_2$ in the 0.5 g of mineral be x.

Then the eq. of $CuFeS_2$ = Eq. of $K_2Cr_2O_7$

$$\Rightarrow \frac{}{183.3/1} = (0.01 \times 6) \times (42 \times 10^{-3})$$

$$\Rightarrow x = 0.462$$

$$\therefore \% \text{ of } CuFeS_2 \text{ in mineral} = \frac{0.462}{0.5} \times 100$$
$$= 92.4\ \%$$

60. [11.00] $S + S^{2} \rightleftharpoons S_2^{2-}, K_1 = 12$ ...(1)

$2S + S^{2-} \rightleftharpoons S_3^{2-}, K_2 = 132$ ...(2)

$S + S_2^{2-} \rightleftharpoons S_3^{2-}, K_3 = ?$

$$K_1 = \frac{(S_2^{-2})}{(S) \times (S^{2-})}$$

$$K_2 = \frac{(S_3^{2-})}{(S)^2 (S^{2-})}$$

Equation (2) / Equation (1)

$$= \frac{K_2}{K_1} = \frac{(S_3^{2-})}{(S)^2 \times (S^{2-})} \times \frac{(S)(S^{-2})}{(S_2^{2-})}$$

$$K_3 = \frac{(S_3^{2-})}{(S)^2 \times (S_2^{2-})} = \frac{K_2}{K_1} = \frac{(S_3^{2-})}{(S) \times (S_2^{2-})}$$

$$K_3 = \frac{(S_3^{2-})}{(S) \times (S_2^{2-})} = \frac{132}{12} = 11$$

Mathematics

61. (3) $I = \int \frac{x}{x}\left(\log(\log x) + \frac{1}{(\log x)^2}\right) dx$

Let $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

and $x = e^t$

$$I = \int e^t \left(\log t + \frac{1}{t^2}\right) dt$$

$$= \int e^t \left(\log t + \frac{1}{t}\right) dt - \int e^t \left(\frac{1}{t} - \frac{1}{t^2}\right) dt$$

$$= e^t \log t - e^t \cdot \frac{1}{t} + c$$

$$= x \log(\log x) - \frac{x}{\log x} + c$$

62. (2) Let $S = \lim_{n \to \infty} \left\{ \left(1 + \frac{1}{n^2}\right)^{\frac{2}{n^2}} \left(1 + \frac{2}{n^2}\right)^{\frac{4}{n^2}} \cdots \left(1 + \frac{n^2}{n^2}\right)^{\frac{2n}{n^2}} \right\}$

Taking logarithm both sides, we get

$$\log S = \frac{2}{n^2} \log\left(1 + \frac{1^2}{n^2}\right) + \frac{4}{n^2} \log\left(1 + \frac{2^2}{n^2}\right)$$
$$+ \ldots\ldots + \frac{2n}{n^2} \log\left(1 + \frac{n^2}{n^2}\right)$$

$$= \sum_{r=1}^{n} \frac{2r}{n^2} \log\left(1 + \frac{r^2}{n^2}\right) = \int_0^1 2x \log(1 + x^2)\, dx$$

put $1 + x^2 = t$

$\Rightarrow$ $2x\,dx = dt$

$$\log_e S = \int_1^2 \log t\, dt = [t \log t - t]_1^2$$

$$= 2 \log 2 - 2 + 1$$

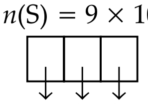
$$\Rightarrow \log S = \log 4 - \log e$$

$$\Rightarrow S = \frac{4}{e}$$

63. (1)

At least two digit are odd no of sample space

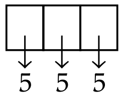
$n(S) = 9 \times 10 \times 10 = 900$



9 10 10

At least two digit are odd

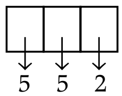
= exactly two digit are odd + exactly three digit are odd



exactly three digit are odd = 5 × 5 × 5 = 125

For exactly two digits are odd

If 0 is used then = 2 × 5 × 5 = 50



If 0 is not used = ${}^3C_1 \times 4 \times 5 \times 5$

= 3 × 4 × 5 × 5 = 300

$$\text{Required Probability} = \frac{300+125+25+7}{900}$$

$$= \frac{475}{900}$$

$$= \frac{19}{36}$$

64. (2) $f'(x) = \log_{1/3}(\log_3(\sin x + a)) < 0 ; x \in R$

$\Rightarrow \quad \log_{3^{-1}}(\log_3(\sin x + a)) < 0$

$\Rightarrow \quad -1 \log_3(\log_3(\sin x + a)) < 0$

$\Rightarrow \quad \log_3(\log_3(\sin x + a)) > 0$

$\Rightarrow \quad \log_3(\sin x + a) > 3^\circ$

$\Rightarrow \quad \log_3(\sin x + a) > 1$

$\Rightarrow \quad (\sin x + a) > 3^1$

$\Rightarrow \quad (\sin x + a) > 3$

Since, $\quad -1 \le \sin x \le 1$

$\Rightarrow \quad -1 + a > 3$ and $1 + a > 3$

$\Rightarrow \quad a > 4$ and $a > 2$

$\Rightarrow \quad a \in (4, \infty)$ and $a \in (2, \infty)$

Most appropriate answer is for all $a \in (4, \infty)$

65. (2)

$$\frac{dy}{dx} = \frac{2xy - 3y^2}{2x^2}$$

Which is Homogeneous linear differential equation of first order

Put $\quad y = x.v$

$$\frac{dy}{dx} = x.\frac{dv}{dx} + v$$

$$x\frac{dv}{dx} + v = \frac{2x.xv - 3x^2v^2}{2x^2}$$

$$x\frac{dv}{dx} + v = \frac{2v - 3v^2}{2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v - 3v^2}{2} - v = \frac{2v - 3v^2 - 2v}{2}$$

$$x\frac{dv}{dx} = \frac{-3v^2}{2}$$

$$\therefore \quad \int\frac{dx}{x} = \int\frac{-2}{3v^2}dv$$

$$\Rightarrow \quad \log x = \frac{-2}{3} \times \frac{-1}{v} + c$$

$$\log x = \frac{2}{3}\frac{x}{y} + c \qquad \text{...(i)}$$

$$\log x - c = \frac{2x}{3y}$$

$$\therefore \quad 3y = \frac{2x}{\log x - c}$$

$$\text{given} \quad y(e) = \frac{e}{3}$$

$$3 \times \frac{e}{3} = \frac{2e}{\log_e e - c}$$

$$\Rightarrow \quad 1 = \frac{2}{1 - c}$$

$$\therefore \quad 1 - c = 2$$

$$c = 1 - 2 = -1$$

$$\log x = \frac{2x}{3y} - 1$$

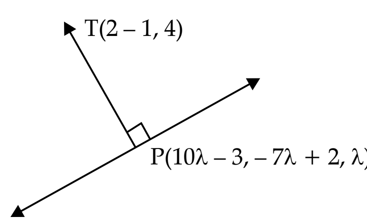
$$\therefore \quad 3y = \frac{2x}{\log x + 1}$$

$$\Rightarrow \quad y = \frac{1}{3}\left(\frac{2x}{\log x + 1}\right)$$

$$\text{Put} \quad x = 1$$

$$y(1) = \frac{1}{3} \times \frac{2 \times 1}{\log 1 + 1} = \frac{2}{3}$$

66. (1) Let P be the foot of perpendicular from point T (2, – 1, 4) on the given line. So P can be assumed as P (10λ – 3, – 7λ + 2, λ)



DR's of TP : 10λ –5, –7λ + 3, λ – 4

∵ TP and given line are perpendicular,

$\Rightarrow 10(10\lambda - 5) - 7(-7\lambda + 3) + (\lambda - 4) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

$\Rightarrow TP = \sqrt{(10\lambda-5)^2+(-7\lambda+3)^2+(\lambda-4)^2}$

$\Rightarrow TP = \sqrt{0+\frac{1}{4}+\frac{49}{4}} = \sqrt{12.5} = 3.54$

Hence, the length of perpendicular is greater than 3 but less than 4.

67. (2) $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -2 & 3 \end{vmatrix}$

$\Rightarrow \quad \vec{a}\times\vec{b} = \hat{i}(9-2)-\hat{j}(6+1)+\hat{k}(-4-3)$

$= 7(\hat{i}-\hat{j}-\hat{k})$

$\therefore \quad \vec{p} = \lambda(\hat{i}-\hat{j}-\hat{k})$

Now, $\vec{p}\cdot(2\hat{i}-\hat{j}+\hat{k}) = \lambda(\hat{i}-\hat{j}-\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})$

$= -6$

$\Rightarrow \lambda\,(2 + 1 - 1) = -6$

$\Rightarrow \quad \lambda = -3$

So, $\quad \vec{p} = -3(\hat{i}-\hat{j}-\hat{k})$

$\Rightarrow \quad \vec{p} = 3(-\hat{i}+\hat{j}+\hat{k})$

68. (4) Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because a^2 + b^2 + c^2 = -2]$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) = (x-1)^2$

Hence, degree = 2.

69. (1) $ax^4 + bx^3 + cx^2 + dx + e$

$$= \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$$

Put $x = 0$ both the sides of above equation

$$\therefore \quad e = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = 0 \text{ (Skew symmetric)}$$

70. (2) $\left(\frac{3}{4}\right)^{6x+10-x^2} < \left(\frac{3}{4}\right)^3$

$\Rightarrow \quad 6x + 10 - x^2 > 3$

$\Rightarrow \quad x^2 - 6x - 7 < 0$

$\Rightarrow \quad (x + 1)(x - 7) < 0$

$\Rightarrow \quad x \in (-1, 7)$

$\Rightarrow$ Number of integral values of $x = 7$

71. (4) $\alpha.\beta = 2$ and $\alpha + \beta = -p$ and $\frac{1}{\alpha}+\frac{1}{\beta} = -q$

$\Rightarrow p = 2q$

Now, $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$= \left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$

$= \frac{9}{2}\left[\frac{5}{2}-\frac{\alpha^2+\beta^2}{2}\right] = \frac{9}{4}[5-(p^2-4)]$

$= \frac{9}{4}(9-p^2) \quad [\because \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta]$

72. (1) $1.3^2 + 2.5^2 + 3.7^2 + \ldots\ldots$ upto 20 terms

$= \sum_{n=1}^{20} n(2n+1)^2 = \sum_{n=1}^{20}(4n^3+4n^2+n)$

$= 4\sum_{n=1}^{20} n^3 + 4\sum_{n=1}^{20} n^2 + \sum_{n=1}^{20} n$

$= 4\left(\frac{20\times 21}{2}\right)^2 + 4\left(\frac{20\times 21\times 41}{6}\right) + \frac{20\times 21}{2}$

$= (420)^2 + 11480 + 210 = 188090$

73. (2) ${}^mC_3 + {}^mC_4 > {}^{m+1}C_3$

$\Rightarrow \quad \frac{{}^{m+1}C_4}{{}^{m+1}C_3} > 1$

$\Rightarrow \quad \frac{\underline{|m+1}}{\underline{|4}\,\underline{|m-3}}\cdot\frac{\underline{|3}\,\underline{|m-2}}{\underline{|m+1}} > 1$

$\Rightarrow \quad \frac{m-2}{4} > 1$

$\Rightarrow \quad m > 6$

$\therefore$ Least value of m is 7

74. (3) Number of all five letter words is $= 10^5$

All five letter words which have no letter repeated is $= {}^{10}P_5$

$= 30240$

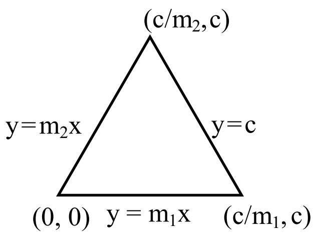
Number of words which have at least one letter repeated is $= 100000 - 30240 = 69760$

75. (2) m_1 and m_2 are roots of

$$x^2 + \left(\sqrt{3}+2\right)x + \left(\sqrt{3}-1\right) = 0$$

$\therefore \quad m_1 + m_2 = -\left(\sqrt{3}+2\right)$

And $\quad m_1 m_2 = \left(\sqrt{3}-1\right)$



$$\Rightarrow \text{Area} = \frac{1}{2}\begin{vmatrix} 0 & \frac{c}{m_1} & \frac{c}{m_2} \\ 0 & c & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{c^2}{m_1} - \frac{c^2}{m_2}\right]$$

$$= \frac{1}{2}c^2\left[\frac{m_2 - m_1}{m_1 m_2}\right]$$

$$= \frac{c^2}{2}\left(\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{m_1m_2}\right)$$

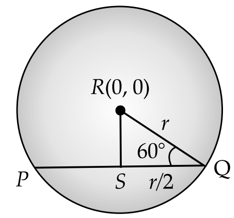
$$= \frac{c^2}{2}\left(\frac{\sqrt{\left(\sqrt{3}+2\right)^2 - 4\left(\sqrt{3}-1\right)}}{\sqrt{3}-1}\right)$$

$$= \frac{c^2}{2}\left(\frac{\sqrt{3+4+4\sqrt{3}-4\sqrt{3}+4}}{\sqrt{3}-1}\right)$$

$$= \frac{c^2}{2}\frac{\sqrt{11}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{c^2}{2} \cdot \frac{\sqrt{33}+\sqrt{11}}{2}$$

76. (4) In right ΔRSQ, $\sin 60° = \frac{RS}{r}$



$$\Rightarrow RS = r \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r}{2}$$

Now, equation of PQ is $y - 2x - 3 = 0$

$$\therefore \frac{\sqrt{3}r}{2} = \frac{|0+0-3|}{\sqrt{5}}$$

$$\Rightarrow \frac{\sqrt{3}r}{2} = \sqrt{\ } \Rightarrow r = \frac{2\sqrt{3}}{5} \Rightarrow r^2 = \frac{12}{5}$$

77. (4) Circle passes through $A(0, 1)$ and $B(2, 4)$. So, its centre is the point of intersection of perpendicular bisector of AB and normal to the parabola at (2, 4).

Perpendicular bisector of AB;

$$y - \frac{5}{2} = -\frac{2}{3}(x-1) \Rightarrow 4x + 6y = 19 \quad \text{...(1)}$$

Equation of normal to the parabola at (2, 4) is,

$$y - 4 = -\frac{1}{4}(x-2) \Rightarrow x + 4y = 18 \quad \text{...(2)}$$

$\therefore$ From (1) and (2), $x = -\frac{16}{5}$, $y = \frac{53}{10}$

$\therefore$ Centre of the circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

78. (3) Given equation of ellipse is

$$x^2 + 4y^2 + 8y - 2x + 1 = 0$$

$$\Rightarrow \quad (x-1)^2 + 4(y^2 + 2y) = 0$$

$$\Rightarrow \quad \frac{(x-1)^2}{4} + \frac{(y+1)^2}{1} = 1$$

$\therefore$ Eccentricity of ellipse is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Foci of the ellipse are given by

$(1 \pm ae, -1)$

where, $ae = \sqrt{a^2 - b^2}$

$\Rightarrow \quad ae = \sqrt{4-1} = \sqrt{3}$

$\Rightarrow \quad$ Foci are $(1 \pm \sqrt{3}, -1)$

Now, latus rectum of the ellipse is given by

$$= \frac{2b^2}{a} = \frac{2\times 1}{2}$$
$$= 1$$

79. (3) $\frac{\text{distance of P from the fous}}{\text{distance of P from the directrix}}$ = eccentricity

Here, P is (x, y) and focus is (1, 2)

$$\because \quad (x-1)^2 + (y-2)^2 = \frac{3(2x+3y+2)^2}{13}$$

Taking square root both sides, we get

$$\sqrt{(x-1)^2+(y-2)^2} = \frac{\sqrt{3}}{\sqrt{13}}(2x+3y+2)$$

$$\Rightarrow \frac{\sqrt{(x-1)^2+(y-2)^2}}{\frac{(2x+3y-2)}{\sqrt{13}}} = \sqrt{3}$$

$$\therefore \quad e = \sqrt{3}$$

80. (4) $y''(1, 3) = 0$

$\Rightarrow \quad 6a(1) + 2b = 0$

$\Rightarrow \quad b = -3a$

$y = 9x^3 + bx^2$

Point (1, 3), then $a+b = 3$

$a - 3a = 3$

$$a = \frac{-3}{2}$$

$$b = \frac{9}{2}$$

81. [9.00] Range of $a\cos x + b\sin x$ is

$$\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$$

Here, $a = \frac{-13}{2}$, $b = -\frac{3\sqrt{3}}{2}$

Now, $b^2 + a^2 = \frac{169}{4} + \frac{27}{4} = \frac{196}{4} = 49$

So, range of $-\frac{13}{2}\sin x - \frac{3\sqrt{3}}{2}\cos x$ is [–7, 7]

∴ Range of $f(x)$ = [–7, 7] + 2 = [–5, 9]

So, maximum value of $f(x)$ = 9

82. [97.00] $$x = \frac{\sin^3\theta}{\cos^2\theta} = \frac{\sin\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$\Rightarrow \quad x = \frac{\sin\theta}{\cos^2\theta} - \sin\theta, \quad y = \frac{\cos\theta}{\sin^2\theta} - \cos\theta$$

$$\Rightarrow \quad x + y = \frac{(\sin^3\theta + \cos^3\theta)}{\cos^2\theta\sin^2\theta} - \frac{1}{2}$$

$$\Rightarrow \quad (x + y) = \frac{\frac{1}{2}(1-\sin\theta\cos\theta)}{\frac{(\sin 2\theta)^2}{4}} - \frac{1}{2}$$

$$\Rightarrow \quad (x + y) = \frac{2\left(1-\frac{\sin 2\theta}{2}\right)}{\sin^2 2\theta} - \frac{1}{2}$$

Given that $\sin\theta + \cos\theta = \frac{1}{2}$

$$\Rightarrow \quad \sin 2\theta = -\frac{3}{4}$$

$$\Rightarrow \quad x + y = \frac{2\left(1+\frac{3}{8}\right)}{\frac{9}{16}} - \frac{1}{2}$$

$$\Rightarrow \quad x + y = \frac{44}{9} - \frac{1}{2}$$

$$\Rightarrow \quad x + y = \frac{79}{18} = \frac{p}{q}$$

$$\Rightarrow \quad p + q = 97$$

83. [4.00] $f(x) = \sin^{-1}[2x] + \cos^{-1}([x]-1)$

$\Rightarrow -1 \le [2x] \le 1 \quad \& \quad -1 \le [x]-1 \le 1$

$\Rightarrow -1 \le 2x < 2 \quad \& \quad 0 \le [x] \le 2$

$\Rightarrow -\frac{1}{2} \le x < 1 \quad \& \quad 0 \le x < 3$

$\Rightarrow 0 \le x < 1$

∴ Domain of $f(x)$ is [0, 1)

$\Rightarrow [x] = 0 : x \in [0, 1)$

$\Rightarrow 0 \le 2x < 2$

$\Rightarrow [2x] = 0 \text{ or } 1$

Now, $f(x) = \sin^{-1}[2x] + \cos^{-1}(-1)$

$$= \left(0 \text{ or } \frac{\pi}{2}\right) + \pi = \pi \text{ or } \frac{3\pi}{2}$$

$$\Rightarrow a + b + \frac{2d}{c} = 1 + 3 = 4$$

84. [3] $\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4}$

$\Rightarrow \sin 2x = -\frac{3}{4}$,

$\therefore \pi < 2x < 2\pi$

$\Rightarrow \frac{\pi}{2} < x \le \pi$ ...(1)

Now, $\frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4}$

$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore \tan x = \dfrac{-8 \pm \sqrt{64-36}}{6} = \dfrac{-4 \pm \sqrt{7}}{3}$

For $\dfrac{\pi}{2} < x < \pi$, $\tan x < 0$

$\therefore \tan x = \dfrac{-4-\sqrt{7}}{3}$

$m + n = -4 + 7$

$= 3$

85. [2.00] Put $x = y = 0$ in given functional equation

$\Rightarrow \quad 2f(0) = f^2(0)$

$\Rightarrow \quad f(0) = 2$

Now, put $x = 0$ in given functional equation

$\Rightarrow f(-y) + f(y) = f(0)\, f(y)$

$\Rightarrow \quad f(y) = f(-y)$

$\therefore f(-2) - f(-1) + f(0) + f(1) - f(2) = f(0) = 2$

86. [1.00] $\lim\limits_{x\to 0} \dfrac{(\sin x)^2}{(e^{\tan^2 x} - 1)}$

$= \lim\limits_{x\to 0} \left(\dfrac{\sin x}{x}\right)^2 \times \dfrac{\tan^2 x}{(e^{\tan^2 x} - 1)} \times \left(\dfrac{x}{\tan x}\right)^2 = 1$

87. [14].

We know that

$|adj\,A| = |A|^{n-1}$

and $|adj\,(adj\,A)| = |A|^{(n-1)^2}$

Given that

$$adj\,(adj\,A) = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$$

$$\therefore \quad |adj\,(adj\,A)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$\Rightarrow |A|^{(3-1)^2} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$\Rightarrow |A|^4 = 14 \times 14 \times 14\,[1\,(1+2) - 2\,(-1-4) - 1(1-2)]$

$= 14 \times 14 \times 14\,[3 + 10 + 1]$

$\Rightarrow \quad |A|^4 = (14)^4$

$\Rightarrow \quad |A| = 14$

88. [1.00] $y = x^{(\sin x)^y}$

$\Rightarrow \quad \ln y = (\sin x)^y \ln x$

$\Rightarrow \quad \ln y = e^{y\ln(\sin x)}\,.\ln x$

$\Rightarrow \quad \dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{x}e^{y\ln(\sin x)} + \ln x . e^{y\ln(\sin x)} \times \left(y.\dfrac{1}{\sin x}.\cos x + y'\ln(\sin x)\right)$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{\pi/2} + 0$

$\Rightarrow \quad \dfrac{dy}{dx} = 1$

since at $x = \dfrac{\pi}{2},\ y = \dfrac{\pi}{2}$

89. [1.00] $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x$

Let $\dfrac{dy}{dx} = t$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = \dfrac{dt}{dx}$

$\Rightarrow \quad \dfrac{dt}{dx} + t = x$

$\Rightarrow \quad te^x = \int xe^x dx + c$

$\Rightarrow \quad te^x = xe^x - e^x + c$

$\Rightarrow \quad \dfrac{dy}{dx} = x - 1 + ce^{-x}$

$\because \quad y'(0) = 1$

$\Rightarrow \quad c = 2$

$\Rightarrow \quad \dfrac{dy}{dx} = 2e^{-x} + x - 1$

$\Rightarrow \quad y = -2e^{-x} + \dfrac{x^2}{2} - x + c$

$\because \quad y(0) = 1$

$\Rightarrow \quad c = 3$

$\Rightarrow \quad y(1) = -\dfrac{2}{e} + \dfrac{1}{2} - 1 + 3$

$\Rightarrow \quad [y(1)] = 1$

90. [4.00] E_1 = Event that he has put on white sock first

E_2 = Event that the second sock is also white.

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$= \frac{{}^6C_1 \times {}^5C_1}{{}^6C_1 \times {}^9C_1} = \frac{5}{9} = \frac{M}{n}$$

$\therefore \quad n - M = 4$

□□