ANSWERS WITH EXPLANATION

Physics

1. (4) Vertical velocity after collision = $e\,u \sin\theta$

$$\text{So, } x = R + R' = \frac{u^2 \sin 2\theta}{g} + \frac{2u\cos\theta(eu\sin\theta)}{g}$$

$$= \frac{u^2 \sin 2\theta}{g}(1+e)$$

2. (1) By conservation of momentum

$$m_1 v_1 = (m_1 + m_2)v$$

or $m_1\sqrt{2gd} = (m_1 + m_2)v$

Let centre of mass rise through a height h

$$\frac{1}{2}(m_1 + m_2)v^2 = (m_1 + m_2)gh$$

$$h = d\left\{\frac{m_1}{m_1 + m_2}\right\}^2$$

3. (2)

$$x = at^2,$$
$$y = bt^2,$$
$$z = 0$$
$$v_x = 2at,$$
$$v_y = 2bt,$$
$$v_z = 0$$
$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
$$= \sqrt{4a^2t^2 + 4b^2t^2}$$
$$= 2t\sqrt{a^2 + b^2}$$

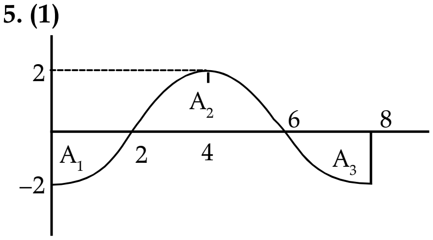
4. (2) Average velocity in time interval when it crosses half of maximum height

vertical displacement = 0 so $\bar{v}_y = 0$

horizontal velocity remains constant

so $v_{av} = u\cos\theta$.

5. (1)



$A_1 \rightarrow$ Area – 1
$A_2 \rightarrow$ Area – 2
$A_3 \rightarrow$ Area – 3

$$A_1 = A_3 : A_2 = A_1 + A_3$$

so net Area = $A_2 - (A_1 + A_3) = 0$

$$F \cdot \Delta t = \Delta p = 0$$

so change in linear momentum is zero

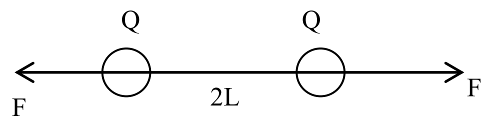
6. (4) mg sin θ = 10 × 3/5 = 6 N

$F_f = \mu mg\cos\theta = 0.8 \times 10 \times 4/5 = 6.4$ N

$\therefore F_f > mg\sin\theta$

$\therefore \quad T = 0$

7. (3)



Due to similar charge, repulsive force acts and angle between them = π

$$\text{Tension} = T = F = \frac{kQ^2}{(2L)^2}$$

or $$T = \left(\frac{kQ^2}{4L^2}\right)$$

8. (2) For non-conducting plate,

$$\vec{E} = \frac{\sigma}{\varepsilon_0} = 10 \text{ V/m}$$

For conducting plate,

$$\vec{E}' = \frac{\sigma}{2\varepsilon_0} = \frac{1}{2} \times 10 = 5 \text{ V/m}$$

9. (2) Given $I = 1.1$ A

$e = 1.6 \times 10^{-19}$ C

$A = \pi r^2 = \pi \times (0.05)^2$

$= 78.5 \times 10^{-4}$ cm^2

$$v_d = \frac{I}{neA}$$

$$n = \frac{6 \times 10^{23}}{7 \text{ cm}^3}$$

$= 0.86 \times 10^{23}$ / cm^3

$$v_d = \frac{1.1}{0.86 \times 10^{23} \times 1.6 \times 10^{-19} \times 78.5 \times 10^{-4}}$$

(volume of 63g Cu)

$v_d = 0.01$ cm/s

$= 0.1$ mm/s

10. (3) Current = i, Length = L

Bent in 1 turn $2\pi r = L$

$$\therefore \quad r = \frac{L}{2\pi}$$

$$B_1 = \frac{\mu_0 i}{2\frac{L}{2\pi}} = \frac{\mu_0 \pi i}{L}$$

When bent in n turns $n(2\pi r') = L$.

or $$r' = \frac{L}{2\pi n}$$

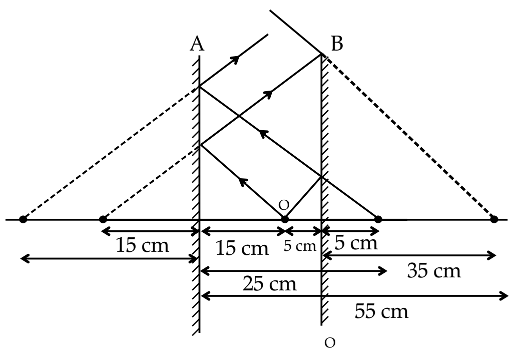
and $$B_2 = \frac{\mu_0 ni}{2\frac{L}{2\pi n}}$$

$$B_2 = \frac{\mu_0 \pi n^2 i}{L}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{1}{n^2}$$

$$= 1 : n^2$$

11. (3)



From ray diagram no image is formed at 45 cm.

12. (3) Fringe width $= \beta = \frac{\lambda D}{d}$

$$\beta \propto \lambda$$

Hence, $$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \quad \frac{1}{\beta_2} = \frac{5000}{6000}$$

$$\beta_2 = \frac{6}{5} = 1.2 \text{ mm}$$

13. (3)

$$M_1 = 2.3 \text{ kg}$$
$$M_2 = 41.15 \text{ g}$$
$$= 41.15 \times 10^{-3} \text{ kg}$$
$$= 0.4115 \text{ kg}$$
$$M_3 = 30.19 \text{ g}$$
$$= 0.03019 \text{ kg}$$
$$M = M_1 + M_2 + M_3 = 2.37134 \text{ kg}$$
$$= 2.4 \text{ kg}$$

14. (2) $$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

$$\Rightarrow \quad \frac{\mu_2}{\mu_1} = \frac{2 \times 10^8}{2.4 \times 10^8} = \frac{2}{2.4} = \frac{5}{6}$$

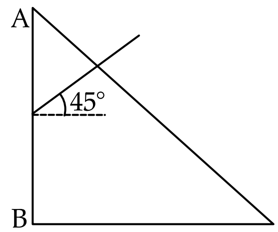
For a ray going from I to II, at critical angle

$$\mu_1 \sin \theta_C = \mu_2 \sin 90°$$

$$\sin \theta_C = \frac{\mu_2}{\mu_1} = \frac{5}{6}$$

$$\Rightarrow \quad \theta_C = \sin^{-1}\left(\frac{5}{6}\right)$$

15. (2)



For TIR at face AB,

$$45° \geq \theta_C$$

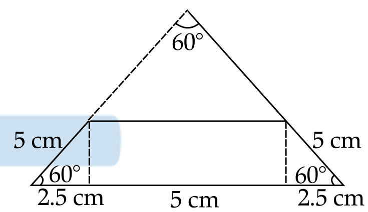
$$\sin 45° \geq \sin \theta_C$$

$$\frac{1}{\sqrt{2}} \geq \frac{1}{\mu}$$

$$\mu \geq \sqrt{2}$$

$$\mu_{min} = \sqrt{2}$$

16. (2) Consider the figure



$\therefore$ Prism formula: $\delta = i + e - A$

$$= 2i - 60°$$
$$= 90 - 60$$
$$= 30°$$

17. (3) $$v = \frac{mz^2e^4}{4\varepsilon_0^2 h^3 n^3}$$

or $$v = \frac{E_0}{h}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{E_0}{h}\left[\frac{n^2 - (n^2 + 1 - 2n)}{[n(n-1)]^2}\right]$$

as $n >> 1$

$$v = \left[\frac{2n}{n^4}\right]\frac{E_0}{h}$$

$$v = \frac{E_0}{h}\left[\frac{2}{n^3}\right]$$

since photon energy is difference in energy of $E_n - E_{(n-1)}$

Hence $$v \propto \frac{1}{n^3}$$

18. (3) From conservation of momentum

$$mv + 0 = mv' + mv'$$

$$\Rightarrow \quad v' = \frac{v}{2}$$

The relative velocity of separation at closest distance of separation is equal to zero.
From energy conservation

$$\frac{1}{2}mv^2 = \frac{mv^2}{8} + \frac{mv^2}{8} + \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad \frac{mv^2}{4} = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \frac{e^2}{\pi\varepsilon_0 m v_0^2}$$

19. (2) From given details

$$1 = h\nu_1 - h\nu_0$$

$$\Rightarrow \quad h\nu_0 = h\nu_1 - 1$$

$$\nu_0 = \nu_1 - \frac{1}{h} \quad ...(1)$$

and

$$k = h\nu_2 - h\nu_0$$

$$k = h\nu_2 - h\nu_1 + 1$$

$$h(\nu_2 - \nu_1) = k - 1$$

$$h = \frac{k-1}{\nu_2 - \nu_1} \quad ...(2)$$

put value of h in equation (1)

$$\nu_0 = \nu_1 - \frac{\nu_2 - \nu_1}{k-1}$$

$$\nu_0 = \frac{k\nu_1 - \nu_1 - \nu_2 + \nu_1}{k-1}$$

$$\nu_0 = \frac{k\nu_1 - \nu_2}{k-1}$$

20. (2) Linear K.E $(K_1) = — mV^2$

Rotational K.E $= \frac{1}{2} I\omega^2$

$$= \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$$

$$(K_2) = \frac{1}{5} mV^2$$

$$\therefore \quad \frac{K_2}{K_1} = \frac{2}{5}$$

21. [5.00] Tangential acceleration, $a_t = r\alpha = 4 \text{ m/s}^2$

Radial acceleration,

$$a_c = \frac{v^2}{r} = \frac{60 \times 60}{1200}$$

$$= 3 \text{ m/s}^2$$

Hence, resultant acceleration of the car

$$a = \sqrt{a_t^2 + a_C^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5 \text{ m/s}^2$$

22. [5] Block will start its motion when

$$F = \mu mg$$

But $\quad \frac{F}{t} = \frac{100}{2}$

$$\Rightarrow \quad F = 50t$$

Therefore $\quad 50t = (0.5)(10(10)\ t = 1 \text{ s}$

For $1 \le t \le 5$, $m\frac{dv}{dt} = F - \mu mg$,

$$\therefore \int_0^v m\,dv = \int_1^5 (F - 50)\,dt$$

$$\Rightarrow \quad mv = 150,$$

$$v = 15 \text{ ms}^{-1}$$

Now work done by all forces

$$W = \Delta K.E. = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2} \times 10 \times (15)^2$$

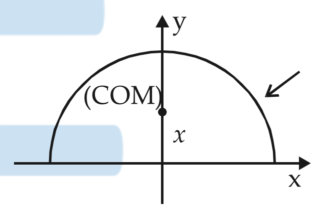
$$1125 = 225\ \alpha$$

$$\alpha = 5$$

$$W = 1125 \text{ J}$$

23. [6.00] For semicircular ring

$$x = \frac{2R}{\theta}\sin\theta$$



$$= \frac{2 \times 3\pi}{\pi} \times \sin\frac{\pi}{2}$$

$$x = 6$$

24. [3.00] $\quad I = \frac{ML^2}{12}\sin^2\theta$

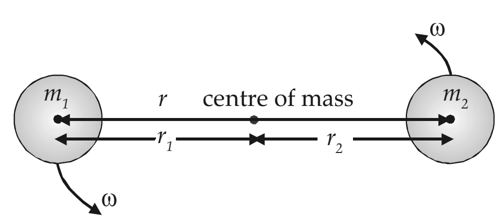
$$\Rightarrow \quad I = \frac{ML^2}{12}\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad ML^2 = 12$$

$$I = \frac{ML^2}{3}\sin^2\theta,$$

at $\theta = \frac{\pi}{3}, I = \frac{ML^2}{3}\left(\frac{3}{4}\right) = \frac{ML^2}{4} = \frac{12}{4} = 3 \text{ kg-m}^2$

25. [125]



$$m_1\omega^2 r_1 = \frac{Gm_1m_2}{(r_1+r_2)^2} = \frac{Gm_1m_2}{r^2} \text{ but}$$

$$\Rightarrow \quad r_1 = \frac{m_2 r}{m_1+m_2}$$

$$\Rightarrow \quad r = \left[\frac{G(m_1+m_2)}{\omega^2}\right]^{1/3}$$

Now applying conservation of mechanical energy (on reduced mass)

$$-\frac{Gm_1m_2}{r}+0 = -\frac{Gm_1m_2}{(R_1+R_2)}+\frac{1}{2}\mu v_r^2$$

where $\mu = \frac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad v_r = \sqrt{\frac{2G(m_1+m_2)}{(R_1+R_2)} - \left\{\frac{\omega^2}{G(m_1+m_2)}\right\}^{1/3}}$$

$$\Rightarrow \quad \alpha = \frac{1}{3},\ \beta = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{\alpha}+\frac{1}{\beta} = 3+2 = 5$$

$$\Rightarrow \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 = 125$$

or

By using dimensional analysis $\beta = \frac{1}{2}$ and

$$\alpha = \frac{1}{3} \text{ So } \frac{1}{\alpha}+\frac{1}{\beta} = 3+2 = 5$$

$$\Rightarrow \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^3 = 125$$

26. [20.00] Tension in cord

$$= \frac{YA}{L}\Delta\ell$$

$$= \frac{5\times10^8\times1\times10^{-6}\times2\times10^{-2}}{10\times10^{-2}}$$

$= 100$ N

When mass is released, stored elastic energy = Kinetic energy of mass

$$\Rightarrow \quad \frac{1}{2}F\times\Delta\ell = \frac{1}{2}mv^2$$

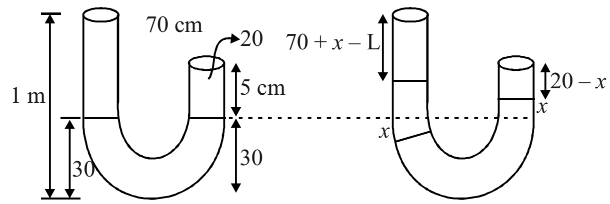
$$v = \sqrt{\frac{F\times\Delta\ell}{m}}$$

$$= \sqrt{\frac{100\times2\times10^{-2}}{5\times10^{-3}}}$$

$= 20\ \text{ms}^{-1}$

27. [6.00] $2x\times\frac{\rho}{2}g = L\times\rho g$

$\Rightarrow \quad x = L$



frequency same

$\Rightarrow \lambda$ = same; $20-x = \frac{\lambda}{4}$; $70 = \frac{5\lambda}{4}$;

$70 = 100-5x$;

$5x = 30$;

$x = 6$ cm

28. [40.00] Maximum acceleration

$= \omega^2 A = g$

$$\Rightarrow \quad A = \frac{g}{\omega^2} = 40 \text{ cm}$$

29. [1.00] $V_0(1+\gamma\times1)-V_0 = h(A_0)$

$$\Rightarrow h = \frac{V_0\gamma}{A_0} = \frac{1.8\times10^{-4}}{1.8\times10^{-4}} = 1 \text{ cm}$$

30. [3.00] Consider a ring of thickness dx

Torque on this ring = QE × x

$$E\times 2\pi x = \pi x^2 \times \frac{dB}{dt}$$

$$E = \frac{x}{2}\times 2kxt - kx^2t$$

$$\text{charge on ring} = \frac{Q}{\pi R^2}\times 2\pi x dx$$

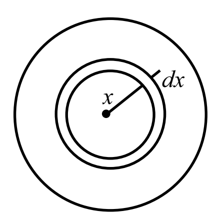
$$\text{Torque on ring} = \frac{2Q}{R^2}x\times kx^2t\times x dx$$

$$= \frac{2kQ}{R^2}x^4 t\, dx$$

$$\text{Total torque} = \int_0^R \frac{2kQ}{R^2}x^4 t\, dx$$

$$= \left[\frac{2kQtx^5}{R^2\times5}\right]_0^R$$

$$= \frac{2kQR^3t}{5} = 3 \text{ N-m}$$



Chemistry

31. (3) $CH_3-\underset{\displaystyle |}{\overset{\displaystyle OH}{C}}H-CH_2-CH=CH-CH_3 \xrightarrow{PCC}$

$CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-CH_2-CH=CH-CH_3$

$\xrightarrow[-CHI_3]{NaOI} {}^{-}O-\overset{\displaystyle O}{\overset{\|}{C}}-CH_2-CH=CH-CH_3 \xrightarrow[lime]{Soda}$

$CH_3-CH=CH-CH_3$
2-butene

32. (1) $N_2(14) = (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2$

$(\pi_{2p_x^2} = \pi_{2p_y{}^2})(\sigma_{2p_z})^2$

Bond order $= \frac{1}{2}$ (No. of electrons in bonding orbital —No. of electrons in anti bonding orbital)

$B.O = \frac{1}{2}(10-4) = 3$

$NO^+(14) = (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2$

$(\pi_{2p_x^2} = \pi_{2py^2})(\sigma_{2p_z})^2$

$B.O = \frac{1}{2}(10-4) = 3$

33. (1) Given:

K_{sp} for bismuth sulphide $(Bi_2S_3) = 1.08 \times 10^{-7}$

$K_{sp}(Bi_2S_3) = [Bi^{3+}]^2 [S^{2-}]^3$

$= (2S)^2 \times (3S)^3$

$K_{sp} = 108\ S^5$

$S^5 = 1.08 \times 10^{-7}/108$

$= 10^{-15}$

34. (1) The electronic configuration of these elements are :

Z = 107 [Rn] $5f^{14}\ 6d^5\ 7s^2$

Z = 108 [Rn] $5f^{14}\ 6d^6\ 7s^2$

Z = 109 [Rn] $5f^{14}\ 6d^7\ 7s^2$

These elements will be place in *d*-block in groups 7th, 8th and 9th respectively as they have incompletely filled *d*-orbital.

35. (4) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-} - Ka_3$

$H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4 - Ka_1$

$HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-} - Ka_2$

$$K_{a_3} = \frac{[H^+]^2[C_2O_4^{2-}]}{[H_2C_2O_4]}$$

$$K_{a_1} = \frac{[H^+][HC_2O_4^-]}{[H_2C_2O_4]}, K_{a_2} = \frac{[H^+][C_2O_4^{2-}]}{[HC_2O_4^-]}$$

$K_{a_3} = K_{a_1} \times K_{a_2}$

36. (2) $\Delta H_{fusion} = 1.435$ kcal mol^{-1}

$= 1435$ cal mol^{-1}

$\Delta S = 5.26$ cal mol^{-1} K^{-1}

$$\Delta S = \frac{\Delta H_{fusion}}{T}$$

$$T = \frac{\Delta H_{fusion}}{\Delta S} = \frac{1435}{5.26} K$$

$= 272.81K \approx 0°C$

37. (2) $Fe^{+3}\ [Fe(CN)_6]^{4-}$

let the oxidation state of Fe in the complex be x

$x + 6(-1) = -4$

$x = -4 + 6$

$x = +2$

38. (1) $C_2H_4 + H_2 \rightleftharpoons C_2H_6$

$\Delta H = -32.7$ k cal

$\Delta n = n_p - n_R$

$= 1 - 2$

$= -1$

Since $\Delta H = -ve$; Exothermic reaction According to Le-Chatelier's principle if the temperature of the system is increased, the system will shift in that direction that consumes the excess heat. Thus, if the temperature increases, reaction goes towards reactant. Thus, concentration of C_2H_4 increases.

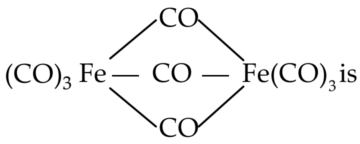
39. (4) $HCl + CH_3COOH \rightleftharpoons Cl^- + CH_3COOH_2^{\ominus}$

Conjugate acid-base pair are differentiated by the presence of one H^+

(HCl, Cl^-) $(CH_3COOH_2^+, CH_3COOH)$

40. (2) $CH_3CH_2-N(CH_3)-CH=CH_2$

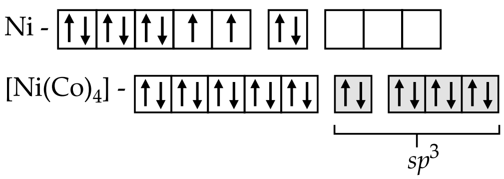
Ethyl methyl vinyl amine

41. (1) 

Tri-μ-carbonyl bis (tricarbonyl ion (0))

42. (4) Ni - Atomic number - 28 - [Ar] $3d^8\ 4s^2$

Ni oxidation state - 0

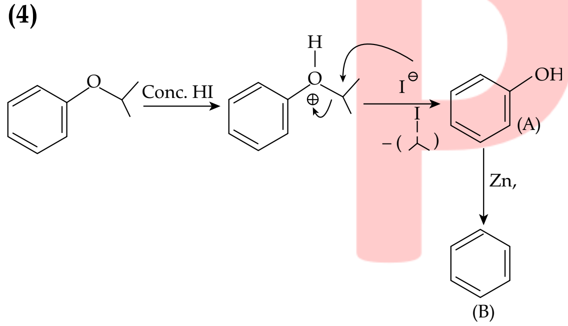


∵ no. of unpaired electron (n) = 0

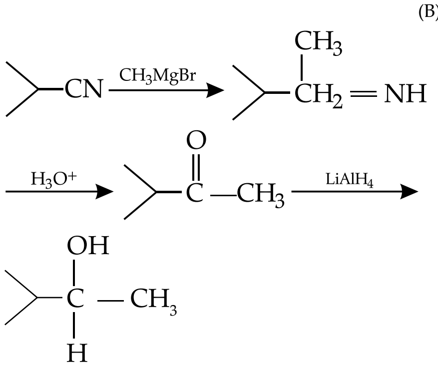
Magnetic moment =

$$\mu = \sqrt{n(n+2)} = \sqrt{0(0+2)} = 0$$

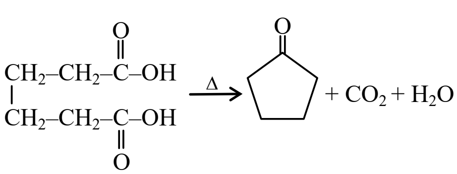
43. (4)



44. (3)



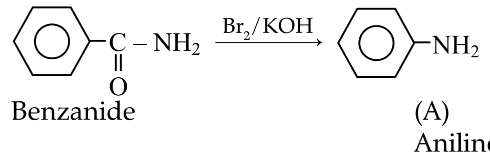
45. (2)



Adipic acid Cyclopentanone

Adipic acid undergo dehydration as well as decarboxylation to give cyclopentanone when it is heated.

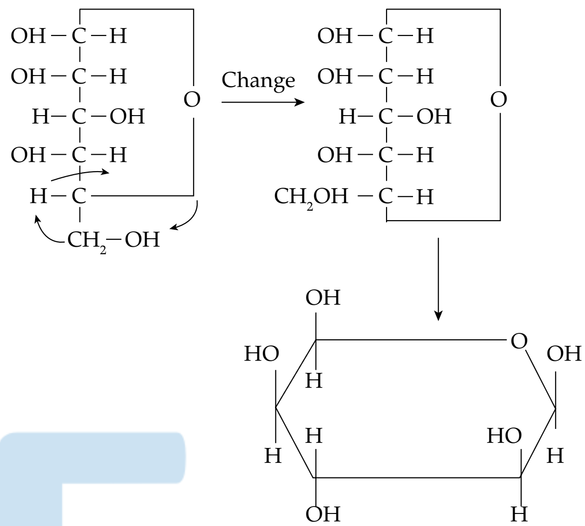
46. (2)



47. (2) $(C_6H_{12}O_5)n + nH_2O \xrightarrow[\text{diastase}]{(x)} C_{12}H_{22}O_{11}$

$C_6H_{22}\,O_{11} + H_2O \xrightarrow[H_2O]{\text{Maltase (y)}} 2C_6H_{12}O_6$

48. (4)



Pyranose ring structure of glucose is due to hemiacetal formation between C–1 and C – 5 carbon atoms. In general, a pyranose is any cyclic isomer that has a five carbon atoms and one oxygen atom in a ring of six atoms. If a hydroxyl at the 5 position of an aldohexose, such as glucose, forms a hemiacetal with the aldehyde (position 1), the resulting isomer is glucopyranose.

49. (1) In the Hunsdiecker reaction →

$CH_3CH_2COOAg + Br_2 \xrightarrow{CCl_4} CH_3CH_2Br + CO_2 + AgBr$

Silver Propionate (No. of C = 3) Ethyl Bromide (No. of C = 2)

No. of carbon atoms decrease.

50. (3)

$CaC_2 + 2H–OH \rightarrow Ca(OH)_2 + H – C \equiv C – H$ (A) Acetylene

$\downarrow Hg^{2+}/H_2SO_4$

$CH_3–CH_2OH \xleftarrow{H_2/Ni} H – C(=O) – CH_3$

(C) Ethanol (B) Acetaldehyde

51. [10.70] $\mu = \dfrac{K_{T+10}}{K} = 1.75$

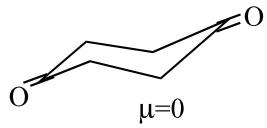
$$\log \frac{K_{T+10}}{K} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$T_1 = 25°C = 298K$$
$$T_2 = 35°C = 308K$$

$$2.303 \times \log \frac{K_{T+10}}{K} = \frac{E_a}{1.987}\left[\frac{308-298}{308\times 298}\right]$$

$$E_a = \frac{2.303\times 308\times 298\times 1.987}{10}\log 1.75 \text{ cal. mol}^{-1}$$
$$= 10.7 \text{ kcal mol}^{-1}$$
$$E_a = 10.7 \text{ kcal mol}^{-1}$$

52. [0.60]



$\mu=0$

Twist boat $\mu = 0$

$$\mu_{net} = \mu_{chair}\, x_{chair} + \mu_{t.b.}\, x_{t.b.}$$
$$1.2 = 0 + \mu_{t.b.}(0.20)$$
$$\because \quad x_{chair} = 0.80$$
$$\therefore \quad x_{t.b.} = 1 - 0.80 \Rightarrow 0.2$$
$$\Rightarrow \quad \mu_{t.b.} = 6.0 \text{ debye}$$

53. (2) Biuret test is given by all proteins and peptides having atleast two peptide linkages. Hence positive test must be given by tripeptide and Biuret. Dipeptides and free amino acids do not give biuret test. Hence glycine and glycylalanine do not give this test.

54. [0.01] According to Nernst Equation,

$$E_{cell} = E^0_{cell} - \frac{0.0591}{n}\log\frac{P}{R}$$

$$E_{cell} = E^0_{Cell} - \frac{0.0591}{n}\log\frac{0.8}{0.2}$$

$$= E^0_{Cell} - \frac{0.0591}{2}\log 2^2$$

$$= E^0_{Cell} - \frac{0.0591}{2}\times 2\times 0.3010$$

$$= 0.04 - 0.03$$
$$= 0.01 \text{ V}$$

55. [266]

Given:

$R = 1750\Omega$

$K = 0.152 \times 10^{-3}\ \Omega^{-1}cm^{-1}$

Conductivity = Cell constant/ Resistance

Cell constant = Conductivity × Resistance

$= 0.152 \times 10^{-3}\ \Omega^{-1}cm^{-1} \times 1750\Omega$

Cell constant = 266×10^{-3} cm^{-1}

The cell constant depends on the distance between the electrodes and their area of cross–section

56. [112.70] $\dfrac{P^o - P_s}{P_s} = \dfrac{n}{N}, \rightarrow x_2$ (mole fraction)

$$\frac{P^o - P_s}{P_s} = \frac{n_2}{n_1} (\text{since } n_1 >> n_2)$$

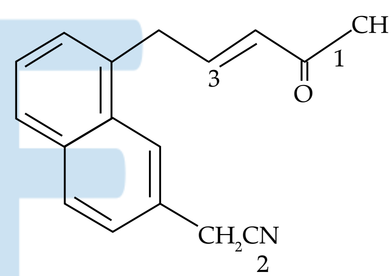
$$= \frac{W_2}{M_2}\times\frac{M_2}{W_1}$$

$$\frac{736-526}{526} = \frac{2.5x}{M_2}\times\frac{18}{1x}$$

$$M_2 = 112.7 \text{ g}$$

57. (3)

Electrophilic centres are those area where electron density is low. These are the atoms which contains the incomplete octet and/or carry a full or a partial positive charge. A partial positive charge can be revealed by writing resonance structures, or by identifying a polar bond.



This compound has 3 electrophilic centers.

58. [530] $dG = V.dP - S.dT$; at const. volume

$$\Delta G = V\cdot\Delta P - \int\left(10+10^{-2}T\right)\cdot dt$$

$$\frac{P_1}{P_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad P_2 = 1\times\frac{400}{300}$$

$$\Delta G = 24.6\times\left(\frac{4}{3}-1\right)\times 100 - \left[10\times 100 + 10^{-2}\times\left(\frac{T_2^2}{2}-\frac{T_1^2}{2}\right)\right]$$

$$\Delta G = 24.6\times\frac{1}{3}\times 100 - \left[1000 + 10^{-2}\times\left(\frac{160000}{2}-\frac{90000}{2}\right)\right]$$

$$= 24.6\times\frac{1}{3}\times 100 - 1000 - 350$$
$$= -530 \text{ J}$$

59. [493] 1 mole of HNO_3 = $\frac{3}{2}$ moles of $NO_2 \rightarrow \frac{3}{2}$ mole of NO $\rightarrow \frac{3}{2}$ mole of NH_3

$$-\left(\frac{3}{2}\times\frac{1}{4}\right)(904)-\left(\frac{3}{2}\times\frac{1}{2}\right)(112)-\left(\frac{1}{2}\right)(140)$$

$$= 493 \text{ kJ / mol}$$

60. [1.122] $Ph-MgBr + HC\equiv C-CH_2-CH_2-CH_2-OH$

$$\downarrow$$

C_6H_6 + $BrMgOC\equiv C-CH_2CH_2CH_2OMgBr$

since, 84 g $HC\equiv C-CH_2-CH_2CH_2-OH$ gives $\rightarrow$ 22.4 L Benzene

$\therefore$ 1 g of Pent – 4 – yne – 1 – ol $\rightarrow \frac{44.8}{84}$

$\rightarrow \frac{22.4}{84}$

$\therefore$ 4.2 g of Pent – 4 – yne – 1 – ol $\rightarrow \frac{22.4}{84}\times 4.2 = 1.12$

Mathematics

61. (2)

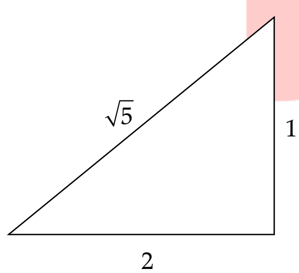
$$\frac{(\sin 7x+\sin 5x)+5(\sin 5x+\sin 3x)+12(\sin 3x+\sin x)}{\sin 6x+5\sin 4x+12\sin 2x}$$

$$\frac{2\sin 6x\cos x+10\sin 4x\cos x+24\sin 2x\cos x}{\sin 6x+5\sin 4x+12\sin 2x}$$

$$= 2\cos x\frac{(\sin 6x+5\sin 4x+12\sin 2x)}{(\sin 6x+5\sin 4x+12\sin 2x)}$$

$$= 2\cos x$$

62. (2)



$$\sec^{-1}\frac{\sqrt{5}}{2}=\tan^{-1}\frac{1}{2}$$

$$\therefore \tan\left[2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right]$$

$$=\tan\left[2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right)+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[2\tan^{-1}\frac{13}{39}+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[\tan^{-1}\frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}}+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{2}\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{4}\cdot\frac{1}{2}}\right)\right]$$

$$=\frac{5}{4}\times\frac{8}{5}=2$$

63. (3) $\sin\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\pi-\cos^{-1}\frac{1}{2}\right]$

$$=\sin\left[\cos^{-1}\frac{1}{2}\right]$$

$$=\sin^{-1}\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

Shortcut Method :

$$\sin\left(\text{arc}\cos\left(-\frac{1}{2}\right)\right)=\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

64. (1) $\frac{\sin(A-B)}{\sin(A+B)}=\frac{\sin^2 A-\sin^2 B}{\sin^2 A+\sin^2 B}$

$$\Rightarrow \frac{\sin(A-B)}{\sin C}=\frac{\sin(A+B)\sin(A-B)}{\sin^2 A+\sin^2 B}$$

$\{\because A + B + C = \pi\}$

$$\Rightarrow \sin(A-B)\left[\frac{1}{\sin C}-\frac{\sin C}{\sin^2 A+\sin^2 B}\right]=0$$

Either $\sin(A-B) = 0$

$\Rightarrow \quad A = B$

or $\quad \sin^2 A + \sin^2 B = \sin^2 C$

$\Rightarrow \quad a^2 + b^2 = c^2$

$\Rightarrow$ Triangle is right angled or isosceles

65. (2) $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

66. (3) $\because a^2 + 4b^2 = 12ab \Rightarrow (a + 2b)^2 = 16\,ab$

Taking logarithm on both sides

$\Rightarrow 2\log(a + 2b) = \log a + \log b + \log 16$

$\Rightarrow \quad \log(a+2b) = \frac{1}{2}(\log a + \log b + 4\log 2)$

67. (2) $\because \quad A = \frac{\sin^2\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin\theta}\cdot\frac{1}{\sin\theta\cos\theta} = 1$

$B = \sin^2\theta + \text{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2 - \tan^2\theta - \cot^2\theta$

$= 5 + 1 + \cot^2\theta + 1 + \tan^2\theta - \tan^2\theta - \cot^2\theta = 7$

and C = 12

$\therefore f(x) = x^2 - 7x + 12$

$\Rightarrow \quad f(x) = (x-3)(x-4)$

68. (1) All even integers less than 200 are

2, 4, 6, ... 198 in AP.

$\Rightarrow \quad n = 99$

$\therefore \quad S_{99} = \frac{99}{2}[2 + 198]$

$= 9900$

integers which are divisible by 6 are

6, 12, 18, ... 198 in AP

$\Rightarrow \quad n = 33$

$\therefore \quad S_{33} = \frac{33}{2}[6 + 198]$

$= 33 \times 102$

$= 3366$

Sum of all integers which are not divisible by 6 is

$= 9900 - 3366$

$= 6534$

69. (1) Given circles $x^2 + y^2 - 2x + 4y - 4 = 0$ and $x^2 + y^2 - 8x - 2y + 8 = 0$

$\Rightarrow \quad \cos\theta = \frac{8 - 4 - ((-8)(-1) + (-2)(2))}{2\cdot 3\cdot 3}$

$= \frac{4-(4)}{2\cdot3\cdot3} = 0$

$\therefore \quad \cos 2\theta = 1$

70. (1) Numerically greatest term

$= \frac{n+1}{\left|\frac{x}{a}\right|+1}$

$= \frac{15+1}{\left|\frac{3}{-5\cdot\frac{1}{5}}\right|+1}$

when $x = \frac{1}{5}$

$= \frac{16}{4} = 4$

Greatest terms are T_4 and T_5

71. (3) We have to find value of $f'(A)$.

$f'(x) = -\sin x - 2x - 1 < 0$

So, $f'(x)$ is decreasing

So, greatest value $= f\left(\frac{\pi}{6}\right)$ &

least value $= f\left(\frac{\pi}{3}\right)$

Now, $f\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)$

And $f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} - \frac{\pi}{3}\left(1+\frac{\pi}{3}\right)$

$= \frac{1}{2} - \frac{\pi}{3}\left(1+\frac{\pi}{3}\right)$

Hence, $f(A)$ in $\frac{\pi}{6} \le x \le \frac{\pi}{3}$ is

$\left[\frac{1}{2} - \frac{\pi}{3}\left(1+\frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$

72. (4) Since, function $f(x)$ is continuous at $x = 1, 3$

$\therefore \quad f(1^-) = f(1^+)$

$\Rightarrow \quad ae + be^{-1} = c \quad$ ...(i)

$f(3^-) = f(3^+)$

$\Rightarrow \quad 9c = 9a + 6c \Rightarrow c = 3a \quad$ ...(ii)

From (i) and (ii),

$b = ae(3 - e) \quad$ ...(iii)

Now, $$f'(x) = \begin{cases} ae^x - be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{cases}$$

$f'(0) = a - b, f'(2) = 4c$

Given, $f'(0) + f'(2) = e$

$a - b + 4c = e \quad$ ...(iv)

From eqs. (i), (ii), (iii) and (iv),

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow \quad 13a - 3ae + ae^2 = e$

$$\Rightarrow \quad a = \frac{e}{e \;\; - 3e + 13}$$

73. (1) Let $I = \int \frac{dx}{3 + \sin 2x}$

$$\Rightarrow I = \int \frac{dx}{3 + \frac{2\tan x}{1 + \tan^2 x}}$$

$$\Rightarrow I = \int \frac{\sec^2 x\, dx}{3.\tan^2 x + 2\tan x + 3}$$

Let $t = \tan x$

$\Rightarrow dt = \sec^2 x\, dx$

$$\therefore \quad I = \int \frac{dt}{3t^2 + 2t + 3}$$

$$\Rightarrow I = \frac{1}{3}\int \frac{dt}{t^2 + \frac{2}{3}t + 1}$$

$$\Rightarrow I = \frac{1}{3}\int \frac{dt}{\left(t + \frac{1}{3}\right)^2 + \left(\frac{2\sqrt{2}}{3}\right)^2}$$

$$\Rightarrow I = \frac{1}{3}\,\frac{1}{\frac{2\sqrt{3}}{3}}\tan^{-1}\left(\frac{t + \frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right) + c$$

$$\left\{\because \int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right\}$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{3\tan x + 1}{2\sqrt{2}}\right) + c$$

74. (2) Let $I = \int_0^{400\pi} \sqrt{1 - \cos 2x}\, dx$

$$\Rightarrow I = \sqrt{2}\int_0^{400\pi} |\sin x|\, dx$$

$\because$ $|\sin x|$ has period π

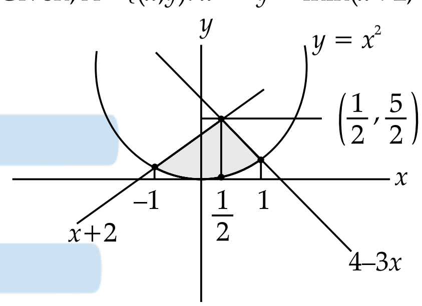
$$\therefore \quad I = 400\sqrt{2}\int_0^{\pi} |\sin x|\, dx$$

$$= 800\sqrt{2}\int_0^{\pi/2} \sin x\, dx$$

$$= 800\sqrt{2}\,[-\cos x]_0^{\pi/2}$$

$$= 800\sqrt{2}$$

75. (2) Given, $A = \{(x, y) : x^2 \le y \le \min(x + 2, 4 - 3x)\}$



$$\text{Area} = \int_{-1}^{\frac{1}{2}}(x + 2 - x^2)dx + \int_{\frac{1}{2}}^{1}(4 - 3x - x^2)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{\frac{1}{2}} + \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$= \frac{1}{8} + 1 - \frac{1}{24} - \frac{1}{2} + 2 - \frac{1}{3}$$

$$+ \left[\left(4 - \frac{3}{2} - \frac{1}{3}\right) - \left(4 \times \frac{1}{2} - \frac{3}{2} \times \frac{1}{4} - \frac{1}{24}\right)\right]$$

$$= \frac{17}{6}$$

76. (1) Given differential equation

$(x - y^2)dx + y(5x + y^2)dy = 0 \quad$ ...(i)

$\Rightarrow \quad (x - y^2)dx = -y(5x + y^2)dy$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x - y^2)}{-(5xy + y^3)}$$

$$\Rightarrow \quad y\frac{dy}{dx} = \frac{x - y^2}{-(5x + y^2)}$$

Let $\quad y^2 = t$

$$y\frac{dy}{dx} = \frac{dt}{dx}$$

$$\therefore \quad \frac{1}{2}\frac{dt}{dx} = \frac{-(x-t)}{(5x+t)}$$

Which is Homogeneous differential equation of first order

Put $\quad t = xv$

$$\Rightarrow \quad \frac{dt}{dx} = x\frac{dv}{dx}+v$$

$$\therefore \quad \frac{1}{2}\left(x\frac{dv}{dx}+v\right) = -\left(\frac{x-xv}{5x+xv}\right) = \frac{-x(1-v)}{x(5+v)}$$

$$\Rightarrow \quad x\frac{dv}{dx}+v = \frac{-2(1-v)}{(5+v)}$$

$$\Rightarrow \frac{xdv}{dx} = \frac{-2(1-v)}{(5+v)} - v = \frac{-2+2v-v(5+v)}{5+v}$$

$$= \frac{-2+2v-5v-v^2}{5+v} = \frac{-2-3v-v^2}{5+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-(v^2+3v+2)}{(v+5)}$$

$$= \frac{-(v+1)(v+2)}{(v+5)}$$

$$\therefore \quad \int\frac{v+5}{(v+1)(v+2)}dv = -\frac{dx}{x}$$

$$\Rightarrow \int\frac{4}{(v+1)}+\frac{3}{(v+2)}dv = \int -\frac{1}{x}dx$$

$\Rightarrow 4\log_e(v + 1) + 3\log_e(v + 2) = -\log_e x + \log_e c$

$$\log_e\left(\frac{(v+1)^4}{(v+2)^3}\right) = \log_e\left(\frac{c}{x}\right)$$

$$\Rightarrow \quad \frac{(v+1)^4}{(v+2)^3} = \frac{c}{x}$$

$$\Rightarrow \quad x(v+1)^4 = (v+2)^3.c$$

Put the value of $\quad v = \frac{t}{x} = \frac{y^2}{x}$

$$x\left(\frac{y^2}{x}+1\right)^4 = \left(\frac{y^2}{x}+2\right)^3 c$$

$$\Rightarrow \quad \frac{x(y^2+x)^4}{x^4} = \frac{(y^2+2x)^3 c}{x^3}$$

$$\Rightarrow \quad \frac{(y^2+x)^4}{x^3} = \frac{(y^2+2x)^3}{x^3}c$$

$$\Rightarrow \quad (y^2+x)^4 = c(y^2+2x)^3$$

77. (3) $\sum_{n=0}^{100} i^{n!} = i^{0!} + i^{1!} + i^{2!} + i^{3!} + i^{4!} + \ldots + i^{100!}$

$= i + i + i^2 + i^6 + i^{4!} + \ldots + i^{100!}$

$= 2i - 2 + (1 + 1 + \ldots$ 97 times)

$= 2i + 95$

78. (1) Given that $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further

Now, $(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also, $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

79. (3) Given $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

Now, $(A - 2I)(A - 3I) = A^2 - 5AI + 6I^2$

Now, $A^2 = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16-2 & 8+2 \\ -4-1 & -2+1 \end{bmatrix}$

$= \begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix}$

$\Rightarrow A^2 - 5AI + 6I^2 = A^2 - 5A + 6I$

$= \begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix} - 5\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 14-20+6 & 10-10+0 \\ -5+5+0 & -1-5+6 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

= Null matrix or Zero matrix

80. (1) $\vec{a}+\vec{b}+\vec{c}$ = P.V. of A

$4\vec{a}+3\vec{b}$ = P.V. of B

$10\vec{a}+7\vec{b}-2\vec{c}$ = P.V. of C

Now, $\quad \overrightarrow{AB} = 3\vec{a}+2\vec{b}-\vec{c}$

And $\quad \overrightarrow{AC} = 9\vec{a}+6\vec{b}-3\vec{c}$

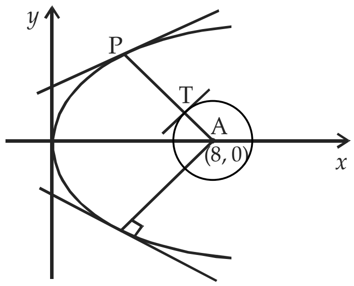
$= 3\left(3\vec{a}+2\vec{b}-\vec{c}\right)$

$\because \quad \overrightarrow{AB} = \frac{1}{3}\overrightarrow{AC}$

$\therefore$ Given points are Collinear.

81. [4.00] $\underline{C} \equiv (8, 0), r = 2$

least distance occurs along common normal



The slope of normal is $-\frac{y}{3}$

$$\therefore \quad -\frac{y}{3} = \frac{y-0}{\left(\frac{y^2}{6}\right)-8}$$

$$\Rightarrow \quad y = 0 \text{ or } y = \pm\sqrt{30}$$

$$\Rightarrow \quad x = 0 \text{ or } x = 5$$

$\therefore$ Points are (0, 0), $\left(5, \sqrt{30}\right)$, $\left(5, -\sqrt{30}\right)$

Hence, least distance is PT

$$= PA - 2$$

$$= \sqrt{39} - 2$$

$$\therefore \quad [k] = 4$$

Shortcut Method :

Let $P(at^2, 2at)$ is on $y^2 = 6x$ $\left(a = \frac{3}{2}\right)$

Normal at P is $xt + y = 3t + \frac{3}{2}t^3$

if it passes through (8, 0), we have

$$8t = 3t + \frac{3}{2}t^3$$

$$\Rightarrow \quad t = 0 \text{ or } \frac{3}{2}t^2 = 5$$

$$t = \sqrt{\frac{10}{3}}$$

$$\therefore \quad \text{Least distance} = \sqrt{(8-5)^2 + 30 - 2}$$

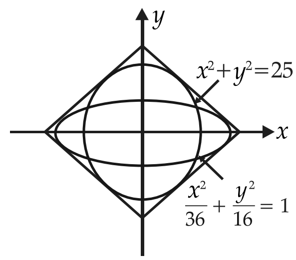
$$k = \sqrt{39} - 2$$

$$\therefore \quad [k] = 4$$

82. [999.00] Equation of tangents to the given ellipse in slope form is given by $y = mx \pm \sqrt{36m^2 + 16}$

Since, it is also tangent to the circle $x^2 + y^2 = 25$

$$\therefore \quad 5 = \left|\frac{0 + 0 \pm \sqrt{36m^2 + 16}}{\sqrt{1 + m^2}}\right|$$



$$\Rightarrow \quad m^2 = \frac{9}{11}$$

$$\therefore \quad y = \pm\frac{3}{\sqrt{11}}x \pm \sqrt{\frac{500}{11}}$$

$$\therefore \quad A = 4 \times \frac{1}{2} \times \frac{\sqrt{500}}{\sqrt{11}} \times \frac{\sqrt{500}}{3}$$

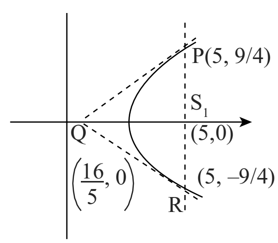
$$= \frac{1000}{3\sqrt{11}}$$

So, $\left(3\sqrt{11}A - 1\right) = 3\sqrt{11}\frac{1000}{3\sqrt{11}} - 1$

$$= 1000 - 1 = 999$$

83. [324.00] $e = \frac{5}{4}$

$$PS_1 = \frac{b^2}{a} = \frac{9}{4}$$



$$Q\left(\frac{16}{5}, 0\right)$$

$$A = \frac{1}{2}\cdot\frac{9}{2}\cdot\frac{9}{5}$$

$$\Rightarrow \quad 80A = 324$$

84. [4.00] Let $O(\vec{O})$, $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$

$\therefore$ D is $\frac{\vec{a}+\vec{b}}{2}$ and E is $\frac{\vec{c}}{2}$

$$\overrightarrow{DE}\cdot\overrightarrow{AC} = \left(\frac{\vec{c}-(\vec{a}+\vec{b})}{2}\right).(\vec{c}-\vec{a})$$

$$= \frac{|\vec{c}|^2}{2}-\frac{\vec{c}.\vec{a}}{2}-\frac{\vec{a}.\vec{c}}{2}-\frac{\vec{b}.\vec{c}}{2}+\frac{|\vec{a}|^2}{2}+\frac{\vec{a}.\vec{b}}{2}$$

$$= \frac{1-\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)+1+\left(\frac{1}{2}\right)}{2} = \frac{1}{2}$$

$\because$ angle between adjacent sides is $\frac{\pi}{3}$

So, $\frac{m}{n}\pi = \frac{\pi}{3}$

$\Rightarrow$ $m = 1, n = 3$

$\Rightarrow$ $m + n = 4$

85. [5.00] Let H : Getting a head

T : Getting a tail.

Required probability $= P(3H$ and $2T$ on first five tosses and sixth is head)

$$= {}^5C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2}$$

$$= \frac{10}{64} = \frac{5}{32}$$

$\Rightarrow$ $\frac{k}{32} = \frac{5}{32}$

$\Rightarrow$ $k = 5$

86. [1.00] $f(x) = [[x] + \{x^2\}] + \{[x^2] + \{x\}\}$

$= [x] + \{x\}$

$\Rightarrow$ $f(x) = x$

$\Rightarrow |f(x)| = |x|$ which is non-derivable at only one point $x = 0$.

87. [5.00] $f(x) = (\ln x)^x + \ln(x^x) + x^{\ln x}$

$f(x) = e^{x\ln(\ln x)} + x\ln x + e^{(\ln x)^2}$

$$f'(x) = e^{x\ln(\ln x)}\left[x.\frac{1}{\ln x}.\frac{1}{x}+\ln(\ln x)\right] +$$

$$[1 + \ln x] + e^{(\ln x)^2}.2\ln x.\frac{1}{x}$$

$\Rightarrow f'(e) = 1.[1 + 0] + (1 + 1) + e.2.1.– = 5$

88. [4.00] Let a point on $y^3 = x^4$ be (t^3, t^4)

So, $3y^2y' = 4x^3$

$\Rightarrow$ $y' = \frac{4x^3}{3y^2}$

$\Rightarrow$ $y' = \frac{4}{3}t$

Equation of tangent is

$$y - t^4 = \frac{4t}{3}(x - t^3)$$

$\because$ it is a normal to $x^2 + y^2 - 2x = 0$

$\therefore$ it must pass through (1, 0)

$\Rightarrow$ $-\frac{3}{4}t^3 = 1 - t^3$

$\Rightarrow$ $\frac{t^3}{4} = 1$

$\Rightarrow$ $t^3 = 4$

Now, $m = \frac{4t}{3}$

$\Rightarrow$ $\left(\frac{3m}{4}\right)^3 = t^3 = 4$

89. [385.00] $P(\bar{A}\cap\bar{B}) = \frac{2}{7}$

$\Rightarrow$ $1 - P(A\cup B)$

$\Rightarrow$ $P(A\cup B) = \frac{5}{7}$

$\Rightarrow$ $P(A)+P(B)-P(A\cap B) = \frac{5}{7}$

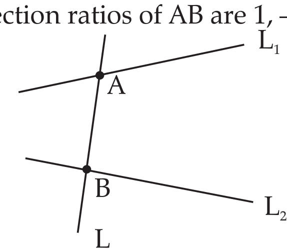
$\frac{200+125-50}{m} = \frac{5}{7}$

$\Rightarrow$ $m = 385$

90. [84] Given lines $L_1 : \frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$

$L_2 : \frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$

And direction ratios of AB are 1, –4, 2



$\because$ Point A lies on L_1

$\therefore$ Coordinates of point A = $(3\lambda + 7, -\lambda + 1, \lambda - 2)$

And Point B lies on L_2

$\therefore$ Coordinates of point $B = (2k, 3k + 7, k)$

Now, direction ratios of $AB = 2k - 3\lambda - 7, 3k + \lambda + 6, k - \lambda + 2$

$$\therefore \frac{2k-3\lambda-7}{1} = \frac{3k+\lambda+6}{-4} = \frac{k-\lambda+2}{2}$$

$\Rightarrow \dfrac{2k-3\lambda-7}{1} = \dfrac{3k+\lambda+6}{-4}$ and $\dfrac{3k+\lambda+6}{-4} = \dfrac{k-\lambda+2}{2}$

$\Rightarrow 11k - 11\lambda - 22 = 0$ and $10k - 2\lambda + 20 = 0$

$\Rightarrow k - \lambda = 2$ and $5k - \lambda = -10$

$\Rightarrow \lambda = -5$ and $k = -3$

$\therefore A = (-8, 6, -7)$ and B = $(-6, -2, -3)$

Now, $AB = \sqrt{(2)^2 + (-8)^2 + (4)^2} = \sqrt{84}$

$\Rightarrow AB^2 = 84$

Hint :

(i) Assume general point on both the given lines and find direction ratios of line which is passing through that points compare these direction ratios with given direction ratios and solve further.

(ii) Direction ratios of the passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Shortcut method:

Coordinates of point $A = (3\lambda + 7, -\lambda + 1, \lambda - 2)$

Coordinates of point $B = (2k, 3k + 7, k)$

Now, direction ratios of the line $AB = 2k - 3\lambda - 7, 3k + \lambda + 6, k - \lambda + 2$

$\because$ Direction ratios of the line AB are 1, –4, 2 (Given)

$\therefore \dfrac{2k-3\lambda-7}{1} = \dfrac{3k+\lambda+6}{-4} = \dfrac{k-\lambda+2}{2}$

$\Rightarrow \lambda = -5$ and $k = -3$

$\therefore A = (-8, 6, -7)$ and B = $(-6, -2, -3)$

Now, $AB = \sqrt{(2)^2 + (-8)^2 + (4)^2} = \sqrt{84}$

$\Rightarrow AB^2 = 84$

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