# ANSWERS WITH EXPLANATION

## Physics

**1. (1)** Rotational Equilibrium

When $\tau = 0 \rightarrow$ Either the body does not rotate or rotate with constant angular Velocity ($\vec{\omega}$), it is said to be in Rotational Equilibrium.

**2. (3)** Work function: $\phi = \frac{1240}{\lambda} \text{eV} = \frac{1240}{620} = 2\,\text{eV}$

Energy of the photon: $U = \frac{1240}{400} = 3.2\,\text{eV}$

Maximum kinetic energy of photoelectrons: $3.2 - 2 = 1.2$ eV

Stopping Potential: 1.2 V

**3. (4)** Due to time Varying Electric field.

$\Rightarrow$ Displacement Current is due to the time Varrying Electric Field.

$\Rightarrow$ It does not depend on the moving charges.

**4. (2)** Binding energy (BE) of atom $E_n = \frac{13.6(z^2)}{(n^2)\text{eV}}$

Now, substituting the values

For hydrogen atom $(z) = 1$

$n = 2$

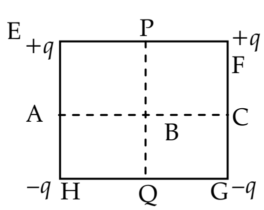
$$E_2 = \left|-13.6 \times \frac{\text{eV}}{2^2}\right|$$

$$= \left|\frac{-13.6\,\text{eV}}{4}\right|$$

$$= |-3.4\text{ eV}|$$

Hence, the required binding energy of the hydrogen atom be 3.4 eV

**5. (2)**




Potential at A $= V_E + V_H$

$$= \frac{kq}{r} - \frac{kq}{r}$$

$$= 0$$

Potential at B $= V_E + V_F + V_G + V_H$

$$= \frac{kq}{r_1} + \frac{kq}{r_1} - \frac{kq}{r_1} - \frac{kq}{r_1}$$

$$= 0$$

Potential at C $= V_F + V_G$

$$= \frac{kq}{r} - \frac{kq}{r} = 0$$

Potential at P $= V_E + V_F$

$$= \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}$$

Potential at Q $= V_H + V_G$

$$= -\frac{kq}{r} - \frac{kq}{r} = -\frac{2kq}{r}$$

So potential will be zero at point A, B and C, therefore correct option is (2)

**6. (4)** Time taken to reach the current from 0 to maximum $= \frac{T}{4}$

Time period (T) $= \frac{1}{f} = \frac{1}{50}$

$$= \frac{\ }{200}$$

$$= 5 \times 10^{-3}\text{s}$$

$$I_{rms} = 10\text{ A}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$I_0 = I_{rms} \times \sqrt{2} = 10\sqrt{2}\text{ A}$$

$$= 14.14\text{ A}$$

**7. (1)** While in air, fringe width

$$\beta = \frac{\lambda D}{d} = 0.4\text{ mm}$$

When immersed in liquid, fringe width

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda}{\mu}\frac{D}{d} = \frac{1}{\mu}\left(\frac{\lambda D}{d}\right)$$

$$= \frac{1}{\left(\frac{4}{3}\right)}(0 \cdot 4)$$

$$= 3\ (0.1) = 0.3\text{ mm}$$

**8. (2)**

$$\frac{1}{f_a} = (1.5-1)\left(\frac{2}{R}\right)$$

$$f_a = R = 10 \text{ cm}$$

when dipped in liquid

$$\frac{1}{f_L} = \left(\frac{1.5}{3}-1\right)\times\frac{2}{R}$$

$$\frac{1}{f_L} = -\frac{1}{R}$$

$$\Rightarrow \quad f_L = -R = -10 \text{ cm.}$$

Diverging lens of focal length 10 cm.

**9. (3)** $\mu_v = 1.5230$, $\mu_r = 1.5145$,

Mean refractive index $(\mu) = \frac{\mu_v + \mu_r}{2}$

$$= \frac{1.5230 + 1.5145}{2}$$

$$\mu = 1.5187$$

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

$$= \frac{1.5230 - 1.5145}{1.5187 - 1} = \frac{0.0085}{0.5187}$$

$$= 0.0163$$

**10. (3)** Let $t_1$ be the thickness of the glass slab when viewed from one side and $t_2$ be the thickness of the glass slab when viewed from other side.

Distance of air bubble $(d) = t_1\left(1-\left(\frac{1}{\mu}\right)\right)$

$$6 = t_1\left(1-\frac{1}{1.5}\right) = \frac{t_1}{3}$$

$$\Rightarrow \quad t_1 = 18 \text{ cm}$$

$$4 = t_2\left(1-\frac{1}{1.5}\right) = \frac{t_2}{3}$$

$$\Rightarrow \quad t_2 = 12 \text{ cm}$$

$$t_{\text{actual}} = \frac{t_1 + t_2}{2} = \frac{18+12}{2}$$

$$t = 15 \text{ cm}$$

**11. (2)**

$$m_I L_f = m_c S_c \,\Delta T_c$$

$$\Rightarrow \quad m_I = \frac{m_c S_C \Delta T_C}{L_f}$$

$$= \frac{2.5 \times 0.39 \times (500-0)}{335}$$

$$= 1.455 \text{ kg}$$

**12. (2)**

$$PV = nRT$$

$$\Rightarrow \quad PV = \frac{N}{N_A}RT$$

Avogadro's number $= 6.0 \times 10^{23}$ particles per mole

760 mm mercury pressure $= 10^5$ Pa

So, $10^{-5}$ mm pressure $= \frac{10^{-5} \times 10^5}{760}$ Pa

$$\Rightarrow \quad N = \frac{PVN_A}{RT}$$

$$= \frac{\left[\frac{10^{-5}}{760}\times 10^5\right]\times\left[1\times 10^{-6}\right]\times 6\times 10^{23}}{8.3\times 273}$$

$$N = \frac{1}{283556}\times 10^{17}$$

$$= 3.5 \times 10^{11}$$

**13. (3)** In the P-V graph, area enclosed by the curve leads to the work done,

In anticlockwise cycle work 13 negative

So area enclosed by the curve(A)

$$= -\frac{1}{2}\times(3P_0 - P_0)\times(3V_0 - V_0)$$

$$= -2P_0V_0$$

**14. (1)** 100°C B $R_B$ T°C C $R_C$ 0°C

$$100 - T°C = \frac{R_B}{R_B + R_C}\times[100-0]$$

$$\Rightarrow \quad T = 100\left[\frac{R_C}{R_B + R_C}\right]$$

We know that $R = \frac{l}{KA}$

$$T = \frac{\frac{1}{K_C}}{\frac{1}{K_B}+\frac{1}{K_C}}100$$

$$= \frac{1}{\frac{K_C}{K_B}+1}\times 100 \quad \left(\text{Where } \frac{K_C}{K_B} = \frac{1}{4}\right)$$

$$= \frac{4}{5}\times 100 = 80°C$$

**15. (4)** Let length of cylinder initially be $\ell_1$, coefficient of linear expansion be $\alpha$, and rise in temperature be $\Delta t$°C, so

$$\Delta\ell = \ell\,\alpha\Delta t \Rightarrow \frac{\Delta\ell}{\ell}\times 100 = \alpha\Delta t\times 100$$

$\therefore$ $2 = \alpha\Delta t \times 100$

Now, for area

$$\Delta A = A\beta\Delta t \Rightarrow \frac{\Delta A}{A} = \beta\Delta t$$

$$\Rightarrow \frac{\Delta A}{A}\times 100 = \beta\Delta t \times 100$$

% change in area $= \beta\Delta t \times 100$

$= \beta\Delta t \times 100$

We know that $2 \times \alpha \times \Delta t = \beta \times \Delta t$

$= 2\alpha\Delta t \times 100$

$= 2(\alpha\Delta t \times 100)$

$= 2 \times 2 = 4\%$

**16. (2)** Cross-sectional area of string (A)

$= 0.8 \times 10^{-6}\ m^2$,

Density of string ($d$) $= 12.5\times\frac{10^{-3}}{10^{-6}}$

$= 12.5 \times 10^3$ Kg/m$^3$

Tension in the string (T) = 64 N

Velocity of the wave ($\nu$) $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$

$$= \sqrt{\frac{64}{12.5\times 10^3\times 0.8\times 10^{-16}}}$$

$$= \sqrt{\frac{32\times 10^3}{5}} = \sqrt{32\times 200}$$

$= 80$ m/s

**17. (1)** $v = \sqrt{\frac{YP}{\rho}}$ velocity of the sound in the gas

$$\nu \propto \frac{1}{\sqrt{\rho}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{d_2}{d_1}}$$

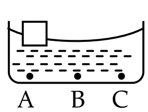
**18. (3)** Ampere

$\Rightarrow$ Henry is the unit of Inductance

$\Rightarrow$ Coulomb is the unit of change.

$\Rightarrow$ The SI unit of displacement current is Ampere.

**19. (1)**




Points A, B and C in the beaker are at the same horizontal level. We know that

$$P = \rho g h$$

where $\rho$ is the density of liquid, $g$ is the gravitational acceleration and $h$ is the height

Hence, the pressure in the beaker will be same at all the mentioned points A, B and C respectively.

So, $P_1 = P_2 = P_3$

**20. (3)** $W = Q.\Delta V$

$\Rightarrow$ $2 = 20 \times \Delta V$

$\Rightarrow$ $\Delta V = 0.1$ volt

**21. [6.00]** Given,

Capacitance of the capacitor (C) = 2μF

$= 2 \times 10^{-6}$F

Initial potential difference across capacitor ($V_0$) = 12 V

Inductance of the inductor (L) = 0.6 mH

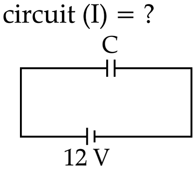
$= 0.6 \times 10^{-3}$ H

$= 6 \times 10^{-4}$ H

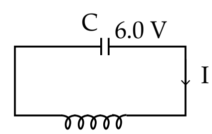
Now potential difference across the capacitor (V) = 6.0 V

Current in the circuit (I) = ?

Initially,




Finally,




Now, applying the conservation of energy

$$\frac{1}{2}CV_0^2 = \frac{1}{2}LI^2 + \frac{1}{2}CV^2$$

Now, substituting the values

$$\frac{1}{2}\times 2\times 10^{-6}\times 12^2$$

$$= \frac{1}{2}\times 6\times 10^{-4}\times I^2 + \frac{1}{2}\times 2\times 10^{-6}\times 6^2$$

$\Rightarrow$ $144 \times 10^{-6}$ J $= 3I^2 \times 10^{-4} + 36 \times 10^{-6}$ J

$\Rightarrow$ $3I^2 \times 10^{-4} = 144 \times 10^{-6}$ J $- 36 \times 10^{-6}$ J

$\Rightarrow$ $I^2 = \frac{108\times 10^{-6}\,J}{3\times 10^{-4}}$

$\Rightarrow$ $I^2 = 36 \times 10^{-2}$

$\Rightarrow$ $I = \sqrt{0.36}$

$\Rightarrow$ $I = 0.6$ A

$\Rightarrow$ $I = 6 \times 10^{-1}$ A

**22. [4.40]** As the voltage across the charging battery,

$$V = E + Ir$$
$$= 30 + 15 \times 0 = 30\text{ V}$$

So, the potential difference across the resistance

$$V_R = 120 - 30$$
$$= 90\text{ V}$$

So, the power wasted in heating the circuit

$$P = VI$$
$$= 90 \times 15$$
$$= 1350\text{ W}$$

So, the energy wasted as heat in time $t$

$$H = P \times t$$
$$= (1350 \times t)\text{ J}$$
$$= \left(\frac{1350}{4.2}\right) \times t\text{ cal.}$$

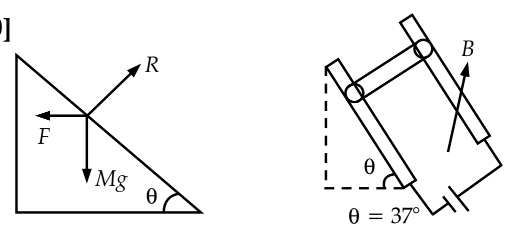
Now, if this heat changes the temperature of 1 kg of water from 15°C to 100°C, then applying the conservation of energy.

Heat lost by the circuit = Heat gain by the water

$$\frac{1350t}{4.2} = mc\Delta\theta$$
$$= 1 \times 10^3 \times 1 \times (100 - 15)$$

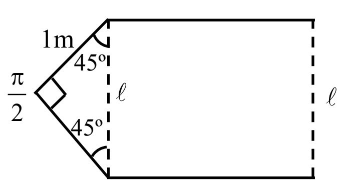
$$\text{i.e., } t = \frac{85 \times 4.2 \times 100}{135}$$
$$= 264.4\text{ s} = 4.4\text{ min.}$$

**23. [3.00]**




$$F\cos\theta = Mg\sin\theta$$
$$BIL\cos\theta = Mg\sin\theta$$
$$B = \frac{Mg}{IL}\tan\theta$$
$$= 0.3 = 3 \times 10^{-1}\text{ Tesla}$$

**24. [1.414]**

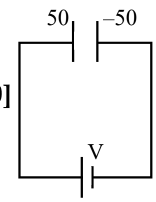



$$V_A - V_D = V \times B \times (\ell \sin 45° + \ell \sin 45°)$$
$$= 1 \times 1 \times \sqrt{2}$$
$$= 1.41\text{ V}$$

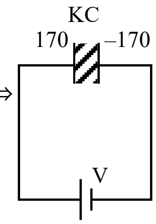
By, sine rule, $\frac{\ell}{\sin 90°} = \frac{1}{\sin 45°}$

$\Rightarrow \quad \ell = \sqrt{2} = 1.414$

**25. [2.40]**




$V = \frac{50}{C} \Rightarrow$




$V = \frac{120}{KC}$

$$\frac{50}{C} = \frac{120}{KC} \Rightarrow K = \frac{120}{50} = 2.4$$

**26. [60.00]**

or $gt^2 - 2ut + 2h = 0$

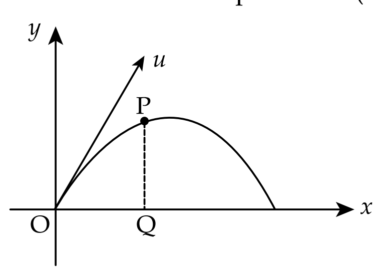
Let $t_1$ and $t_2$ are the roots of the above equation.

For any general quadratic equation $ax^2 + bx + c = 0$, which have the root $x$ and $y$ then $(x + y) = -\frac{c}{a}$ product of root $(xy) = \frac{b}{a}$,

So, product of roots $t_1 t_2 = \frac{2h}{g}$

$$\therefore \quad (t_2 - t_1)^2 = (t_1 + t_2)^2 - 4t_1t_2$$
$$16 = 64 - 4 \times \frac{2h}{g}$$
$$\Rightarrow \quad h = 60\text{ m}$$

**27. [30.00]** Let the velocity of the projectile be $u$ co-ordinate of the point P = (30 m, 40 m)




Trajectory of the projectile

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos\theta}$$

Now, $y = 40$, $x = 30$

Substituting the values

$$40 = 30\tan\theta - \frac{1}{2} \times 10 \times \frac{30^2}{4^2\cos^2\theta}$$

$$40 = 30\tan\theta - \frac{10 \times 900}{2u^2}\left(1 + \tan^2\theta\right)$$

$\Rightarrow 8u^2 = 2 \times 3u^2 \tan\theta - 900(1 + \tan^2\theta)$

$\Rightarrow 900\tan^2\theta - 6u^2\tan\theta + (8u^2 + 900) = 0$

For real value of θ,

if $b^2 - 4ac \geq 0$

$\Rightarrow (6u^2)^2 \geq 4 \times 900 \times (8u^2 + 900)$

$\Rightarrow 36u^4 \geq 3600\,(8u^2 + 900)$

$\Rightarrow u^4 \geq 100(8u^2 + 900)$

$\Rightarrow u^4 \geq 800u^2 + 90000$

$\Rightarrow u^4 - 800u^2 \geq 90000$

$\Rightarrow u^4 - 2 \times 400u^2 + 400^2 \geq 250000$

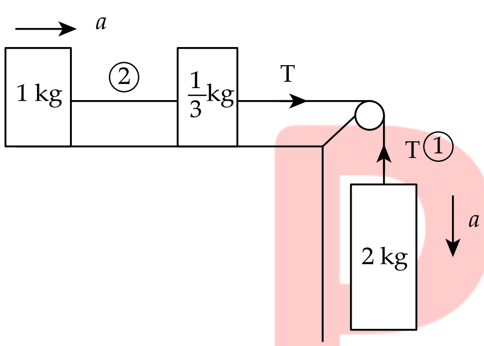
$\Rightarrow (u^2 - 400)^2 \geq 250000$

$\Rightarrow u^2 - 400 \geq 500$

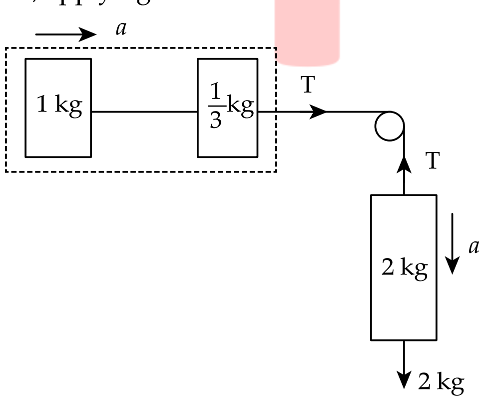
$\Rightarrow u^2 \geq 900$

$\Rightarrow u \geq 30$ m/s

**28. [8.00]** Let the tension in the string 1 be represented as T and the acceleration of the system be represented




Now, applying newton's law of motion




$$2g - T = 2a \qquad ...(1)$$

$$T = \left(1 + \frac{1}{3}\right)a \qquad ...(2)$$

From equation (1) and (2)

$$\Rightarrow \quad 2g = \left(2 + \frac{4}{3}\right)a$$

$$\Rightarrow \quad 2g = \frac{10}{3}a$$

$$\Rightarrow \quad a = \frac{2g \times 3}{10}$$

$$\Rightarrow \quad a = \frac{2 \times 3 \times 10}{10} \qquad (\because g = 10 \text{ m/s}^2)$$

$$\Rightarrow \quad a = 6 \text{ ms}^2$$

So, tension in the string

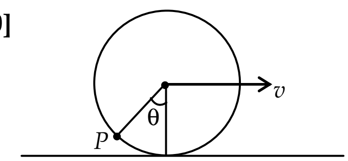
$$T = \left(1 + \frac{1}{3}\right) \times 6$$

$$\Rightarrow \quad T = \frac{4}{3} \times 6$$

$$\Rightarrow \quad T = 8 \text{ N}$$

Hence, the tension in string 1 is 8 N.

**29. [4.00]**




$$V_P = \sqrt{V^2 + V^2 + 2V^2 \cos(\pi - \theta)}$$

$$V_P = 2V \sin\left(\frac{\theta}{2}\right)$$

$$\text{Now } V_P = \frac{ds}{dt} = \frac{ds}{dv} \cdot \frac{dv}{dt}$$

$$\Rightarrow \quad V_P = \omega \frac{ds}{d\theta} = \frac{V}{R}\frac{ds}{d\theta}$$

$$\Rightarrow \quad \frac{V}{R}\frac{ds}{d\theta} = 2V \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad ds = 2R \sin\left(\frac{\theta}{2}\right) d\theta$$

$$\Rightarrow \quad S = 2R \int_0^{2\pi} \sin\left(\frac{\theta}{2}\right) d\theta = 8\,R = 4 \text{ m}$$

**30. [843.75]** Work done by motor

$$= mgh + \frac{1}{2}mv^2 = 225 \times 10000 \text{ J}$$

$$\text{Power of motor} = \frac{225 \times 10000}{1800} \text{W}$$

∴ Electric power used

(P) = 67.5 % of power of motor

$$= \frac{67.5 \times 225 \times 10000}{(1800 \times 100)\text{W}}$$

$$= \frac{67.5 \times 225}{18\text{W}} = 843.75 \text{ W}$$

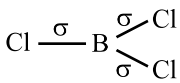
# Chemistry

**31. (4)** Grams of $H_2O = \frac{22.2}{77.8} \times 126 = 35.95$

No. of molecules of $H_2O = \frac{35.95}{18} \simeq 2$

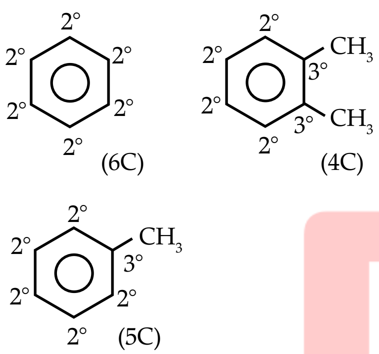
Hence, hydrate is $Na_2SO_3 . 2H_2O$

**32. (3)**

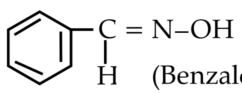



In $BCl_3$, B is the central atom which has three hybrid orbitals and no lone pair, hybridisation of $BCl_3$, is $sp^2$ and shape is trigonal planar.

**33. (1)**

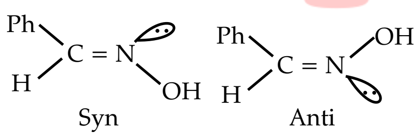



**34. (3)**



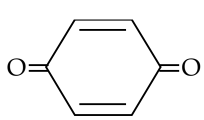

(Benzaldoxime)

**Geometrical isomerism :**




**35. (3)**

An organic compound having two ketone groups present in double-single conjugated molecule.




**36. (2)**

A-II, B-III, C-IV, D-I

Zymase obtained from yeast

A-II

Diastase is obtained from digestive tract of animals and also in malt extract.

B-III

Urease is obtained from soyabean.

C-IV

Pepsin is obtained from Stomach.

D-I

**37. (1)**

$$\underset{\text{Propan-1-ol}}{CH_3-CH_2-CH_2-OH} \xrightarrow{SOCl_2} CH_3-CH_2-CH_2-Cl \xrightarrow{KCN}$$

$$CH_3-CH_2-CH_2-CN \xrightarrow[Na(Hg)/C_2H_5OH]{H_2/Ni} \underset{n\text{-butylamine}}{CH_3-CH_2-CH_2-CH_2-NH_2}$$

**38. (2)** $H_2O \rightleftharpoons H^+ + OH^-$

$$K_h = \frac{[H^+] \cdot [OH^-]}{[H_2O]}$$

$$\underbrace{[H_2O].K_h}_{K_w} = [H^+].[OH^-] \quad [H_3O^+ = OH^-]$$

$$\text{at } 90°C \underbrace{K_\omega}_{10^{-12}} = \underset{10^{-6}}{[H^+]}.\underset{10^{-6}}{[OH]} \quad [\because H^+ = H_3O^+ = 10^{-6}]$$

$$K_\omega = 10^{-12}$$

**39. (1)** $H_2 + I_2 \rightleftharpoons 2HI$

At equ. $\frac{0.4}{5} \quad \frac{0.4}{5} \quad \frac{2.4}{5}$

$$K_c = \frac{\left(\frac{2.4}{5}\right)^2}{\left(\frac{0.4}{5}\right)\left(\frac{0.4}{5}\right)}$$

$$K_c = \frac{2.4 \times 2.4}{0.4 \times 0.4}$$

$$K_c = \frac{5.76}{0.16}$$

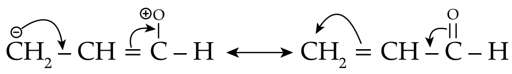
$$K_c = 36$$

**40. (1)** $[\overset{-2}{N} = \overset{+4}{C} = \overset{-2}{S}]^-$

N > C & S > C

(in Electronegativity)

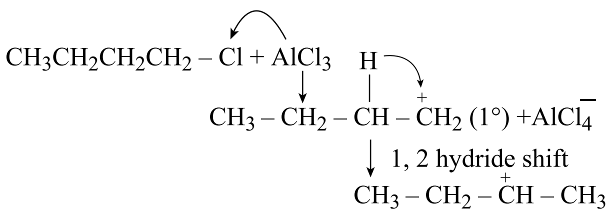
**41. (2)** Resonating structures (II) and (III) will be unstable as there is positive charge on electronegative atom (O). Thus (I) will be maximuum stable. Between (II) and (III), compound (III) will be more stable as negative charge present on carbon can further take part in resonance which can be shown below.



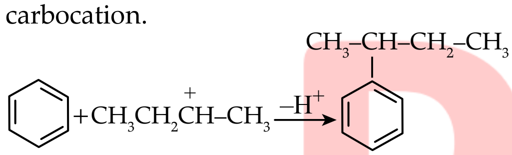
Therefore, order of stability of the given resonating structure is I>III>II.

**42. (2)** $2CH \equiv CH \xrightarrow[NH_4Cl]{Cu_2Cl_2} CH \equiv C{-}CH{=}CH_2$ (Vinylacetylene)

**43. (4)** The reaction is Friedel Craft's reaction and it is an electrophilic substitution reaction the reaction occurs as follows:

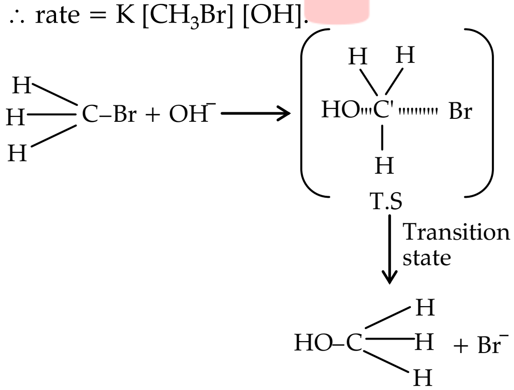


2° carbocation is more stable than 1° carbocation.

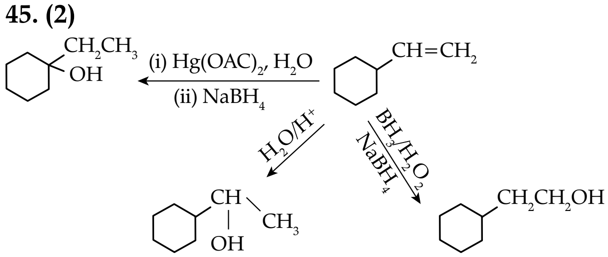


**44. (3)** Since the given alkyl halide is a 1° alkyl halide, so it will undergo $S_N2$ mechanism and thus, the rate of the reaction is dependent on both the reactants.

$\therefore$ rate = K $[CH_3Br]$ [OH].



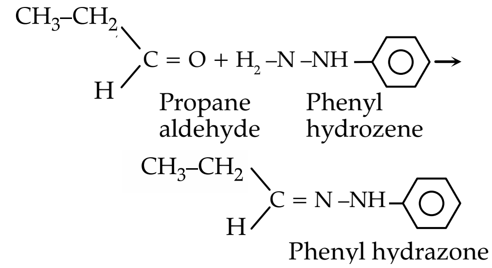
**45. (2)**

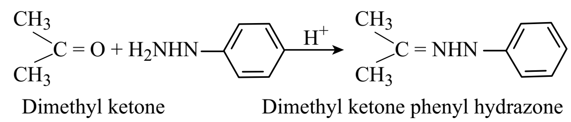


**46. (2)** $CH_2 = CH - CH_2 - OH \rightarrow CH_3CH_2 - CHO$

Allyl alcohol (Propanol)

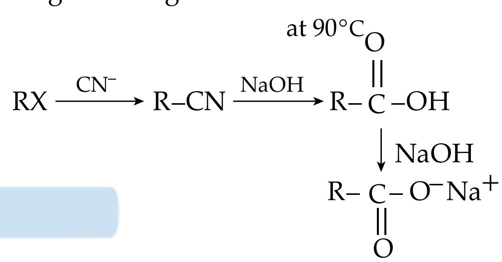
Propanol is an isomer of Allyl alcohol.




Both of the hydrazone contain same percentage of nitrogen.

**47. (3)**



Sodium salt of carboxylic acid.

**48. (1)**

$CH_3NH_2 + CHCl_3 \xrightarrow{3NaOH} CH_3NC + 3NaCl + 3H_2O$

methylamine, chloroform, methylisocyanide (Carbylamine)

**49. (1)** $LiAlH_4$ can only reduce COOH group and not the double bond.

$CH_2 = CH\,COOH \xrightarrow{LiAlH_4} CH_2 = CH - CH_2OH$

**50. (1)**

The depression in the freezing point of a solution is directly proportional to the molal concentration of the solution.

$\Delta T_f = i\, k_f \times m$

$$\frac{(\Delta T_f)_A}{\Delta T_f\, B} = \frac{1}{4}$$

Put the value of $\Delta T_f$ in the above the equation

$$\frac{(\Delta T_f)_A}{(\Delta T_f)_B} = \frac{k_f \dfrac{1\,g}{M_A \times 1\,kg}}{k_f \times \dfrac{1\,g}{M_B \times 1\,kg}}$$

$$\frac{1}{4} = \frac{M_A}{M_B}$$

$$\text{or } \frac{M_A}{M_B} = 1:4$$

**51. [17.49]** Given, $k = 0.008\ min^{-1}$

From unit of k, the reaction is of the first order.

$$k = \frac{2.303}{20}\log\frac{V_\infty}{V_\infty - V_t}$$

$$\Rightarrow \quad 0.008 = \frac{2.303}{20}\log\frac{V_\infty}{V_\infty - 16}$$

$$\Rightarrow \quad 0.0695 = \log\frac{V_\infty}{V_\infty - 16}$$

$$\Rightarrow \quad V_\infty = 17.49\ mL$$

**52. [4.50]** $_{92}U^{238} \longrightarrow {}_{82}Pb^{206} + 8\,{}_2He^4 + 6\,{}_{-1}e^0$

$$\text{Pb present} = \frac{20.6}{206} = 0.1\ \text{g atom} = \text{U decayed}$$

$$\text{U present} = \frac{23.8}{238} = 0.1\ \text{g atom}$$

Thus, N = 0.1 g atom

$N_0$ (U present + U decayed)

= 0.1 + 0.1 = 0.2 g atom

The radioactive decay reaction follows first order kinetics:

$$\therefore t = \frac{2.303}{K}\log\frac{N_0}{N}$$

$$\text{Also, } K = \frac{0.693}{t_{1/2}} = \frac{0.693}{10^9}$$

$$\text{Now, } t = \frac{2.303 \times 4.5 \times 10^9}{0.693}\log_{10}\frac{0.2}{0.1}$$

$$t = 4.5 \times 10^9 \text{ year}$$

**53. [6.17]** $E^\circ = E^\circ_{Zn/Zn^{2+}} - E^\circ_{Ag/Ag^+}$

0.76 – (– 0.80) = 1.56

$$E^0_{cell} = \frac{0.0591}{n}\log k$$

$$\frac{1.56 \times 2}{0.0591} = \log k$$

$$\log k = 52.79$$

$$k = 6.17 \times 10^{52}$$

**54. [20].**

$Fe^{3+} + 3e^- \rightarrow Fe$

3F → 1 mole is deposited

For 56 g → 3 × 96500 (required charge)

For 1 g → $\frac{3 \times 96500}{56}$ (required charge)

For 0.3482 g → $\frac{3 \times 96500}{56} \times 0.3482$

= 1800.06

Q = it

1800.06 = 1.5t

t = 20 min

**55. [353]** $\Delta T_f = \frac{1000 \times K_f \times w}{W \times m}$ ...(1)

$= K_f \times$ molality

= 1.86 × 1 = 1.86 ...(2)

Again from above equation

$$\frac{w}{W} = \frac{\Delta T_f \times m}{K_f \times 1000}$$

$$= \frac{1.86 \times 342}{1.86 \times 1000} = 0.342 \quad ...(3)$$

Again $w$ + W = 1000 ...(4)

[As weight of solution = 1000 g]

From equation (3) & (4) we get

$w$ = 254.84 g

W = 754.16 g

As the solution freezed up to –3.534, the wt. of sucrose remains unchanged.

So, from (i) we get

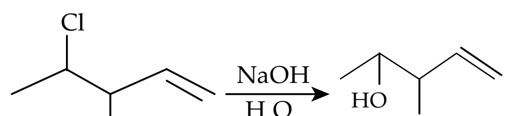
$$3.534 = \frac{1.86 \times 1000 \times 254.84}{342 \times W_1}$$

$W_1$ = 392.18 g

∴ Ice separated = 745.16 – 392.18

= 352.98 g ≃ 353 g

**56. [2].**



P (Major Product)

The given reaction undergoes nucleophilic substitution.

Number of π electrons present in P = 2

**57. [5.00]** Sodium ethane -1-2 -diaminetetracetato chromate (II), $Na_2[Cr(EDTA)]$ contains $2Na^+$.

Sodium hexanitrito cobaltate (III), $Na_3[Co(NO_2)_6]$ contains $3Na^+$.

**58. [75.00]** $Fe^{2+}$ and $Fe^{3+}$

Number of $Fe^{3+}$ = 2 × Number of $Fe^{2+}$

$\therefore$ EF = $2Fe^{3+} . Fe^{2+}O_4^{2-} = Fe_{0.75}O$

$Fe_xO = Fe_{0.75}O$

$x = 0.75$

Value of ($x$ × 100) = 0.75 × 100 = 75

**59. [1216]** $\frac{1}{\lambda} = R_H Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Here, $n_1 = 1 \quad n_2 = 2$

$$= R_H\left[1 - \frac{1}{4}\right] \qquad [\because \frac{1}{R_H} = 912 \text{ Å}]$$

$$\frac{1}{\lambda} = R_H \times \frac{3}{4}$$

$$\lambda = \frac{4}{3R_H} = \frac{4}{3} \times 912 = 1216 \text{ Å}$$

**60. [26.00]** $\mu = \sqrt{n(n+2)} = 5.92$

$$\Rightarrow n^2 + 2n - 35 = 0$$

$$\therefore \quad n = \frac{-2 \pm \sqrt{2^2 - 4 - 35.1}}{2}$$

$$n = 5$$

$\therefore$ Thus, there are 5 unpaired electrons in +3 oxidation state. The electronic configuration of element is [Ar] $3d^6\, 4s^2$

Atomic number of element = 26

## Mathematics

**61. (1)** Given : $|z_1| = |z_2| = |z_3| = 1$

$$\Rightarrow \quad |z_1|^2 = 1 = z_1\bar{z}_1$$

$$|z_2|^2 = 1 = z_2\bar{z}_2$$

$$|z_3|^2 = 1 = z_3\bar{z}_3$$

$$\because \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$$\Rightarrow \left|\frac{z_1\bar{z}_1}{z_1} + \frac{z_2\bar{z}_2}{z_2} + \frac{z_3\bar{z}_3}{z_3}\right| = 1$$

$$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$$

$$\Rightarrow |\overline{z_1 + z_2 + z_3}| = 1$$

$$\Rightarrow |z_1 + z_2 + z_3| = 1$$

**62. (2)** $I = \int 1 \cdot (\log_e x)^2\, dx$

Using integration by patrs, take 1 as second function

$$\therefore \quad I = x(\log_e x)^2 - \int x \cdot \frac{2\log_e x}{x} dx$$

$$= x(\log_e x)^2 - 2\int \log_e x dx$$

Again by parts, we get

$$I = x(\log_e x)^2 - 2\left[x\log_e x - \int x \cdot \frac{1}{x} dx\right]$$

$$= x(\log_e x)^2 - 2x\log_e x + 2x + c$$

**63. (1)**

At least two digit are odd no of sample space

$$n(S) = 9 \times 10 \times 10 = 900$$

9 10 10

At least two digit are odd

= exactly two digit are odd + exactly three digit are odd

5 5 5

exactly three digit are odd = 5 × 5 × 5 = 125

For exactly two digits are odd

If 0 is used then = 2 × 5 × 5 = 50

5 5 2

If 0 is not used = $^3C_1 \times 4 \times 5 \times 5$

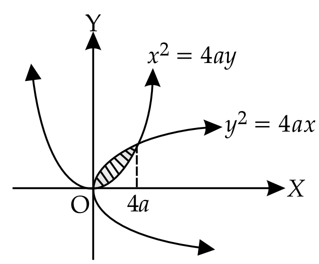
= 3 × 4 × 5 × 5 = 300

$$\text{Required Probability} = \frac{300 + 125 + 25 + 7}{900}$$

$$= \frac{475}{900}$$

$$= \frac{19}{36}$$

**64. (4)** Curve is $y^2 = 4ax$ and $x^2 = 4ay$




Intersection points are (0, 0) and (4a, 4a)

$$\text{So, Area} = \int_0^{4a}\left(\sqrt{4ax} - \frac{x^2}{4a}\right)dx$$

Given that, Area = 1

$$\Rightarrow \int_0^{4a}\left(\sqrt{4ax} - \frac{x^2}{4a}\right)dx = 1$$

$$\Rightarrow \left[\sqrt{4a} \cdot \frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{12a}\right]_0^{4a} = 1$$

$$\Rightarrow \quad \frac{2}{3}(4a)^2 - \frac{(4a)^3}{12a} = 1$$

$$\Rightarrow \quad a = \frac{\sqrt{3}}{4}$$

**65. (2)**

$$\frac{dy}{dx} = \frac{2xy - 3y^2}{2x^2}$$

Which is Homogeneous linear differential equation of first order

Put $y = x.v$

$$\frac{dy}{dx} = x.\frac{dv}{dx} + v$$

$$x\frac{dv}{dx} + v = \frac{2x.xv - 3x^2v^2}{2x^2}$$

$$x\frac{dv}{dx} + v = \frac{2v - 3v^2}{2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2v - 3v^2}{2} - v = \frac{2v - 3v^2 - 2v}{2}$$

$$x\frac{dv}{dx} = \frac{-3v^2}{2}$$

$$\therefore \quad \int \frac{dx}{x} = \int \frac{-2}{3v^2} dv$$

$$\Rightarrow \quad \log x = \frac{-2}{3} \times \frac{-1}{v} + c$$

$$\log x = \frac{2}{3}\frac{x}{y} + c \qquad \text{...(i)}$$

$$\log x - c = \frac{2x}{3y}$$

$$\therefore \quad 3y = \frac{2x}{\log x - c}$$

given $y(e) = \frac{e}{3}$

$$3 \times \frac{e}{3} = \frac{2e}{\log_e e - c}$$

$$\Rightarrow \quad 1 = \frac{2}{1 - c}$$

$$\therefore \quad 1 - c = 2$$

$$c = 1 - 2 = -1$$

$$\log x = \frac{2x}{3y} - 1$$

$$\therefore \quad 3y = \frac{2x}{\log x + 1}$$

$$\Rightarrow \quad y = \frac{1}{3}\left(\frac{2x}{\log x + 1}\right)$$

Put $x = 1$

$$y(1) = \frac{1}{3} \times \frac{2 \times 1}{\log 1 + 1} = \frac{2}{3}$$

**66. (3)**

$$y = \frac{ax + b}{x^2 - 5x + 4}$$

$$\Rightarrow \quad y' = \frac{(ax^2 - 5ax + 4a) - (ax + b)(2x - 5)}{(x-1)^2(x-4)^2}$$

$$= \frac{-ax^2 + 4a - 2bx + 5b}{(x-1)^2(x-4)^2}$$

Since, $y$ has extremum at (2, –1)

$\therefore \quad y'_{(2,-1)} = 0$

$\Rightarrow -4a + 4a - b = 0$

$\Rightarrow \quad b = 0$

and $2a + b = 2$

($\because$ (2, –1) lies on the curve)

Put $b = 0$ in above equation

$\Rightarrow a = 1, b = 0$

**67. (1)** $f(x) = x^3 + 4x^2 + \lambda x + 1$

Differentiating w.r.t. $x$, we get

$f'(x) = 3x^2 + 8x + \lambda + 0 < 0$ …(1)

Also, $f(x)$ is a monotonically decreasing function for $x \in (-2, -2/3)$

$$\Rightarrow (x+2)\left(x + \frac{2}{3}\right) < 0$$

$$\Rightarrow x^2 + 2x + \frac{2}{3}x + \frac{4}{3} < 0$$

$$\Rightarrow x^2 + \frac{8}{3}x + \frac{4}{3} < 0$$

$\Rightarrow 3x^2 + 8x + 4 < 0$ … (2)

Comparing equation (1) and (2),

we get, $\lambda = 4$

**68. (1)** $y = x^{\log_e x}$

$\Rightarrow \quad \log y = \log x . \log n$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\log n}{x}$$

$$= \frac{2\log_e x \; x^{\log_e x}}{x}$$

**69. (2)** $f(x) = \dfrac{\tan x \log(x-2)}{(x^2 - 4x + 3)}$; $x \in (2, 4) - \{3, \pi\}$

$$\text{Now, } \lim_{x \to \pi} f(x) = \lim_{x \to \pi} \frac{\tan x \log(x-2)}{x^2 - 4x + 3}$$

$$= \frac{\tan\pi \log(\pi-2)}{\pi^2-4\pi+3} = 0$$

and $\quad f(\pi) = \frac{1}{6}\tan\pi = 0$

$\therefore \quad f(x)$ is continuous at $x = \pi$

Now, $f(3) = \frac{1}{6}\tan 3$ and $\lim_{x\to3}\frac{\tan x \log(x-2)}{x^2-4x+3}$

$$= \lim_{x\to3}\frac{\tan x \log(x-2)}{(x-1)(x-3)}$$

i.e., $\lim_{x\to3} f(x) = \lim_{x\to3}\frac{\tan x \log(1+x-3)}{(x-1)(x-3)}$

$$= \lim_{x\to3}\frac{\tan x}{x-1}\times\lim_{x\to3}\frac{\log(1+x-3)}{x-3}$$

$\therefore \quad \lim_{x\to3} f(x) = \frac{\tan 3}{2}\times 1 = \frac{1}{2}\tan 3$

$\Rightarrow \quad \lim_{x\to3} f(x) \neq f(3)$

Hence, $f(x)$ is discontinuous at $x = 3$.
Number of values of $x$, where $f(x)$ is discontinuous, is 1.

**70. (4)** Let $A = \lim_{x\to\infty}\left(1+\frac{x+7}{x+2}\right)^{x+4}$

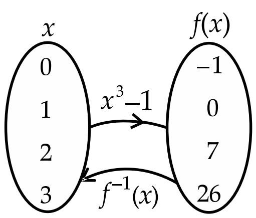
Above limit has indeterminate form of $1^\infty$

So, $A = e^{\lim_{x\to\infty}\left[\frac{x+7}{x+2}-1\right](x+4)}$

$\Rightarrow A = e^{\lim_{x\to\infty}\left[\frac{5}{x+2}\right](x+4)}$

$\Rightarrow A = e^5$

**71. (3)**




$\therefore$ Domain of $f^{-1}(x)$
$=$ range of $f(x)$
$= \{-1, 0, 7, 26\}$

**72. (3)** $\log_{1/3}(x^2 + x + 1) > -1$

$\Rightarrow x^2+x+1 < \left(\frac{1}{3}\right)^{-1}$

$x^2 + x + 1 > 0 \Rightarrow x \in R$

$\Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0$

$\Rightarrow (x-1)(x+2) < 0$

$\Rightarrow x \in (-2, 1)$

**73. (2)** $(y-1)(y-3)\ldots\ldots(y-99)$

Total term is 50

So, coefficient of $y^{49}$ is $-(1+3+5+\ldots..+99)$

$$= -\frac{50}{2}[2+49\times2]$$

$$= -2500$$

**74. (4)** $\alpha, \beta$ are the roots of

$$x^2 + px - q = 0$$

$\Rightarrow \quad \alpha + \beta = -p,$

$\alpha\beta = -q$

$\gamma, \delta$ are the roots of

$$x^2 + px + r = 0$$

$\Rightarrow \quad \gamma + \delta = -p,$

$\gamma\delta = r$

Therefore,

$(\alpha - \gamma)(\alpha - \delta)$

$= \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta$

$= \alpha^2 + p\alpha + r$

$\because \alpha$ is root of

$$x^2 + px - q = 0$$

$\Rightarrow \quad \alpha^2 + p\alpha = q$

$\Rightarrow (\alpha - \gamma)(\alpha - \delta) = q + r$

**75. (3)** $d = \frac{30}{n+1}$

$\because \quad \frac{1+7d}{1+(n-1)d} = \frac{5}{9}$

$\Rightarrow \quad \frac{1+7\frac{30}{n+1}}{1+(n-1)\frac{30}{n+1}} = \frac{5}{9}$

On solving, we get

$$146n = 2044$$

$\Rightarrow \quad n = 14$

**76. (4)** Required ratio $= -\frac{3(1)+4(2)-7}{3(-2)+4(1)-7}$

$$= -\frac{3+8-7}{-6+4-7}$$

$$= -\frac{4}{-9} = \frac{4}{9}$$

$$= 4:9$$

**77. (4)** Mean : $\frac{28+16+9+\lambda}{8} = 7$

$\Rightarrow \quad \lambda = 3$

Variance :

So, $\quad \sigma^2 = \frac{2^2+1^2+4^2+1^2}{8}$

$\Rightarrow \quad \sigma^2 = \frac{11}{4}$

Variance $= \frac{11}{4}$

**78. (4)** Use $t_1 + t_2 = -\frac{2}{t_1}$ ...(1)

Here, $2at_1 = 2$

$\Rightarrow 2 \cdot 1 \cdot t_1 = 2$

$\Rightarrow t_1 = 1$ and

And $2at_2 = 2t$

$\Rightarrow 2 \cdot 1 \cdot t_2 = 2.5$

$\Rightarrow t_2 = t$

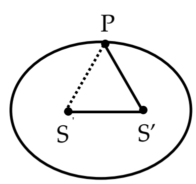
From (1), $1 + t = -\frac{2}{1}$

$= -2$

$\Rightarrow t = -3$

**79. (4)** Given, $2a = 6$ and $2b = 4$

$\therefore a = 3$ and $b = 2$




Distance between two pins $= SS' = 2ae$

Here, $e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$

$\therefore 2ae = 2\times 3\times \frac{\sqrt{5}}{3} = 2\sqrt{5}$

Length of string = SPSS′

= SP + S′P + SS′

$= 2a + 2ae$

$= 6+2\sqrt{5}$

$\therefore$ Length of string $= 6+2\sqrt{5}$ and distance between pins $= 2\sqrt{5}$

**80. (1)** Foci of ellipse (–4, 0) (4, 0)

$9 = 25(1-e^2)$

$e = \frac{4}{5}$

focus (4, 0) of hyperbola

$e = 2$

$b^2 = a^2(e^2-1)$

$ae = 4$

$a = 2$

$\Rightarrow a^2 = 4$

$b^2 = 12$

equation of hyperbola

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

**81. [7.00]** Use $\frac{\pi}{7} + \frac{5\pi}{14} = \frac{\pi}{2}$ and

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

Now, $f(x)+g(x) = \sin x\cos x(\cos^2 x+\sin^2 x)$

$= \sin x \cos x$

$\Rightarrow f\left(\frac{\pi}{7}\right)+g\left(\frac{\pi}{7}\right) = \sin\frac{\pi}{7}\cdot\cos\frac{\pi}{7}$

$= \sin\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)$

$= \cos\frac{5\pi}{14}\sin\frac{5\pi}{14}$

$= g\left(\frac{5\pi}{14}\right)+f\left(\frac{5\pi}{14}\right)$

Now, $7\left[\frac{f\left(\frac{\pi}{7}\right)+g\left(\frac{\pi}{7}\right)}{g\left(\frac{5\pi}{14}\right)+f\left(\frac{5\pi}{14}\right)}\right] = 7$

**82. [8.00]** $y = 1 + (\tan^2\theta + \cot^2\theta) + (\sec^2\theta + \text{cosec}^2\theta)$

$\Rightarrow y = 1 + \sec^2\theta - 1 + \text{cosec}^2\theta - 1 + \sec^2\theta + \text{cosec}^2\theta$

$\Rightarrow y = 2(\sec^2\theta + \text{cosec}^2\theta) - 1$

$= \frac{2}{\sin^2\theta\cos^2\theta} - 1$

$\Rightarrow y = 8\,\text{cosec}^2 2\theta - 1$

$y$ is least if $\text{cosec}^2 2\theta = 1$

$\Rightarrow \cos 2\theta = \pm 1,\ 2\theta \in (0, 8\pi)$

$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$

Total = 8

**83. [7.00]** From cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$

$16 = 9 + c^2 - 2\cdot 3\cdot c\cos 60°$

$\Rightarrow c^2 - 3c - 7 = 0$

**84. [3.00]** Given: $\theta = \sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); x \geq \frac{3}{2}$

$\Rightarrow \theta = \pi - 2\tan^{-1}x + 2\tan^{-1}x$

$\Rightarrow \theta = \pi$

$\therefore \frac{\cos\pi+\sin\pi+4}{\sec\pi} = \frac{-1+4}{-1} = -3$

**85. [7.00]** Required number of ways

= Total when all A's seperated – Total when A's separated and H's are together

$= \frac{7!}{2!}({}^8C_4) - 6!({}^7C_4)$

$= \frac{7!.6!}{4!.3!}(6) = 4^1.5^2.6^3.7^1$

Now, $4^a . 5^b . 6^c . 7^d$

$= 4^1 . 5^2 . 6^3 . 7^1$

$\Rightarrow a + b + c + d = 1 + 2 + 3 + 1 = 7$

**86. [1.00]** $n(S) = 120$

Let E be the event that the number formed is divisible by 4.

So, $n(E) = 3! \times 4 = 24$

$$\therefore \ P = \frac{24}{120} = \frac{1}{5}$$

$$\Rightarrow \ 5P = 5 \times \frac{1}{5} = 1$$

**87. [14].**

We know that

$$|adj A| = |A|^{n-1}$$

$$\text{and} |adj(adj A)| = |A|^{(n-1)^2}$$

Given that

$$adj\,(adj\,A) = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$$

$$\therefore \quad |adj(adj A)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$\Rightarrow \ |A|^{(3-1)^2} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$\Rightarrow |A|^4 = 14 \times 14 \times 14\,[1\,(1+2) - 2\,(-1-4) - 1(1-2)]$$

$$= 14 \times 14 \times 14\,[3 + 10 + 1]$$

$$\Rightarrow \quad |A|^4 = (14)^4$$

$$\Rightarrow \quad |A| = 14$$

**88. [4.00]** $\dfrac{a^{2008} + b^{2008} + 2006}{2008} \geq \left(a^{2008} . b^{2008} \times 1\right)^{\frac{1}{2008}}$

$\Rightarrow a^{2008} + b^{2008} + 2006 \geq 2008|a|\,|b|$

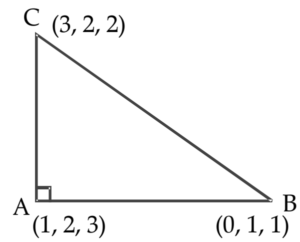
$\therefore a^{2008} + b^{2008} + 2006 = 2008\,|a|\,|b|$

if $a^{2008} = b^{2008} = 1$

$\Rightarrow a = 1, -1$ and $b = 1, -1$

$\therefore$ Total solutions are 4.

**89. [1.00]**




$\because AC^2 + AB^2 = BC^2$

$\Rightarrow \Delta ABC$ is right angled triangle

$\therefore \quad 2 = \sqrt{9+1+1}$

$\Rightarrow \quad 2R = \sqrt{11}$

$\Rightarrow \quad 4R^2 = 11$

$\Rightarrow \quad \dfrac{4R^2}{11} = 1$

**90. [900]** Volume of cube,

$V = x^3$

Differentiate above equation w.r.t. time, we get

$$\frac{dV}{dt} = 3x^2 \times \frac{dx}{dt}$$

Given, $\dfrac{dx}{dt} = 3$ cm/s

At $x = 10$ cm

$$\frac{dV}{dt} = 3 \times 100 \text{ cm}^2 \times 3 \text{ cm/s}$$

$$\frac{dV}{dt} = 900 \text{ cm}^3\text{/s}$$

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