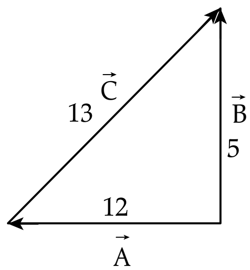
ANSWERS WITH EXPLANATION

Physics

1. (3) The Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be represented as shown in figure.



Clearly angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$.

2. (2) $y = a\sin(At - Bx + C)$

Angle has no dimensions so

Dimensions of At $= [M^0L^0T^0]$

$\Rightarrow \quad A = [T^{-1}]$

Dimensions of Bx $= [M^0L^0T^0]$

$\Rightarrow \quad B = [L^{-1}]$

Dimensions of C $= [M^0L^0T^0]$

3. (1) $h = \dfrac{\text{Energy}}{\frac{c}{\lambda}}$

Where c is speed of light

h = Planks Constant

λ = Wave length

$$\Rightarrow \quad h = \frac{M^1L^2T^{-2}}{\left(\frac{LT^{-1}}{L}\right)} = M^1L^2T^{-1}$$

$\Rightarrow \quad h = \text{kg m}^2\text{s}^{-1}$

$\Rightarrow \quad = (10^3 \text{ g})(10^2 \text{ cm})^2 \text{ s}^{-1}$

$\Rightarrow \quad h = 10^7 \text{ g cm}^2 \text{ s}^{-1}$

Hence, the required ratio will be 10^7

4. (2) **Given :** $m = 5$ kg, $v > 330$ m/s, $\Delta t = 0.01$ s

$F.\Delta t = \Delta p = m\Delta v = 5 \text{ kg} \times 330 \text{ m/s}$

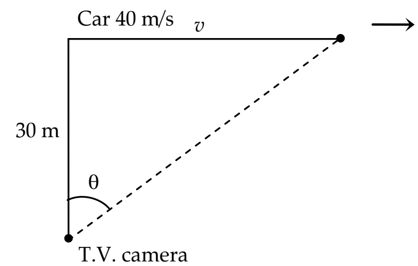
$\Delta p = 1650$ kg m/s

$$F = \frac{\Delta}{\Delta} = \frac{1650}{0.01} = 1650 \times 100$$

$$= 1.65 \times 10^5 \text{ N}$$

So, the applied force should by greater than calculated value, so correct option will be (2) i.e., 2×10^5 N.

5. (4)



We have

$$\tan\theta = \frac{x}{30}; \quad \frac{d}{dt}[\tan\theta] = \frac{d}{dt}\left[\frac{x}{30}\right]$$

$$\frac{d}{d\theta}(\tan\theta)\cdot\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{30} \times 40$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{4}{3\sec^2\theta}$$

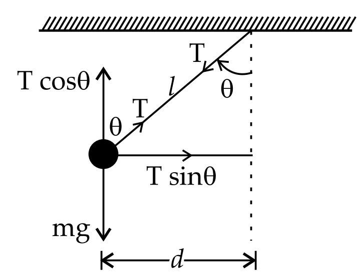
At the given instant, $\theta = 30°$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{4}{3}\cos^2 30°$$

$$= \frac{4}{3} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{4}{3} \times \frac{3}{4} = 1 \text{ rad s}^{-1}$$

or the angular speed with which camera should be rotated = 1 rad/s

6. (4)



T sinθ = ma

T cosθ = mg ...(1)

$\therefore \tan\theta = \dfrac{a}{g}$

$dW = -T\sin\theta . l d\theta$

$$W = \int_0^{\tan^{-1}(a/g)} -T\sin\theta . l d\theta$$

$$= Tl[\cos\theta]_0^{\tan^{-1}(a/g)}$$

$= Tl[\cos[\tan^{-1}(a/g)] - 1]$

$= \dfrac{mg}{\cos\theta}[\cos[\tan^{-1}(a/g)]-1]$ from (1)

7. (3) $v = \omega r$ since ω is same for both

when radius is r, $v_1 = \omega r$

when radius is $2r$, $v_2 = \omega(2r)$

$$\frac{v_1}{v_2} = \frac{\omega}{\omega(2\ )} = \frac{1}{2};$$

(Here x and y are displacement of particles A & B in the same time interval.)

8. (3) $v_1^2 = \omega^2(A^2 - x_1^2)$ ...(1)

$v_2^2 = \omega^2(A^2 - x_2^2)$ ...(2)

Substituting equation (1) from equation (2)

$$v_2^2 = v_1^2 + \omega^2\left(x_1^2 - x_2^2\right)$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \omega^2\left(x_1^2 - x_2^2\right)$$

$$\Rightarrow \quad \omega^2 = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)^2 = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}$$

$$\Rightarrow \quad \frac{T^2}{2\pi^2} = \frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

9. (1) $F = T \times \ell$

$= 60$ dyne/cm $\times$ 15 cm $\times$ 4

$= 3600$ dyne

10. (2) $I = nAevd = nAe\mu E$

$= nAe\mu\dfrac{V}{L} = (8.5 \times 10^{28}) \times (10^{-2})^2 \times (1.6 \times 10^{-19}) \times (4.5 \times 10^{-6}) \times \dfrac{4}{0.2}$

$\Rightarrow \quad = 1.22$ A

11. (3) $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\therefore \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = 25$$

$$\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = 5$$

$$\Rightarrow \quad \frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} = 5$$

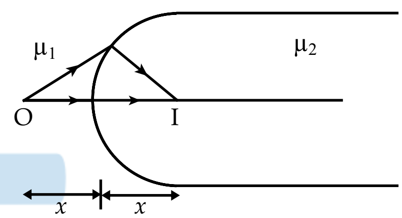
$$\Rightarrow \quad \sqrt{\frac{I_1}{I_2}} + 1 = 5\sqrt{\frac{I_1}{I_2}} - 5$$

$$\Rightarrow \quad 6 = 4\sqrt{\frac{I_1}{I_2}}$$

$$\Rightarrow \quad \sqrt{\frac{I_1}{I_2}} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{9}{4}$$

12. (2)



μ_1 = Refractive index of object medium

μ_2 = Refractive index of image medium

R = Radius of curvature

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{x} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_1 + \mu_2}{x} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad x = \left(\frac{\mu_1 + \mu_2}{\mu_2 - \mu_1}\right)R$$

13. (1) **Given**

Flint glass	**Crown glass**
$\omega_f = 0.053$	$\omega_c = 0.034$
$\mu_f = 1.68$	$\mu_c = 1.53$
$A_f = ?$	$A_c = 4°$

For no dispersion

$$\omega_f d_f - \omega_c d_c = 0$$

or $\omega_f A_f(\mu_f - 1) - \omega_c A_c(\mu_c - 1) = 0$

$$A_f = \frac{\omega_c A_c(\mu_c - 1)}{\omega_f(\mu_f - 1)}$$

$$= \frac{0.034 \times 4°(1.53 - 1)}{0.053 \times (1.68 - 1)}$$

$$= \frac{0.034 \times 4 \times 0.53}{0.053 \times 0.68}$$

$$\Rightarrow \quad A_f = 2°$$

14. (1) $I = 4\,I_0 \cos^2\left(\frac{\phi}{2}\right)$

$$I_0 = 4\,I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$$

$$\text{or} \quad \frac{\phi}{2} = \frac{\pi}{3}$$

$$\text{or} \quad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\cdot \Delta x$$

$$\text{or} \quad \frac{1}{3} = \left(\frac{1}{\lambda}\right) y \cdot \frac{d}{D}\left(\Delta x = \frac{yd}{D}\right)$$

$$\therefore \quad y = \frac{\lambda}{3 \times \frac{d}{D}} = \frac{6\times 10^{-7}}{3\times 10^{-4}} = 2\times 10^{-3}$$

$$m = 2 \text{ mm}$$

15. (3) $v_d = \mu E$

$$= \frac{\mu v}{l}$$

$$= (1000 \times 10^{-4}) \times \frac{2}{10\times 10^{-2}}$$

$$= 2 \text{ m/s}$$

16. (4) $E = \frac{ch}{\lambda}$ or $E \propto \frac{1}{\lambda}$ or λE = constant

Given $E_1 = E$, $E_2 = \frac{4E}{3}$, $E_3 = 2E$

$$E_3 - E_1 = 2E - E = E$$

wavelength of the emitted photon is

$$\therefore \quad E_2 - E_1 = \frac{4E}{3} - E = \frac{E}{3}$$

$$\lambda E = \lambda' \frac{E}{3}$$

$$\therefore \quad \lambda' = 3\lambda$$

17. (1) Graph between $\log \frac{R}{R_0}$ and $\log_e A$ is always a straight line.

$$R = R_0 A^{1/3}$$

$$\frac{R}{R_0} = A^{1/3}$$

$$\log_e \frac{R}{R_0} = \frac{1}{3} \log_e A$$

18. (4) When the coil is within the field there is no change in magnetic flux passing through it. Thus, no current will be induced and the acceleration will be g. But according to Lenz's law the induced current will oppose its motion when it enters or leaves the field. Therefore, acceleration will be less than g when it enters and comes out of the magnetic field.

19. (1) Power of plate 1 : $A \sigma T^4$

Power of plate 3 : $81 A \sigma T^4$

$\therefore$ Power absorbed by Middle plate $\frac{A\sigma T^4}{2} + \frac{81 A \sigma T^4}{2} = 41 A \sigma T^4$ Power emitted by middle plate; $\sigma A(x)^4$

At steady state,

$$\sigma A x^4 = 41 A \sigma T^4$$

$$x = 41^{1/4}\, T$$

20. (1) $R = \frac{u^2}{g} = d$ (given)

$$H_{max} = \frac{u^2}{2g} = \frac{d}{2}$$

21. [3.00] $\frac{1}{2} v'^2 = \frac{1}{2}\left(\frac{1}{2} v^2\right)$

$$\therefore \quad v' = \frac{1}{\sqrt{2}} v$$

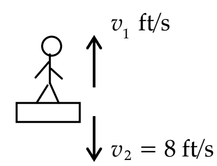
$$\therefore \quad e = \frac{1}{\sqrt{2}}$$

Now $d = h\left[\frac{1+e^2}{1-e^2}\right]$

$$= 1\left[\frac{1+(1/\sqrt{2})^2}{1-(1/\sqrt{2})^2}\right]$$

$$= \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3.00$$

22. [2.22]



$$m_1 v_1 = m_2 v_2$$

$$\Rightarrow \quad 50\, v_1 = 5 \times 8$$

$$\Rightarrow \quad v_1 = 0.8 \text{ ft/s}$$

Now $d_p - 2 \times (d_f) = d_M$

$$\Rightarrow \quad v_2 t - 2 \times 8 = v_1 \times t$$

$$\Rightarrow \quad t = \frac{16}{v_2 - v_1} = \frac{16}{7.2} \text{ s}$$

$$= 2.22 \text{ s}$$

23. [1.67] $I_1 = 1 = \frac{E}{R+r} = \frac{10}{8+r} \Rightarrow r = 2\Omega$

$I_2 = \frac{E}{[8\times8/8+8]+2} = \frac{10}{6} = 1.67 \text{ A}$

24. [780] If m is mass of single drop then as it drops

$mg = 2\pi rT$

If number of drops in M = 10 grams is N then,

$$N = \frac{M}{m} = \frac{Mg}{mg} = \frac{Mg}{2\pi rT} \approx 779.86 \approx 780$$

25. [0.06] Weight of cylinder

$= 300 \times 10^{-4} \times 10 \times 10^{-2} \times 800$ g

$= 2.4$ g

Let x is the length of cylinder inside water. Then

$2.4 \text{ g} = 300 \times 10^{-4} \times x \times 1000$ g

$x = 0.08$ m

When completely immersed buoyant force

$F_b = 300 \times 10^{-4} \times 0.1 \times 1000$ g

$F_b = 3$ gN

Therefore to immerse the cylinder inside water external agent has to push it by 0.02 m. against average upward thrust.

Increase in upward thrust

$= 3 \text{ g} - 2.4 \text{ g} = 0.6$ g N.

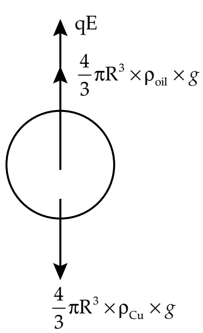
Since this increase takes place gradually, so we take average upward thrust against which work done $= 0.3$ g N

$\therefore$ Work done $= 0.3 \text{ g} \times 0.02$

$= 0.3 \times 10 \times 0.02$

$= 0.06$ J

26. [34.00]



For equilibrium

$$qE + \frac{4}{3}\pi R^3 \times \rho_{oil} \times g = \frac{4}{3}\pi R^3 \times \rho_{Cu} \times g$$

$$q = \frac{\frac{4}{3}\pi R^3(\rho_{Cu} - \rho_{oil})g}{E}$$

$$q = \frac{\frac{4}{3} \times 3.14 \times (0.5 \times 10^{-2})^3 (7.8) \times 10^3 \times 10}{3600}$$

$$q = \frac{\frac{4}{3} \times 3.14 \times 7.8 \times 0.125 \times 10^{-2}}{3600}$$

$q = 3.4 \times 10^{-5}$ C

$= 34 \times 10^{-6}$ C

$= 34.00\ \mu$C

27. [5.00]

$$S_1P - S_2P = \frac{d^2}{2D}$$

$$= \frac{2\times10^{-3}\times2\times0^{-3}}{2\times\frac{8}{5}}$$

$$= \frac{5}{2}\lambda$$

$(\lambda = 500 \text{ nm})$

So, when S is at ∞ there is 1st minima and when S is at S_2 there is last minima because $d/\lambda = 4000$

So the number of minimas will be 4001 and number of maximas will be 4000 = 3995 + 5

i.e., $n = 5$

28. [2.00] As in case of discharging of a capacitor through a resistance

$q = q_0 e^{-t/CR}$

$$i = -\frac{dq}{dt} = \frac{q_0}{CR}e^{-t/CR}$$

Here, $CR = \left(\frac{\varepsilon_0 kA}{d}\right)\left(\rho\frac{d}{A}\right) = \frac{\varepsilon_0 k}{\sigma}$ [as $\rho = 1/\sigma$]

i.e., $CR = \frac{8.846\times10^{-12}\times5}{7.4\times10^{-12}} = 6$

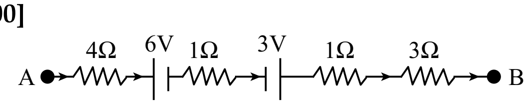
So, $i = \frac{8.85\times10^{-6}}{6}e^{-12/6}$

$$= \frac{8.85\times10^{-6}}{6\times7.39}$$

$[\text{As } e = 2.718,\ e^2 = 7.39]$

$= 0.20 = 2 \times 10^{-1}\ \mu$A

29. [6.00]



$$E_{eg} = \frac{\frac{9}{1}+\frac{0}{2}}{\frac{1}{1}+\frac{1}{2}}$$

$$= 9 \div \frac{3}{2}$$

$$= 6.00 \text{ V}$$

Potential difference across $2\Omega = 6.00$ V

30. [139]

$$I = I_0 e^{-\mu x}$$

$$\frac{I_0}{2} = I_0 e^{-\mu x}$$

$$\mu x = \ell n 2$$

$$x = \frac{\ell n 2}{\mu}$$

$$= \frac{0.693}{50}$$

$$= 0.01386 \text{ cm}$$

$$= 138.6\ \mu\text{m} \simeq 139\ \mu\text{m}$$

Chemistry

31. Option (2) is correct.

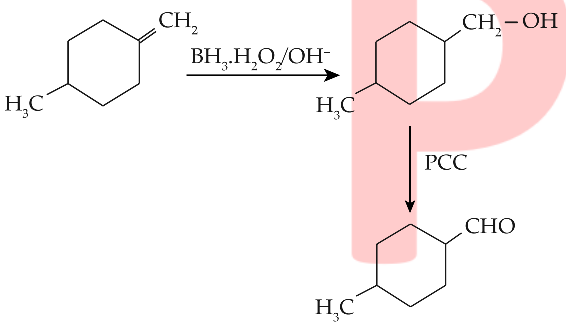
$[PtCl_4]^{2-}$ has square planar geometry and dsp^2 hybridisation.

BrF_5 has $sp^3\ d^2$ hybridisation and square bipyramidal geometry

PCl_5 has sp^3d hybridisation and triginal bipyramidal geometry

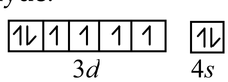
$[CO(NH_3)_6]^{3+}$ has d^2sp^3 hybridisation & octahedral geometry.

32. Option **(3) is correct.**

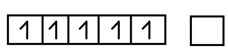


In the reaction, first step involves addition of H_2O to alkene according to anti–markovnikov's rule while the second step involves oxidation of 1° alcohol to aldehyde.

33. (4) Fe(26)

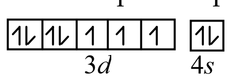


Fe^{+2}

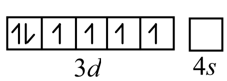


CN^- is strong field ligand, it pair the electron and form low spin complex.

Co(27)

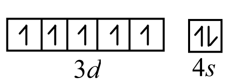


Co^{+3}

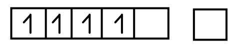


NO_2^- being a strong field ligand is able to pair up the pairing of electrons, so low spin complex will form.

Mn (25)



Mn^{+3}



CN^- strong field ligand, causes the pairing of electron so low spin complex will be formed. Thus, all the given complexes are low spin complex.

34. (3) $Na_3PO_4 + 3AgNO_3 \rightarrow \underset{\text{yellow}}{Ag_3PO_4} \downarrow + 3NaNO_3$

Yellow precipitate is soluble in dilute nitric acid as well as in ammonium hydroxide.

35. (4) Given:

Molar mass of A = 93 g mol^{-1}

Molal depression constant of water is 1.86 K kg mol^{-1}

$$\Delta T_f = 0.2$$

$$\Delta T = ik_f m$$

$$m = \frac{0.7}{93} \times \frac{1000}{42}$$

$$0.2 = \frac{0.7}{93} \times \frac{1000}{42}$$

$$0.2 = i \times 1.86 \times \frac{0.7}{93} \times \frac{1000}{42}$$

$$i = 6$$

$$\alpha = \frac{i - 1}{\frac{1}{n} - 1}$$

Put the value $i = 6$

$$\alpha = \frac{6 - 1}{\frac{1}{2} - 1}$$

$$= 0.8$$

The percentage association of solute A in water is 80%.

36. (2) $\Lambda_{m_1} = \dfrac{k_1 \times 1000}{M_1} = \dfrac{k_1 \times 1000}{\frac{10}{0.02}}$

$\Lambda_{m_2} = \dfrac{k_2 \times 1000}{\frac{20}{0.08}}$

It is given that $k_1 = k_2$

$k_1 = \dfrac{\wedge_{m_1}}{2} k_2 = \dfrac{\wedge_{m_2}}{4}$

Applying the given condition on conductivity.

$\dfrac{\wedge_{m_1}}{2} = \dfrac{\wedge_{m_2}}{4}$

$\wedge_{m_2} = 2 \wedge_{m_1}$

37. (2) (A) $R\text{–}CN \xrightarrow{\text{reduction}} R\text{–}CH_2NH_2$

1° Amine

(B) $R\text{–}CN \xrightarrow[H_2O]{CH_3MgBr} R\text{–}\underset{}{\overset{CH_3}{\overset{|}{C}}}{=}N\text{–}MgBr$

$\xrightarrow{H_2O} R\text{–}\underset{\underset{O}{\|}}{C}\text{–}CH_3 + NH_3 + Mg\begin{smallmatrix}Br\\OH\end{smallmatrix}$

Methyl ketone

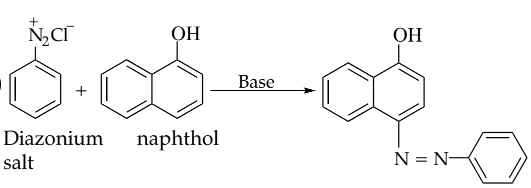
(C) $R\text{–}NC \xrightarrow{\text{hydrolysis}} R\text{–}NH\text{–}CH_3$

2° Amine

(D) $R\text{–}NH_2 \xrightarrow{HNO_2} R\text{–}OH$

Alcohol

38.(1)



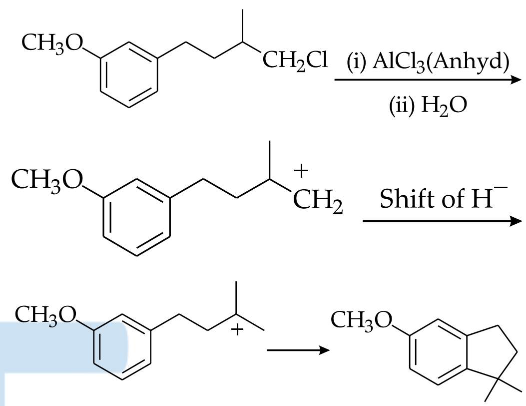
4 - (Phenylazo)-1-naphthol
(orange red dye)

39. (1)

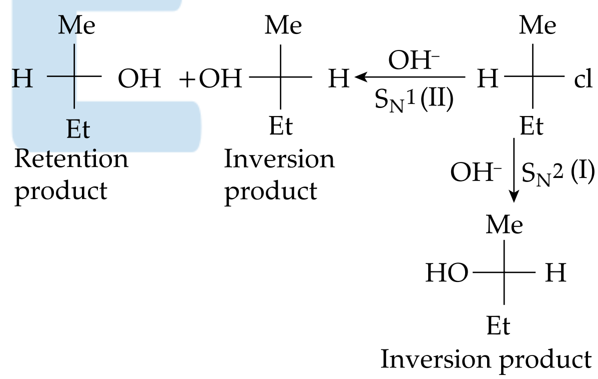
Element	%	Atomic weight amount	Relative number	Simple ratio
C	18.5%	12	$\frac{18.5}{12} = 1.54$	1
H	1.55%	1	$\frac{1.55}{1} = 1.55$	1
O	24.81%	6	$\frac{24.81}{16} = 1.55$	1
Cl	55.04%	35.5	$\frac{55.04}{35.5} = 1.55$	1

$\therefore$ Empirical formula = CHClO

40. (4)



41. (4)



Step 1 is a S_N2 reaction, since as inverted product is formed i.e., the attack of nucleophile occurs from the back side of the halogen (leaving group).

Step 2 is a S_N1 reaction, since both retention and inverted product is obtained. This is because in this mechanism, a carbocation is generated thus an attack of nucleophile from both rear and front side of the halogen is possible.

42. (2) The bond strength is directly proportional to bond order,

	Total no. of electrons	Molecular orbital configuration	$B.O=\frac{1}{2}(N_b-N_a)$	Magnetic properties
O_2^+	15	$(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2$ $(\pi 2p_x{}^2=\pi 2p_y{}^2)\ (\pi^*2p_x{}^1=\pi^*2p_y{}^0)$	$\frac{1}{2}(10-5)=2.5$	Paramagnetic
O_2	16	$(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2$ $(\pi 2p_x{}^2=\pi 2p_y{}^2)\ (\pi^*2p_x{}^1=\pi^*2p_y{}^1)$	$\frac{1}{2}(10-6)=2$	Paramagnetic
O_2^-	17	$(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2$ $(\pi 2p_x{}^2=\pi 2p_y{}^2)\ (\pi^*2p_x{}^1=\pi^*2p_y{}^1)$	$\frac{1}{2}(10-7)=1.5$	Paramagnetic
O_2^{2-}	18	$(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2$ $(\pi 2p_x{}^2=\pi 2p_y{}^2)\ (\pi^*2p_x{}^1=\pi^*2p_y{}^1)$	$\frac{1}{2}(10-8)=1$	Diamagnetic

According to this, the correct order of bond strength is : $O_2^{2-}<O_2^-<O_2<O_2^+$

43. (2) $Cl_2(g)\rightarrow 2Cl(g)$ 242.3 kJ mol^{-1}

$I_2(g)\rightarrow 2I(g)$ 151 kJ mol^{-1}

$I_2(s)\rightarrow I_2(g)$ 62.76 kJ mol^{-1}

$2I(g)+2Cl(g)\rightarrow 2ICl(g)\ 2\times(-242.3\text{ kJ mol}^{-1})$

The required equation will be

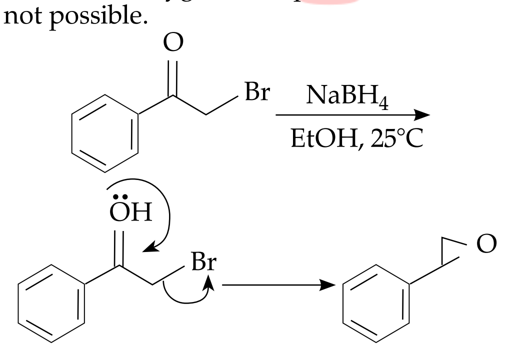
$I_2(s)+Cl_2(g)\rightarrow 2ICl(g)$

$\Delta H=242.3+151+62.76-(2\times 242.3)=-28.54$

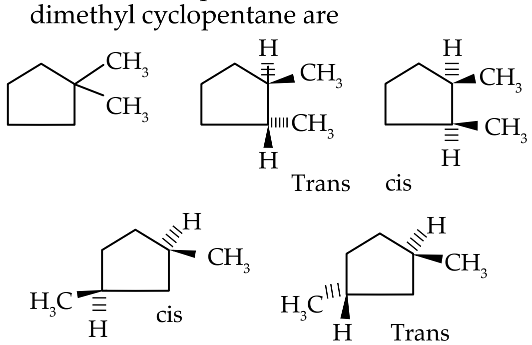
$$\Delta H=\frac{-28.54}{2}=-14.27\text{ kJ/mol}$$

44. (4) Expansion of octet is not possible for second period elements e.q. NO etc,. in option (1), (2) if positive charge delocalised expansion of octet of atom is required which is not possible where as in option (3) explanation of octet of oxygen is required which also not possible.

45. (2)



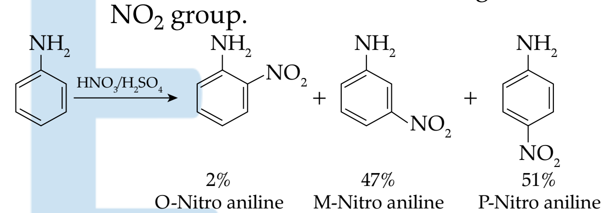
46. (3) The structure possible for stereoisomer of dimethyl cyclopentane are

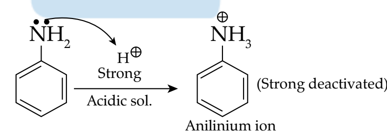


∴ Thus there are 5 stereoisomers possible.

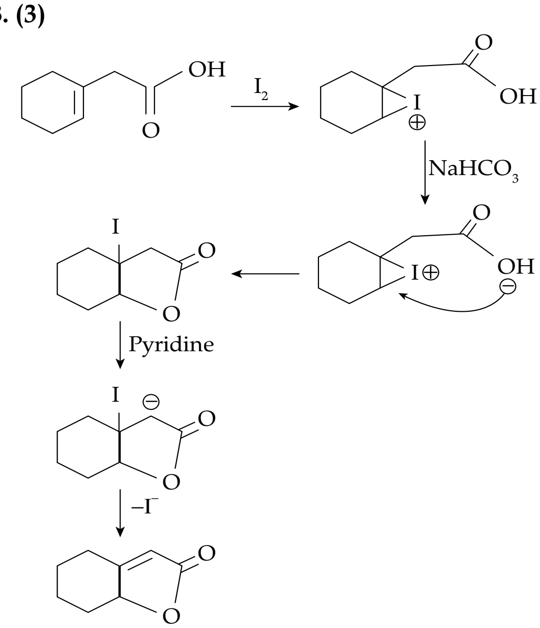
47. (1) Nitration of aniline involves acidic medium. So protonation of aniline takes place forming anilium ion which being meta directing forms mainly m-nitro aniline. Hence, direct nitration of aniline forms the following products.

Nitrating mixture is a strong acidic mixture because electron-withdrawing is the NO_2 group.

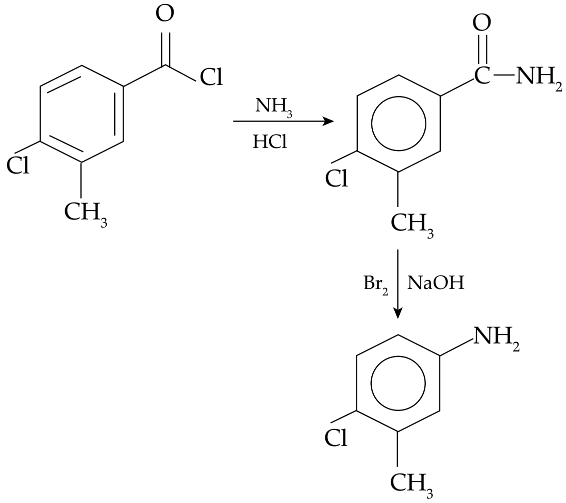




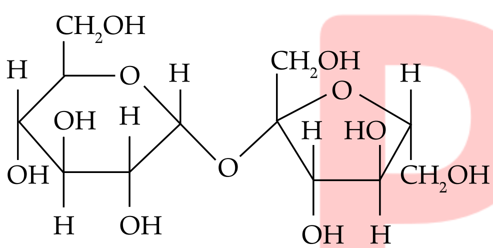
48. (3)



49. (3)



50. (2) Glycosidic is a head to head linkage. Sucrose is composed of a molecule of glucose joined to a molecule of fructose by an α–1, β–2–glycosidic linkage.



51. [6.00] Concentration of drop

$$= \frac{\text{Mole}}{\text{volume in ml}} \times 1000$$

$$= \frac{3.0 \times 10^{-6}}{0.05} \times 1000 = 0.06 \text{ mol L}^{-1}$$

Disappearance of 10^7 mol L^{-1} takes 1 s

Disappearance of 0.06 mol L^{-1} will take

$$\frac{0.06}{10^7} = 6.00 \times 10^{-9} \text{ s}$$

52. [23080]

$$\frac{N}{N_0} = \frac{1}{16}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^4$$

$$n = 4$$

Total time = $n \times t_{1/2}$

= 4 × 5770

= 23080 years.

53. [8.23] $[Co(CN)_6]^{4-} \rightarrow [Co(CN)_6]^{3-} + e^-, E^\circ_{RP} = 0.83V$

$Co^{3+} + e^- \longrightarrow Co^{2+}\ E^\circ_{RP} = 1.82$ V

$[Co(CN)_6]^{4-} + Co^{3+} \rightleftharpoons Co^{2+} + [Co(CN)_6]^{3-}$

$$\text{Ecell} = \text{E°cell} + \frac{0.059.}{1}\log_{10}\frac{[Co^{3+}]\,[Co\,(CN)_6]^{4-}}{[Co^{2+}]\,[Co\,(CN)_6]^{3-}}$$

$$= \text{E°cell} + \frac{0.59}{1}\log_{10}\frac{[Co^{3+}]\,[Co\,(CN)_6]^{4-}[CN^-]^6}{[Co^{2+}]\,[Co\,(CN)_6]^{3-}[CN^-]^6}$$

$$= \text{E°cell} + \frac{0.59}{1}\log_{10}\frac{[Co^{3+}]\,[Co\,(CN)_6]^{4-}[CN^-]^6}{[Co^{2+}]\,[Co\,(CN)_6]^{3-}[CN^-]^6}$$

$$= \text{E°cell} + \frac{0.059}{1}\log_{10}\frac{[Co^{3+}]\,[CN^-]^6}{[Co\,(CN)_6]^{3-}} \times \frac{1}{\frac{[Co^{2+}]\,[CN]^6}{[Co\,(CN)_6]^{-4}}}$$

$$= \text{E°cell} + \frac{0.059}{1}\log_{10}\frac{K_{f_1}}{K_{f_2}}\quad K_{f_1} = \left[\frac{(Co\,(CN)_6]^{4-}}{[Co^{2+}]\,[CN^-]^6}\right.$$

$$\left[K_{f_1} = \frac{(Co\,(CN)_6)^{3-}}{[Co^{3+}]\,[CN^-]^6}\right]$$

$$0 = 0.83 + 1.82 + \frac{0.059}{1}\log_{10}\frac{10^{19}}{K_{f_2}}$$

$$\Rightarrow \quad \frac{K_{f_2}}{10^{19}} = 8.23 \times 10^{44}$$

$$\Rightarrow \quad K_{f_2} = 8.23 \times 10^{63}$$

54. [3]

The reaction of an acid and a base, which forms water and a salt is called neutralization.

Acid + Base → salt + H_2O

0.1 M $M(OH)_2$

10ml 0.05M

30ml

At equivalence point equivalent of acid = equivalent of base

= 0.1 × 10 × n

= 30 × 0.05 × 2

= 3

The neutralization occurs when 10 ml of 0.1M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is 3

55. [0.733] Observed Molecular mass

$$= \frac{1000 \times k_f \times w}{w \times \Delta T}$$

$$= \frac{1000 \times 5.12 \times 20 \times 10^{-3}}{1 \times 0.69}$$

= 148.4

Normal Molecular mass of phenol = 94

$$i = \frac{\text{Normal Molecular mass}}{\text{Observed Molecular mass}}$$

$$2C_6H_5OH \rightleftharpoons (C_6H_{5_o}OH)_2$$

Initially 1 α/2

equilibrium 1 – α

$\therefore$ Total no. of moles $= 1 - \alpha + \frac{\alpha}{2}$

$$= 1 - \frac{\alpha}{2}$$

Thus, $i = 1 - \frac{\alpha}{2}$

$\Rightarrow \quad 1 - \frac{\alpha}{2} = 0.633$

$$\alpha = 0.733$$

56. [14].

The Dumas method is appropriate to determine the molecular weights of volatile organic substances that are liquids at room temperature.

Given:

Mass of organic compound = 0.2 g

Volume of N_2 gas evolved at STP = 22.4 ml

Mass of N_2 gas evolved $= \frac{22.4 \times 10^{-3}}{22.4} \times 28$

$$= 0.028 \text{ g}$$

% of nitrogen in the compound $= \frac{0.028 \times 100}{0.2}$

The percentage of nitrogen in the compound is = 14%

57. [4].

Total number of peptide bonds = number of amino acids −1

= 5 – 1

= 4

ala–gly–leu–ala–val 4 peptide bonds

There are five amino acids and four peptide linkages.

58. [1.823] (E.N.) Allred Rochow

$$= 0.744 + \frac{0.359}{r^2} Z_{eff}$$

Z_{eff} of $Si^+ \Rightarrow Si = 1s^2 2s^2 2p^6 3s^2 3p^2$

$\therefore \quad Z_{eff} = 14 - (3 \times 0.35 + 8 \times 0.85 + 2 \times 1)$

$= 4.15$

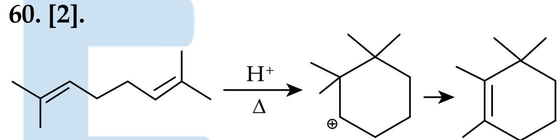
$\therefore$ E.N. $= 0.744 + \frac{0.359}{(1.175)} \times 4.15 = 1.823$

59. [2.00] The configuration of C_2 molecule according to Molecular orbital theory is

$KK\,[\sigma_{(2s)}]^2\,[\sigma_{(2s)}]^2\,[(\pi_{2px})]^2\,[\pi_{2py}]^2$

As two π MO are filled. ∴ Two π–bond

60. [2].

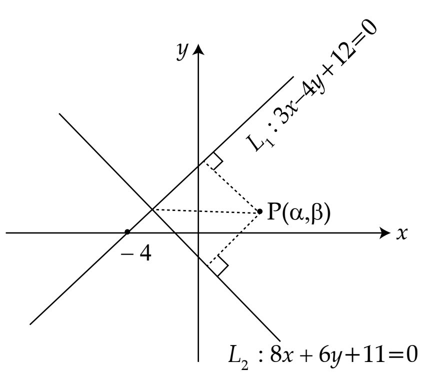


2, 7-Dimethylocta-2, 6-diene

1, 2, 3, 3-tectro methyl cyclohex-1-ene

Mathematics

61. (4)



Given: $L_1 : 3x\text{-}4y+12=0$

$L_2 : 8x+6y+11=0$

Equation of angle bisector of L_1 and L_2 containing origin

$$\frac{3x-4y+12}{\sqrt{(3)^2+(4)^2}} = \frac{8x+6y+11}{\sqrt{64+36}}$$

$\Rightarrow 2(3x - 4y + 12) = 8x + 6y + 11$

$\Rightarrow 2x + 14y - 13 = 0$ (i)

$\therefore$ Distance of $p\ (\alpha,\beta)$ from L_1 is 1

$\therefore \frac{3\alpha - 4\beta + 12}{5} = 1$

$\Rightarrow 3\alpha - 4\beta + 7 = 0$ (ii)

$\therefore$ P(α,β) also lies on $2x+14y-13=0$

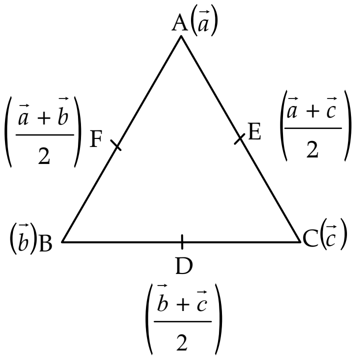
$\therefore 2\alpha + 14\beta - 13 = 0$ (iii)

on solving equation (ii) and (iii), we get

$\alpha = \frac{-23}{25}$ and $\beta = \frac{53}{50}$

$$\therefore 100(\alpha+\beta) = 100\left(\frac{-23}{25}+\frac{53}{50}\right) = 14$$

62. (2)



$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$$

$$= \left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\left(\frac{\vec{a}+\vec{c}}{2}-\vec{b}\right)+\left(\frac{\vec{a}+\vec{b}}{2}-\vec{c}\right)$$

$$= \left(\vec{a}+\vec{b}+\vec{c}\right)-\left(\vec{a}+\vec{b}+\vec{c}\right)$$

$$= 0$$

Hence, $\left|\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}\right| = 0$

63. (3) $I = \int \frac{e^x}{e^{2x}+1}\,dx$

Let $e^x = t$

$\Rightarrow e^x dx = dt$

$\Rightarrow I = \int \frac{1}{t^2+1}\,dt$

$= \tan^{-1} t + C$

$= \tan^{-1} e^x + C$

64. (1) $I = -\int_0^{1/3}(3x-1)\,dx + \int_{1/3}^{1}(3x-1)\,dx$

$$= \left[x-\frac{3x^2}{2}\right]_0^{1/3} + \left[\frac{3x^2}{2}-x\right]_{1/3}^{1}$$

$$= \frac{1}{3}-\frac{1}{6}+\frac{3}{2}-1-\frac{1}{6}+\frac{1}{3}$$

$$= \frac{5}{6}$$

65. (1) Area $= 2\int_0^{\pi} \sin x\,dx$

$= 2\left[-\cos x\right]_0^{\pi}$

$= 2[1 + 1] = 4$ square units

66. (2) We have $\sin\left(\frac{dy}{dx}\right) = \frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}x + c$

$\because$ The curve passes through (0, 1) $\Rightarrow$ c = 1

$\therefore y = \frac{\pi}{6}x + 1 \Rightarrow \frac{y-1}{x} = \frac{\pi}{6} = \sin^{-1}\frac{1}{2}$

$\Rightarrow \sin\left(\frac{y-1}{x}\right) = \frac{1}{2}$

67. (2) Given: $y = a\log_e|x| + bx^2 + x$

$$\Rightarrow y = \begin{cases} a\log_e x + bx^2 + x; & x > 0 \\ a\log_e(-x) + bx^2 + x; & x < 0 \end{cases}$$

$$\Rightarrow \frac{dy}{dx} = \begin{cases} \frac{a}{x} + 2bx + 1, & x > 0 \\ \frac{a}{x} + 2bx + 1, & x < 0 \end{cases}$$

Since, $y = f(x)$ has it's extremum values at $x = -1$ and $x = 2$

$\therefore \left(\frac{dy}{dx}\right)_{x=-1} = 0$ and $\left(\frac{dy}{dx}\right)_{x=2} = 0$

$\Rightarrow -a - 2b + 1 = 0$ and $\frac{a}{2} + 4b + 1 = 0$

$\Rightarrow 2b + a = 1$ and $a + 8b = -2$

$\Rightarrow a = 2$ and $b = -\frac{1}{2}$

68. (3)
$$\frac{dy}{dx} = \frac{a\cos\theta}{-a\sin\theta + \frac{a\sec^2\frac{\theta}{2}}{\tan\frac{\theta}{2}}\times\frac{1}{2}}$$

$$= \frac{a\cos\theta}{-a\sin\theta + \frac{a}{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}}$$

$$= \frac{a\cos\theta}{\frac{a}{\sin\theta} - a\sin\theta}$$

$$= \frac{a\cos\theta\sin\theta}{a - a\sin^2\theta}$$

$$= \frac{\cos\theta\sin\theta}{\cos^2\theta}$$

$$= \tan\theta$$

69. (3) $y = \dfrac{1}{2 - \sin 3x}$

$-1 \le \sin\theta \le 1$

$-1 \le \sin 3x \le 1$

$1 \ge -\sin 3x \ge -1$

$3 \ge 2 - \sin 3x \ge 1$

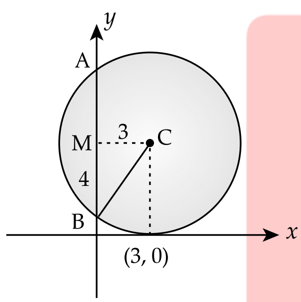
$\dfrac{1}{3} \le \dfrac{1}{2-\sin 3x} \le 1$

$\therefore$ range $= \left[\dfrac{1}{3}, 1\right]$

70. (2) $\lim\limits_{x\to -1} \dfrac{x^3 - 2x - 1}{x^5 - 2x - 1}$

$= \lim\limits_{x\to -1} \dfrac{(x+1)(x^2 - x - 1)}{(x+1)(x^4 - x^3 + x^2 - x - 1)} = \dfrac{1}{3}$

71. (1) Let centre of circle is C and circle cuts the y-axis at B and A. Let mid-point of chord BA is M.



$CB = \sqrt{MC^2 + MB^2}$

$\Rightarrow CB = \sqrt{3^2 + 4^2} = 5$ = radius of circle

$\therefore$ Equation of circle is,

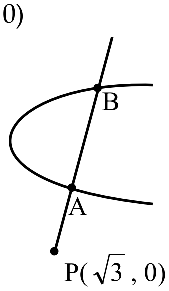
$$(x-3)^2 + (y-5)^2 = 5^2$$

(3, 10) satisfies this equation.

Although, there will be another circle satisfying the same conditions that will lie below the x-axis having equation

$(x-3)^2 + (y+5)^2 = 5^2$

72. (1) Given, parabola is $y^2 = x + 2$ and given line is $y = \sqrt{3}x - 3$ and co-ordinates of P are $(\sqrt{3}, 0)$



AB makes an angle of 60° with the positive direction of x-axis. Co-ordinates of any point on this line may be taken as

$$(\sqrt{3} + r\cos 60°, 0 + r\sin 60°)$$

or $\left(\sqrt{3} + \dfrac{r}{2}, \dfrac{r\sqrt{3}}{2}\right)$

If this point lies on $y^2 = x + 2$ then

$$\frac{3}{4}r^2 = \sqrt{3} + \frac{r}{2} + 2$$

or $\quad 3r^2 = 4\sqrt{3} + 2r + 8$

or $\quad 3r^2 - 2r - 4(2 + \sqrt{3}) = 0 \quad$ ...(i)

Let r_1 and r_2 be the roots of equation (i)

then, $r_1 r_2 = -\dfrac{4(2+\sqrt{3})}{3}$

Now, $PA \cdot PB = |r_1|\ |r_2|$

$= |r_1 r_2| = \dfrac{4}{3}(2 + \sqrt{3})$

73. (1) Let the eccentric angle of the point P be θ. So, equation of tangent is

$\dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{b} = 1 \quad$ ...(i)

and equation of normal is

$\dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = a^2 - b^2 \quad$ ...(ii)

These lines meet the major axis $y = 0$ at points T and T′ such that $TT' = a$

So, T is $\left(\dfrac{a}{\cos\theta}, 0\right)$ and T′ is $\left(\dfrac{(a^2 - b^2)\cos\theta}{a}, 0\right)$

$\therefore TT' = \dfrac{a}{\cos\theta} - \dfrac{(a^2 - b^2)\cos\theta}{a} = a$ (given)

or $\quad a^2 - a^2e^2\cos^2\theta = a^2\cos\theta$

or $\quad 1 - e^2\cos^2\theta = \cos\theta$

or $\quad e^2\cos^2\theta + \cos\theta - 1 = 0$

or $\quad e^2\cos^2\theta + \cos\theta = 1$

74. (4) Equation of ellipse

$\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1 \quad$ **....(i)**

And equation of hyperbola

$\dfrac{x^2}{144} - \dfrac{y^2}{\alpha} = \dfrac{1}{25} \quad$ **....(ii)**

$\Rightarrow \dfrac{x^2}{\frac{144}{25}} - \dfrac{y^2}{\frac{\alpha}{25}} = 1$

Have same foci, then

$$a^2 - b^2 = l^2 + m^2$$

$$\Rightarrow \quad 16 - 7 = \frac{144}{25} + \frac{\alpha}{25}$$

$$\Rightarrow \quad 9 = \frac{144}{25} + \frac{\alpha}{25}$$

$$\Rightarrow \quad 9 \times 25 = 144 + \alpha$$

$$\Rightarrow \quad \alpha = 225 - 144 = 81$$

$$\therefore \Rightarrow \quad \alpha = 81$$

Latus Rectum of hyperbola $= \dfrac{2b^2}{a} = \dfrac{2\times\frac{\alpha}{25}}{\frac{12}{5}}$

$$\Rightarrow \quad \frac{2\times 81}{25}\times\frac{5}{12} = \frac{27}{10}$$

75. (3) Let roots be α and β then from equation

$$-x^2 - bx + a = 0$$

We have $\alpha\beta = -a < 0$

⇒ Both the roots are in opposite sign and

$$\alpha + \beta = -b < 0$$

⇒ Both roots are in opposite sign and greater root in magnitude is negative.

76. (2)
$$N = \left(\frac{5}{3}\right)^{-100}$$

$\Rightarrow \log_{10} N = -100\,(\log_{10} 5 - \log_{10} 3)$
$= -100\,(1 - 0.3010 - 0.4771)$
$= -100\,(0.2219)$
$= -22.19$
$= -23 + 0.81$

So, characteristic = – 23

⇒ Number of ciphers after decimal is 22

77. (1) ∵ Sum of n terms of A.P with first terms a and common difference d is given by

$$S_n = \frac{n}{2}\,[a + (n-1)\,d]$$

$$\therefore \quad \frac{S_{n_1}}{S_{n_2}} = \frac{\frac{n}{2}\left[2a_1 + (n-1)d_1\right]}{\frac{n}{2}\left[2b_1 + (n-1)d_2\right]} = \frac{2n}{n+1}$$

$$\Rightarrow \quad \frac{a_1 + \frac{(n-1)}{2}d_1}{b_1 + \frac{(n-1)}{2}d_2} = \frac{2n}{n+1} \qquad \ldots\text{(i)}$$

For $T_{8,}$ we know $\dfrac{n-1}{2} = 7$

$$\Rightarrow \quad n = 15$$

Put $n = 15$ in (i), we get

$$\frac{(T_8)_1}{(T_8)_2} = \frac{30}{16} = \frac{15}{8}$$

78. (2)
$$T_4 = {}^nC_3 (ax)^{n-3}\left(\frac{1}{x}\right)^3$$

$$= {}^nC_3\, a^{n-3}\, x^{n-6} = \frac{5}{2} \text{ (given)}$$

$$\therefore \quad n - 6 = 0$$

$$\Rightarrow \quad n = 6$$

$$\Rightarrow \quad T_4 = {}^6C_3 a^{6-3} = \frac{5}{2}$$

$$\Rightarrow \quad a^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

$$\Rightarrow \quad a = \frac{1}{2}$$

$$\Rightarrow \quad a = \frac{1}{2},\ n = 6$$

79. (1)
$${}^{(n+1)}C_{(n-2)} - {}^{(n+1)}C_{(n-1)} \le 100$$

$$\Rightarrow \quad \frac{(n+1)!}{(n-2)!(n+1-n+2)!} - \frac{(n+1)!}{(n-1)!(n+1-n+1)!} \le 100$$

$$\Rightarrow \quad \frac{(n+1)!}{(n-2)!3!} - \frac{(n+1)!}{(n-1)!2!} \le 100$$

$$\Rightarrow (n+1)!\left[\frac{1}{3!(n-2)!} - \frac{1}{2!(n-1)!}\right] \le 100$$

$$\Rightarrow \quad (n+1)!\left[\frac{(n-1)! - 3(n-2)!}{6(n-1)!(n-2)!}\right] \le 100$$

$$\Rightarrow \frac{(n+1)n(n-1)!\left[(n-1)(n-2)! - 3(n-2)!\right]}{6(n-1)!(n-2)!} \le 100$$

$$\Rightarrow \frac{(n+1)\,n\,(n-1)!(n-2)!(n-1-3)}{6(n-1)!(n-2)!} \le 100$$

$$\Rightarrow \quad \frac{(n+1)\,n\,(n-4)}{6} \le 100$$

$$\Rightarrow \quad n(n+1)(n-4) \le 600$$

Put $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\ldots$

For $n = 5$, $600 \ge 5 \times 6 \times 1$ holds good

For $n = 8$, $600 \ge 8 \times 9 \times 4 = 288$ holds good

For $n = 9$, $600 \ge 9 \times 10 \times 5 = 450$ holds good

For $n = 10$, $600 \le 10 \times 11 \times 7 = 770$ does not hold

Hence, the number of positive integer which is satisfying the given inequality must be 9.

Shortcut Method :

$${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$$

$$\Rightarrow {}^{n+1}C_3 - {}^{n+1}C_2 \le 100$$

$$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{(n+1)n}{2} \le 100$$

$\Rightarrow$ $n\,(n + 1)\,(n - 4) \le 600$

Consider

$f(x) = x\,(x + 1)\,(x - 4)$; $x \ge 1$

$= x^3 - 3x^2 - 4x$

$\Rightarrow$ $f'(x) = 3x^2 - 6x - 4$

If $f'(x) > 0$

$$x \in \left(1 + \frac{\sqrt{21}}{3}, \infty\right)$$

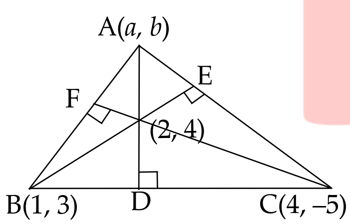
It is easy to check that

$f(10) = 10 \times 11 \times 6 > 660 > 600$

and $f(9) = 9 \times 10 \times 5 = 450 < 600$

$\therefore$ $x = 9$ is the last integer so that $f(x) < 600$.

80. (3)



From the figure

Slope of $BE = 1$

Slope of $AC = -1$

$\because$ (slope of BE $\times$ slope of $AC = -1$)

$\therefore$ Equation AC is $x + y = -1$ ...(i)

Slope of $CF = \dfrac{-9}{2}$

Slope of $AB = \dfrac{2}{9}$

$\because$ (slope of CF $\times$ Slope of $AB = -1$)

$\therefore$ Equation AB is $2x - 9y = -25$ ...(ii)

Solving (i) and (ii), we get

$$(a, b) = \left(\frac{-34}{11}, \frac{23}{11}\right)$$

$\therefore$ The value of $33b + 22a = 1$

81. [2.00] $f_4(x) - f_6(x)$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - 2\sin^2 x\cos^2 x\right) - \frac{1}{6}\left(1 - 3\sin^2 x\cos^2 x\right)$$

$$= \frac{1}{4} - \frac{1}{6}$$

$$= \frac{1}{12}$$

$$\Rightarrow 24\left(f_4(x) - f_6(x)\right) = 2$$

82. [130] Given $x = \dfrac{2\alpha}{1+\alpha^2}$

let $y = \sin\theta$

$$\Rightarrow 2\theta = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \tan 2\theta = \frac{4}{3}$$

$$\Rightarrow \frac{2t}{1-t^2} = \frac{4}{3} \Rightarrow 3t = 2 - 2t^2$$

$\Rightarrow$ $2t^2 + 3t - 2 = 0$

$\Rightarrow$ $(2t - 1)\,(t + 2) = 0$

$\Rightarrow$ $\tan\theta = \dfrac{1}{2}$

$\Rightarrow$ $\sin\theta = \dfrac{1}{\sqrt{5}}$

$\therefore$ $y = \dfrac{1}{\sqrt{5}}$

$\therefore$ $y^2 = 1 - x$

$$\Rightarrow \frac{1}{5} = 1 - \frac{2\alpha}{1+\alpha^2}$$

$\Rightarrow$ $1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha$

$\Rightarrow$ $4\alpha^2 - 10\alpha + 4 = 0$

$\Rightarrow$ $(2\alpha - 1)\,(\alpha - 2) = 0$

$\Rightarrow$ $\alpha = \dfrac{1}{2}$, $\alpha = 2$

$$\therefore \sum_{\alpha \in s} 16\alpha^3 = 16\left[\frac{1}{8} + 8\right] = 16 \times \frac{65}{8} = 130$$

83.[1.00] $$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) = \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1 + 2^n \cdot 2^{n-1}}\right)$$

$$= \sum_{n=1}^{\infty} (\tan^{-1} 2^n - \tan^{-1} 2^{n-1})$$

$= \tan^{-1} 2 - \tan^{-1} 1 + \tan^{-1} 4 - \tan^{-1} 2 + \ldots + \tan^{-1} \infty$

$= -\tan^{-1} 1 + \tan^{-1} \infty$

$$= -\frac{\pi}{4} + \frac{\pi}{2}$$

$$= \frac{\pi}{4} = 1.00 \times \frac{\pi}{4}$$

84. [8.00] Number of five digit numbers with 2 at 10^{th} place = 8 × 8 × 7 × 6 = 2688

Q It is given that, number of five digit number with 2 at 10^{th} place = 336 k

$\therefore$ 336 k = 2688 $\Rightarrow k = 8$

85. [17/8] Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27\times17}{64\times8}$$

$$\Rightarrow \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$$

$$\therefore \quad k = \frac{17}{8}$$

86. [5.00] $f(x) = \begin{cases} \dfrac{a\sin x + b\tan x - 3x}{x} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$

$$\lim_{x\to0} \frac{a\left(x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}\right) + b\left(x + \dfrac{x^3}{3} + \dfrac{2x^5}{15}\right) - 3x}{x^3}$$

Limit exist if $a + b - 3 = 0$ ...(i)

and $\lim_{x\to0} f(x) = f(0) = 0$

$$-\frac{a}{6} + \frac{b}{3} = 0 \quad \text{...(ii)}$$

From (i) and (ii), we get

$$a = 2, b = 1$$

$$\therefore \quad a^2 + b^2 = 5.00$$

87. [7.00] 4 must be obtained on atleast one dice. x = number of dice on which 4 is obtained required probability and the rest of the number same from 1, 2, 3

Now, $P_{(x = 1)} + P_{(x = 2)} + P_{(x = 3)} + P_{(x = 4)}$

$$= \frac{{}^4C_1 \times 3\times 3\times 3}{6^4} + \frac{{}^4C_2 \times 3^2}{6^4} + \frac{{}^4C_3 .3}{6^4} + \frac{1}{6^4}$$

$$= \frac{175}{1296} = \frac{25\times7}{1296}$$

Now, $\dfrac{25a}{1296} = \dfrac{25\times7}{1296}$

or, $a = 7$

88. [7.00] Given equation

$$4x^3 - 24x^2 + 47x - 30 = 0$$

$$\Rightarrow \quad (x-2)(2x-5)(2x-3) = 0$$

So, $a = 2,\ b = \dfrac{5}{2},\ c = \dfrac{3}{2}$

$$\Rightarrow \quad s = \frac{a+b+c}{2} = 3$$

Now substituting values of a, b, c in the determinant D,

$$D = \begin{vmatrix} 4 & 1 & 1 \\ \frac{1}{4} & \frac{25}{4} & \frac{1}{4} \\ \frac{9}{4} & \frac{9}{4} & \frac{9}{9} \end{vmatrix} = \frac{81}{2} = \frac{9^2}{2}$$

$$\therefore \quad p - q = (9 - 2) = 7.00$$

89. [600] Since, D is a skew symmetric matrix

$\therefore \quad D + O^T = 0$

$\Rightarrow \quad a\alpha^2 + b\alpha + c = 0,$

$a\beta^2 + b\beta + c = 0$

$a\gamma^2 + b\gamma + c = 0$

$\Rightarrow \quad ax^2 + bx + c = 0$ has three roots

$\Rightarrow \quad a = b = c = 0$

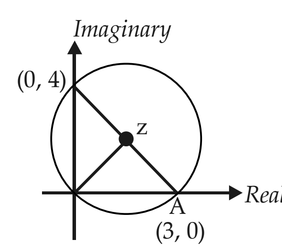
$$\therefore \quad \lambda \begin{vmatrix} 1 & 1 & 2 \\ 3 & 4 & 1 \\ 9 & 0 & 3 \end{vmatrix}$$

$$= -60$$

$\Rightarrow \quad |10\lambda| = |10 \times -60| = 600$

90. [5.00] z is equidistant from (0, 0), (3, 0) and (0, 4)

$\Rightarrow \quad z$ is at the circumcentre of ΔOAB



$$\Rightarrow \quad |z| = \frac{5}{2}$$

$$\Rightarrow \quad |2z| = 5.00$$

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