**PAPER CODE :- FULL TEST**
**FULL SYLLBUS TEST**

## ANSWER KEY

### PHYSICS

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (A) | **2.** | (B) | **3.** | (D) | **4.** | (D) | **5.** | (C) | **6.** | (C) | **7.** | (C) |
| **8.** | (D) | **9.** | (C) | **10.** | (C) | **11.** | (D) | **12.** | (C) | **13.** | (C) | **14.** | (B) |
| **15.** | (D) | **16.** | (B) | **17.** | (A) | **18.** | (D) | **19.** | (C) | **20.** | (B) | **21.** | 3 |
| **22.** | 61.6 | **23.** | 25 | **24.** | 26.4 | **25.** | 33.6 | **26.** | 4 | **27.** | 7 | **28.** | 2 |
| **29.** | 2 | **30.** | 2 | | | | | | | | | | |

### CHEMISTRY

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **31.** | (B) | **32.** | (C) | **33.** | (D) | **34.** | (A) | **35.** | (C) | **36.** | (A) | **37.** | (C) |
| **38.** | (D) | **39.** | (A) | **40.** | (B) | **41.** | (A) | **42.** | (D) | **43.** | (A) | **44.** | (B) |
| **45.** | (B) | **46.** | (B) | **47.** | (C) | **48.** | (B) | **49.** | (D) | **50.** | (A) | **51.** | 0.70 |
| **52.** | 2606 | **53.** | 455 | **54.** | 24 | **55.** | 25 | **56.** | 6 | **57.** | 1 | **58.** | 248 |
| **59.** | 728 | **60.** | 6 | | | | | | | | | | |

### MATHEMATICS

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **61.** | (C) | **62.** | (C) | **63.** | (D) | **64.** | (C) | **65.** | (D) | **66.** | (D) | **67.** | (A) |
| **68.** | (D) | **69.** | (D) | **70.** | (A) | **71.** | (D) | **72.** | (D) | **73.** | (C) | **74.** | (D) |
| **75.** | (D) | **76.** | (B) | **77.** | (D) | **78.** | (C) | **79.** | (D) | **80.** | (C) | **81.** | 2 |
| **82.** | 500 | **83.** | 2 | **84.** | 3 | **85.** | 60 | **86.** | 192 | **87.** | 3 | **88.** | 3 |
| **89.** | 2401 | **90.** | 50 | | | | | | | | | | |

## SOLUTIONS

## PHYSICS

**1.** (A)

**Sol.** $\lambda_m$ T = const.

ln $\lambda_m$ + lnT = C

$\frac{d\lambda_m}{\lambda_m} + \frac{dT}{T} = 0 \quad \therefore \quad \frac{d\lambda_m}{\lambda_m} = -\frac{dT}{T}$

Now $\frac{d\lambda_m}{\lambda_m}$ = – 1 % = (–ve sign indicates decrease)

$= -1\,\% = -\frac{1}{100}$ (–ve)

dT = 1 (given)

∴ T = 100 K.

**2.** (B)

**Sol.** Due to rotation, the effective value of g is $g' = g - \omega^2 R \cos^2\lambda$. Now ω increases so g' will decrease & so weight = mg' will reduce

**3.** (D)

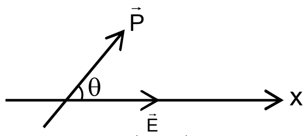
**Sol.** a sinθ = λ

∴ sinθ ≃ tanθ = x/D

$\lambda = \frac{ax}{D} = 5000$ Å

**4.** (D)

**Sol.**




$PE\sin\theta = P(\sqrt{3}E)\sin(90° - \theta)$

$\tan\theta = \sqrt{3}$

θ = 60°

**5.** (C)

**Sol.** $v \propto \frac{1}{n}$

**6.** (C)

**Sol.** $N_1 = mg\cos 30 = \frac{\sqrt{3}}{2} mg$

$N_2 - mg\sin 30 = \frac{m}{R} 4g\sin 30 R \quad \Rightarrow N_2 = \frac{5mg}{2}$

$N = \sqrt{N_1^2 + N_2^2} = \frac{mg}{2}\sqrt{28} = mg\sqrt{\frac{28}{4}} = \sqrt{7}mg$

**7.** (C)

**8.** (D)

**Sol.** Magnetic field due to circular segment = $\frac{3}{4} \cdot \frac{\mu_0 i}{2a}$

Due to one straight wire segment = $\frac{\mu_0 i}{4\pi b}(\sin 45° + \sin 0°) = \frac{\mu_0 i}{4\sqrt{2}\,\pi\, b}$

Net field

$= \frac{3\mu_0 i}{8a} + 2 \times \frac{\mu_0 i}{4\sqrt{2}\,\pi b} = \frac{\mu_0 i}{4\pi}\left(\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right)$

**9.** (C)

**Sol.** $dR = \frac{c\,d\ell}{\sqrt{\ell}}$

Acording to Question

$$\int_0^{\ell} c\frac{d\ell}{\sqrt{\ell}} = \int_{\ell}^{1} c\frac{d\ell}{\sqrt{\ell}}$$

$$\left(2\sqrt{\ell}\right)_0^{\ell} = \left(2\sqrt{\ell}\right)_{\ell}^{1}$$

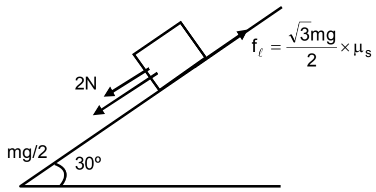
$$2\sqrt{\ell} = 2 - 2\sqrt{\ell}$$

$$4\sqrt{\ell} = 2$$

$$\ell = \frac{1}{4} = 0.25 \text{ m.}$$

**10.** (C)

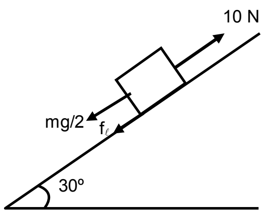
**Sol.** For just equilibrium:




$$2 = \frac{\sqrt{3}mg}{2}\mu_s - \frac{mg}{2} \qquad \text{...........(1)}$$

In the other case:

$$\frac{\sqrt{3}mg}{2}\mu_s + \frac{mg}{2} = 10 \qquad \text{...........(2)}$$




equation (1) / equation (2)

$$\frac{1}{5} = \frac{\sqrt{3}\mu_s - 1}{\sqrt{3}\mu_s + 1}$$

$$\mu_s = \frac{\sqrt{3}}{2}$$

**11.** (D)

**Sol.** $j = \frac{i}{A}$ and $i = \frac{V}{R}$

$\Rightarrow j = \frac{V}{RA}$ using $R = \frac{1}{\sigma}.\frac{\ell}{A}$

$j = \frac{V\sigma}{\ell}$ (or as $J = \sigma E$)

using values $j = 20 \times 10^6$ A/m$^2$

**12.** (C)

**Sol.** Here path difference will be :

$\Delta x = (\mu_2 - \mu_1)t \quad \Rightarrow \quad \delta = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)t$

Hence (C)

**13.** (C)

**Sol.** For solenoid

$$\frac{B}{\mu_0} = H$$

$B = \mu_0 nI$

$H = nI$

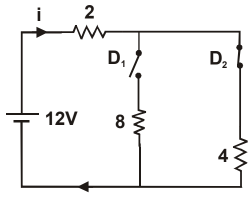
$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I$

$I = 3A$

**14.** (B)

**Sol.** The diode $D_1$ is reverse-biased and acts as an open switch as shown in Fig. So, there is no current through $D_1$ and the 8Ω resistor.

However, $D_2$ is forward biased and acts like a short-current or closed switch. The current drawn is $I = 12/(2 + 4) = 2A$.




**15.** (D)

**Sol.** Vertical components of velocities of A and B are same. So they collide.

Time to collide, $t = \frac{6}{10\sin 37^\circ} = 1$ sec

Height of balls at time of collision

$H = 8 \text{ x } 1 - \frac{1}{2} \text{ x } 10 \text{ x } 1^2 = 3m$

**16.** (B)

**Sol.** Initially, the frqeuency of the wire,

$$\frac{1}{2\ell}\sqrt{\frac{mg}{\mu}} = 110 \text{ Hz}$$

Finally, : $\frac{1}{2\ell}\sqrt{\frac{mg\left(1-\frac{\rho_\ell}{\rho_b}\right)}{\mu}} = 100 \text{ Hz}$

Dividing get $\frac{\rho_b}{\rho_l} = \frac{121}{21}$.

**17.** (A)

**Sol.** Parallel axes theorem is applicable only between two parallel axes.

By symmetry $I_{CC'} = I_{AA'}$

Parallel axes theorem $\Rightarrow I_{BB'} = I_{CC'} + MR^2 = I_{AA'} + MR^2$

**18.** (D)

**Sol.** At the end of one complete vibration, the particle returns to the initial position.

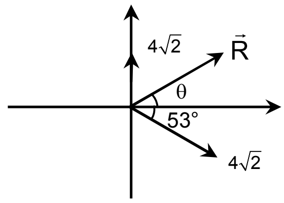
**19.** (C)

**Sol.** $i_1 = \frac{200\sqrt{2}}{\sqrt{30^2 + 40^2}}\sin\left(\omega t - \tan^{-1}\left(\frac{40}{30}\right)\right)$

$= 4\sqrt{2}\ \sin(100\pi t - 53^\circ)$

$i_2 = \frac{200\sqrt{2}}{50}\sin\left(\omega t + \frac{\pi}{2}\right)$

Phasor diagram

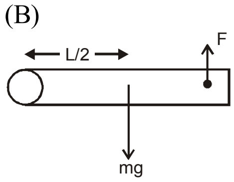



$$\vec{R} = \frac{12\sqrt{2}}{5}\hat{i} + \frac{4\sqrt{2}}{5}\hat{j}$$

$\Rightarrow \tan\theta = \frac{R_y}{R_x} = \frac{1}{3}$ $\qquad \Rightarrow \cos\theta = \frac{3}{\sqrt{10}}$

**20.** (B)

**Sol.**




Taking torque about hinge :

Mg L/2 = FL

$$mg\frac{L}{2} = 2\left(\frac{mV}{10}\right)10L$$

V = 2.5 m/sec

**21.** 3

**Sol.** $\Delta Q = \Delta U + W$

$10 = \Delta U + 5 \Rightarrow \Delta U = 5J$

$\Rightarrow \quad nC_V\Delta T = 5$ ....(1)

Now, $\Delta Q = nC\Delta T$

$10 = C \times \frac{5}{C_V}$ [from equation (1)]

or $\quad C = 2C_V$

**22.** 61.6

**Sol.** Before collision, linear momentum of the system is zero. Therefore, after collision momentum of B will be equal and opposite to momentum of C (since $P_A = 0$)

**23.** 25

**Sol.** We have $K_\alpha = \frac{m_y}{m_y + m_\alpha}.Q \qquad \Rightarrow K_\alpha = \frac{A-4}{A}.Q \Rightarrow 48 = \frac{A-4}{A}.50 \qquad \Rightarrow A = 100$

**24.** 26.4

**Sol.** $T = 2\pi\sqrt{\frac{\ell}{g}}$

when the temperature is raised length changes to $l(1 + \alpha\,\Delta T)$

when the lift acceleration upwards

$g_{eff} = g + a$

new period of pendulum,

$$T' = 2\pi\sqrt{\frac{\ell\ (1+\alpha\ \ \Delta T)}{g+a}}$$

$$T' = T \Rightarrow g\frac{(1+\alpha\ \ \Delta T)}{(g+a)} = 1$$

or $\quad a = g\,\alpha\,\Delta T = 10 \times 20 \times 10^{-4} \times 5.0$

$= 1\ m/sec^2$

$w_{eff} = m\,(g + a)$

$= 60\,(10 + 1) = 660$ N

**25.** 33.6

**Sol.** $W = h\upsilon_{th} = 0.8$ eV.

**26.** 4

**Sol.** % error in x = x error in $(a^3 - b^2) + \frac{1}{2}$ % error in (c + d)

% error in $(a^3 - b^2) = \frac{3a^2\Delta a + 2b\Delta b}{a^3 - b^2} \times 100$

% error in $(c + d) = \frac{\Delta c + \Delta d}{c + d} \times 100$

**27.** 7

**Sol.** $\frac{5}{3}$ sin 30° = n . $\frac{5}{6} \Rightarrow n = 1$

$\sin\theta_c = \frac{3}{5} \Rightarrow \theta_c = 37°$

and so 37 – 30 = 7

**28.** 2

**Sol.** $\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi R^3}$

$= \frac{12 \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi(2.7 \times 10^{-15})^3}$

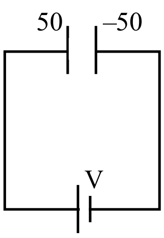
**29.** 2

**Sol.** Shortest wavelength of Brackett corresponds to n = 4 and n = ∞ and shortest wavelength of Balmer series corresponds to n = 2 and n = ∞

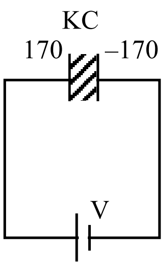
$\therefore (Z^2)\left(\frac{13.6}{16}\right) = \frac{13.6}{4} \Rightarrow Z = 2$

**30.** 2

**Sol.**




$V = \frac{50}{C}$




$V = \frac{120}{KC}$

$\frac{50}{C} = \frac{120}{KC} \Rightarrow K = \frac{120}{50} = 2.4$

**31.** (B)

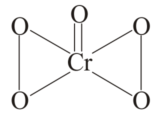
**Sol.** K on account of lower IE1[st] can easily form K+ ion losing one electron.

**32.** (C)

**Sol.** (A) and (B) have negative overlap while (C) has positive overlap. Thus (C) will show effective overlapping.

**33.** (D)

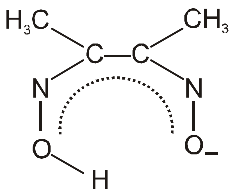
**Sol.**



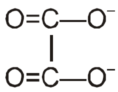

Butterfly structure

**34.** (A)

**Sol.**




(B)




(C) $NH_2CH_2CH_2NH_2$

**35.** (C)

**Sol.** Factual

**36.** (A)

**37.** (C)

**38.** (D)

**Sol.** Cyclohexene do not show geometrical isomerism and the compound given is incapable of showing optical isomerism]

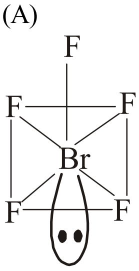
**39.** (A)

**40.** (B)

**Sol.** Diels elder's Reaction

**41.** (A)

**Sol.**




**42.** (D)

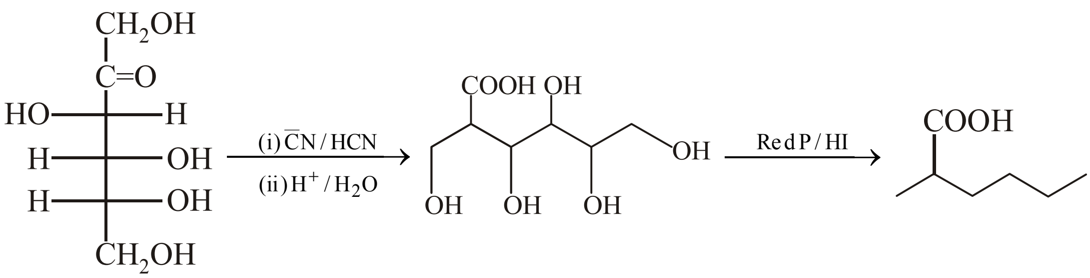
**Sol.** Due to highest oxidation state.]

**43.** (A)

**44.** (B)

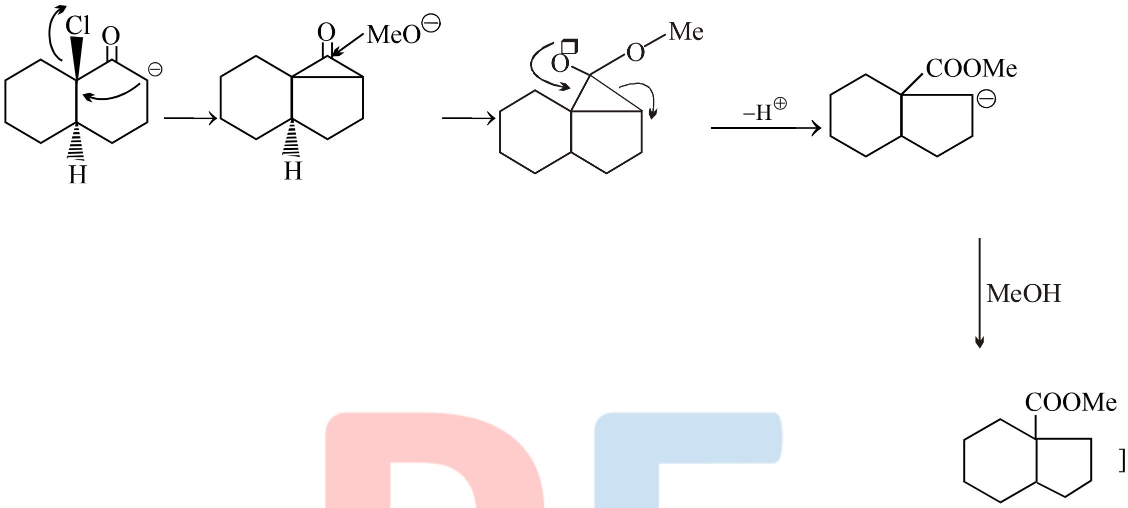
**45.** (B)

**Sol.**



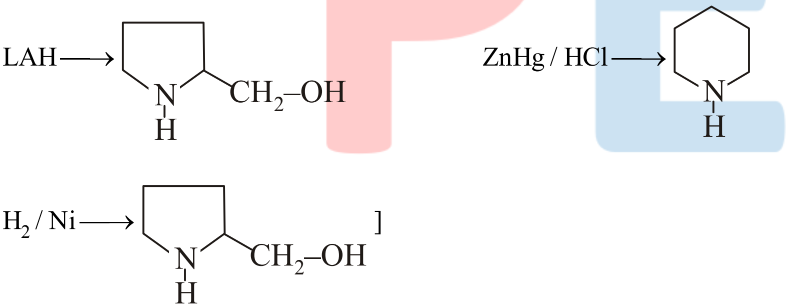

**46.** (B)

**Sol.**



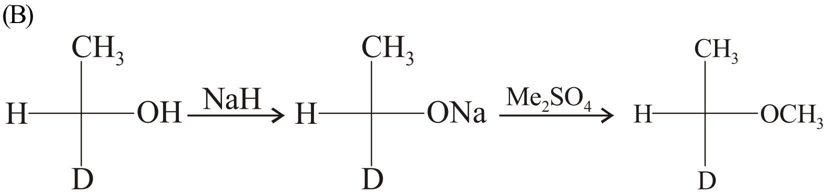

**47.** (C)

**Sol.**




**48.** (B)

**Sol.**

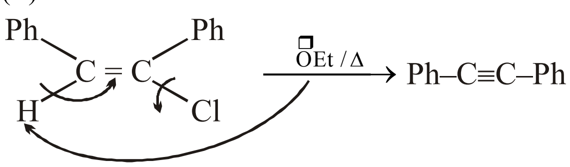



(X)

**49.** (D)

**50.** (A)

**Sol.**




$E_2$ syn. reaction take place. Syn. elimination take place in rigid systems [Vinyl halide, bicyclo etc.] where anti periplaner conformation is not possible.

**51.** 0.70

**Sol.** $2AB(s) \rightleftharpoons A_2(g) + B_2(g)$

$\qquad\qquad\qquad\quad 0.5 + x \quad x$

$K_p = P_{A_2}.P_{B_2} \Rightarrow 0.06 = (0.5 + x)x$

$x^2 + 0.5x - 0.06$

$x = \dfrac{-0.5 \pm \sqrt{(0.5)^2 + 4(0.06)}}{2} \Rightarrow \dfrac{-0.5 + 0.7}{2} \Rightarrow 0.1$

$P_{total} = P_{A_2} + P_{B_2} \Rightarrow 0.6 + 0.1$

$\Rightarrow$ **0.70 atm**

**52.** 2606

**Sol.** $ln\, K_{eq} = \dfrac{-\Delta H°}{RT} + \dfrac{\Delta S°}{R}$

$\therefore$ Slope of $ln\, K_{eq}$ v/s $\dfrac{1}{T} = \dfrac{-\Delta H°}{R} \Rightarrow \Delta H° = -300\, R \Rightarrow -600$ Cal

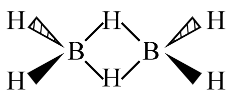
at 300 K $K_{eq} = 10 \quad \therefore\ \Delta G° = -RT\, ln\, 10 \Rightarrow -2.303 \times 300 \times 2$

$\therefore\ \Delta S° = \dfrac{\Delta H° - \Delta G°}{T} = \dfrac{-600 + 600 \times 2.303}{300} \Rightarrow 2.606$ Cal / K

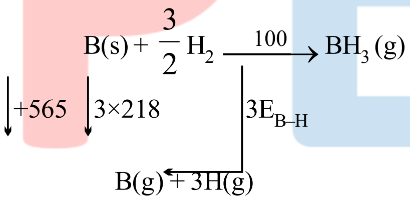
$\therefore\ 1000\, \Delta S° = 2606$ Cal / K

**53.** 455

**Sol.**

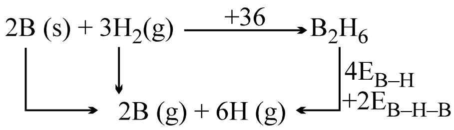



B. E. of B–H:




$3E_{B-H} = 565 + 654 - 100$

$E_{C-H} = 373$




$2E_{B-H-B} = 2 \times 565 + 6 \times 218 - 36 - 4 \times 373$

$E_{B-H-B} = 455$ kJ/mol $= 4.55 \times 10^2$ kJ/mole

**54.** 24

**Sol.** $n = 1 \qquad P_1 = $ atm $\; T = 300$ K

$\qquad P_2 = 2$ atm, monoatomic $C_{v_m} = \dfrac{3}{2}R$

$\Delta U = nC_{v_m}\Delta T$

$900 = 1 \times \dfrac{3}{2} R\,(T_2 - 300)$

$\Rightarrow\ T_2 = 600$ K

$V_2 = \dfrac{nRT_2}{P_2} = \dfrac{1 \times 0.08 \times 600}{2} = 24$ litre

**55.** 25

**Sol.** $^{238}_{92}X \longrightarrow ^{A}_{Z}Y + ^{4}_{2}He + 232.75\,MeV$

$^{234}_{90}Y$

$Z = 90$

$\Delta M = \frac{232.75}{931} \times 100 = 25$ **Ans.**

**56.** 6

**Sol.** In first oxide : $\frac{E_A}{E_O} = \frac{x}{y}$ $\Rightarrow \frac{32/3}{8} = \frac{x}{y}$ i.e. $\frac{x}{y} = \frac{4}{3}$

$\therefore$ In second oxide : $\frac{E_A}{E_O} = \frac{3}{4}$ $\Rightarrow \frac{E_A}{8} = \frac{3}{4}$

$\therefore$ $E_A = 6$ **Ans.**

**57.** 1

**Sol.** $\pi = CRT \times i$ $\alpha = \frac{1}{5} = 0.2$

$Ci = 0.6$
$i = 1.2$
$[H^+] = C\alpha = 0.5 \times 0.2 = 0.1$
$pH = 1$ **Ans.**

**58.** 248
**Sol.** Henry Law

$P_{N_2} = k_{N_2} X_{N_2}$

$P_{O_2} = k_{O_2} X_{O_2}$

$X_{O_2} = \frac{0.2\,bar}{2.5 \times 10^4\,bar} = 0.08 \times 10^{-4} = 8 \times 10^{-6}$

$X_{N_2} = \frac{0.8\,bar}{5 \times 10^4\,bar} = 1.6 \times 10^{-5}$

$X = m\,M'_A$

$\Rightarrow$ $m_{O_2} = \frac{X_{O_2}}{M'_A} = \frac{8 \times 10^{-6}}{18 \times 10^{-3}}$ $\Rightarrow$ $m_{over\ all} = \frac{2.4 \times 10^{-5}}{18 \times 10^{-3}}$

$m_{N_2} = \frac{X_{N_2}}{M'_A} = \frac{1.6 \times 10^{-5}}{18 \times 10^{-3}}$ $\Rightarrow$ $m_L = \frac{4}{3} \times 10^{-3}$

$\Delta T_f = 1.86 \times \frac{4}{3} \times 10^{-3} = 0.62 \times 4 \times 10^{-3} = 2.48 \times 10^{-3}$

**59.** 728

**Sol.** $8e^- + 2\,Ph-NO_2 \longrightarrow Ph-N=N-Ph$

$$OH^- \longrightarrow \frac{1}{4}O_2 + \frac{1}{2}H_2O + e^-$$

$$C_{60} + 60\,O_2 \longrightarrow 60\,CO_2$$

$$\frac{96\times10^{-3}}{60\times12}\times60\times4\times\frac{1}{8}\times182 \Rightarrow 0.728 \text{ gm}$$

**60.** 6

**Sol.** $\lambda_A - \lambda_B = 3\lambda_1$

$n_2 - n_1 = 3$

So maximum number of spectral lines $= \dfrac{(n_2-n_1)(n_2-n_1+1)}{2} = \dfrac{3\times4}{2} = 6$ ]

**61.** (C)

**Sol.** Let $\vec{b} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix} = \hat{j} - \hat{k}$

$\Rightarrow \beta - \gamma = 0, \alpha - \gamma = 1, \alpha - \beta = 1$

$\Rightarrow \beta = \gamma, \alpha = 1 + \gamma, \alpha = 1 + \beta$

$\vec{a} . \vec{b} = 1 \Rightarrow \alpha + \beta + \gamma = 1$

$\Rightarrow \beta + 1 + \beta + \beta = 1 \Rightarrow \beta = 0$

$\therefore \alpha = 1, \gamma = 0$

$\therefore \vec{b} = \hat{i}$.

**62.** (C)

**Sol.** $y = u^m$

$dy/dx = mu^{m-1}\dfrac{du}{dx}$

The given differential equation becomes

$2x^4.u^m.\ mu^{m-1}\dfrac{du}{dx} + u^{4m} = 4x^6$

$\Rightarrow \dfrac{du}{dx} = \dfrac{4x^6 - u^{4m}}{2mx^4x^{2m-1}}$

For homogeneous equation degree should be same in numerator & denominator so,

$6 = 4m = 4 + 2m - 1 \Rightarrow m = 3/2$

**63.** (D)

**Sol.** The L.H.S. of the above inequality is equal to

$a^2\left(\dfrac{\sin 3x}{12} + \dfrac{3}{4}\sin x\right) - a\cos x - 20\ \sin x\Big|_0^{\pi/2}$

$= a^2\left(-\dfrac{1}{12} + \dfrac{3}{4}\right) - a(0 - 1) - 20 = \dfrac{2a^2}{3} + a - 20.$

Thus the given inequality is $(2a^2/3) + a - 20 \le -a^2/3$

i.e. $a^2 + a - 20 \le 0 \Leftrightarrow -5 \le a \le 4$

Since a is *a* positive integer so a = 1, 2, 3, 4.

**64.** (C)

**Sol.** $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1, P \equiv (2\cos\theta, 3\sin\theta)$.

$\therefore 4\cos^2\theta + 9\sin^2\theta = \dfrac{31}{4}$.

$\Rightarrow \cos^2\theta = \dfrac{1}{4} \quad \Rightarrow \cos\theta = \pm\dfrac{1}{2}$.

$\therefore |12\cos\theta| = 6$.

**65.** (D)

**Sol.** Given,y cosα – x sinα = p

and ysin(30º – α) – x cos(30º – α) = p

are inclined at 60º so line ax + by = 1 can be acute angle bisector ....(i)

i.e., y cosα – x sinα – p

= –(ysin(30º – α) – xcos(30º – α) – p)

⇒ y[cosα+sin(30º–α)]

–x[sinα + cos(30º–α)] = 2p ....(ii)

From Eqs.(i) and (ii), we ge

$$\frac{b}{\cos\alpha+\sin(30^\circ-\alpha)}=\frac{a}{(\sin\alpha+\cos(30^\circ-\alpha))}=\frac{1}{2p}$$

$$\Rightarrow \frac{\sqrt{a^2+b^2}}{\sqrt{2+1}}=\frac{1}{2p}$$

$$\Rightarrow a^2+b^2=\frac{3}{4p^2}$$

**66.** (D)

**Sol.** (a cos α, b sin α), (a cos β, b sin β), (ae, 0) are collinear.

$$\frac{b(\sin\beta-\sin\alpha)}{a(\cos\beta-\cos\alpha)}=\frac{b\sin\alpha-0}{a\cos\alpha-ae}$$

∴ (cosα – e) (sinβ – sinα) = sin α(cosβ – cosα)

$$\therefore\ e=\frac{\cos\alpha(\sin\beta-\sin\alpha)-\sin\alpha(\cos\beta-\cos\alpha)}{\sin\beta-\sin\alpha}=\frac{\sin(\alpha-\beta)}{\sin\alpha-\sin\beta}=\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$$

**67.** (A)

**Sol.** $$I=\int\frac{\left(1-\frac{\sin^2x}{\cos^2x}\right)}{\frac{1}{\cos^2x}}\frac{dx}{\sin 2x}$$

$$=\int\cot 2x\,dx=\frac{\ell n|\sin 2x|}{2}+C$$

**68.** (D)

**Sol.** $$f(x)=\lim_{t\to0}\tan^{-1}\left(\frac{e^{xt}-1}{tx}\times\tan^{-1}x\right)$$

$$\frac{\Delta}{3}=\frac{1}{2}(AB)h$$

$$\frac{\Delta}{3}=h$$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

$$=\sqrt{4(4-3)(4-3)(4-2)}=\sqrt{8}$$

$$h=\frac{\sqrt{8}}{3}=\frac{2\sqrt{2}}{3}$$

**69.** (D)

**Sol.** $n(A) = 2, n(B) = 4 \Rightarrow n(B \times A) = 4 \times 2 = 8$

$\because$ Relation $(R) \subset B \times A$

$\Rightarrow$ Total number of subsets of $B \times A = 2^8 = 2^{2^3}$

**70.** (A)

**Sol.** We have,

$$\bar{X} = \frac{8+12+13+15+22}{5} = 14$$

Calculation of Variance

| $x_i$ | $x_i - \bar{X}$ | $(x_i - \bar{X})^2$ |
|---|---|---|
| 8 | –6 | 36 |
| 12 | –2 | 4 |
| 13 | –1 | 1 |
| 15 | 1 | 1 |
| 22 | 8 | 64 |
| | | $\sum(x_i - \bar{X})^2 = 106$ |

Here n = 5 and $\Sigma\left(x_1 - \bar{X}\right)^2 = 106$

$$\therefore \text{Var}(X) = \frac{1}{n}\Sigma\left(x_1 - \bar{X}\right)^2 = \frac{106}{5} = 21.2$$

**71.** (D)

**Sol.** Inequality $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$ ...(1)

L.H.S. is valid if :

$x^2 - 10x + 22 > 0 \quad \frac{x}{2} > 0$

$x < 5 - \sqrt{3}$ or $x > 5 + \sqrt{3} \quad x > 0$

eq$^n$ (1) will be solved for two cases

(1) $0 < \log_2\left(\frac{x}{2}\right) < 1$

$\Rightarrow 1 < \frac{x}{2} < 2 = \Rightarrow 2 < x < 4$

$\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$

$x^2 - 10x + 22 < 1$

$x^2 - 10x + 21 < 0 \Rightarrow 3 < x < 7$

The common solution $3 < x < 4$

(2) $\log_2\left(\frac{x}{2}\right) > 1 \Rightarrow \frac{x}{2} > 2$

$x > 4$

$\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$

$x^2 - 10x + 22 > 1 \Rightarrow x^2 - 10x + 21 > 0$

$x < 3$ or $x > 7$ common sol$^n$ $x > 7$

two cases $x \in (3, 4) \cup (7, \infty)$

Now common solution with initial values

$x \in (7, \infty)$

**72.** (D)

**Sol.** $I = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$= \frac{1}{4} \int (1+\cos^2 2x + 2\cos 2x)\ dx$

$= \frac{1}{4} \int \left(1 + \frac{1+\cos 4x}{2} + 2\cos 2x\right) dx$

$= \frac{1}{8} \int (3 + \cos 4x + 4\cos 2x)\, dx$

$= \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{4\sin 2x}{8\times 2}$

**73.** (C)

**Sol.** $\alpha = \lambda(1-\beta) = \lambda - \lambda(\lambda - \lambda c)$

Also, $(1+\lambda^3)\beta = \lambda - \lambda^2 + \lambda^3$, $(1+\lambda^3)\,\gamma = \lambda - \lambda^2 + \lambda^3$

$\therefore\ (1+\lambda^3)\,\alpha = (1+\lambda^3)\beta = (1+\lambda^3)\gamma,\ \therefore \lambda^3 = -1,\ \therefore\ \lambda = -1, -\omega, -\omega^2$

**74.** (D)

**Sol.** $p = \frac{3}{4}, q = \frac{1}{4}, n = 5$

$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5 = \frac{459}{512}$

**75.** (D)

**Sol.** $[y + [y]] = 2\cos x \Rightarrow [y] = \cos x$

$y = \frac{1}{3}[\sin x + [\sin x]]] = [\sin x] \Rightarrow [\sin x] = \cos x$

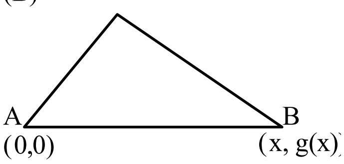
Number of solution in $[0, 2\pi]$ is 0

Hence total solution is 0.

$\therefore$ both are periodic with period $2\pi$.

**76.** (B)

**Sol.**




area $= \frac{\sqrt{3}}{4}(AB)^2$

$\Rightarrow \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}[x^2 + g(x)^2]$

$\Rightarrow g(x) = \pm\sqrt{1-x^2}$

$g(x) = \sqrt{1-x^2} \rightarrow g(x)$ is a $f^n$

$g(x) = -\sqrt{1-x^2}$ $g(x)$ is a $f^n$

$g(x) = \pm\sqrt{1-x^2}$ not a $f^n$

**77.** (D)

**Sol.** Let term be a – 2d, a – d, a, a + d, a + 2d

$\therefore\ 3a = -12 \Rightarrow a = -4,\ (a-2a)(a-d)(a) = 8$

$\therefore\ d = -3$

$\therefore\ a_2 + a_4 + a_6 = -21$

**78.** (C)

**Sol.** $\dfrac{4}{\tan 2\theta} = \dfrac{1}{\tan^2\theta} - \tan^2\theta$

$\Rightarrow \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} \qquad \Rightarrow \dfrac{4(1-\tan^2\theta)}{2\tan\theta} = \dfrac{1-\tan^4\theta}{\tan^2\theta}$

$\Rightarrow (1-\tan^2\theta)(\tan^2\theta - 2\tan\theta + 1) = 0 \qquad \Rightarrow \tan\theta = \pm 1 \qquad \Rightarrow \theta = n\pi \pm \dfrac{\pi}{4}.$

**79.** (D)

**Sol.** For $x \to 0$

$2x^2 + \dfrac{2}{x^2} \to \infty$ also $2\left(x^2 + \dfrac{1}{x^2}\right) \geq 4$

**80.** (C)

**Sol.** ${}^nC_r : C_{r+1} : {}^nC_{r+2} \equiv 1 : 7 : 35$

$\Rightarrow \dfrac{r+1}{n-r} = \dfrac{1}{7} \Rightarrow 8r = n-7$ and $\dfrac{r+2}{n-(1+r)} = \dfrac{1}{5} \qquad \therefore\ n = 23$

**81.** 2

**Sol.** $\sin 2x \cos y = (a^2-1)^2 + 1$

$\Rightarrow a^2 - 1 = 0 \Rightarrow a^2 - 1 = 0$

$\Rightarrow a^2 = 1$ ...(i)

$\cos 2x \sin y = a + 1 \Rightarrow a < 0$ ...(ii)

from (i) and (ii) $a = -1$

$\therefore\ \sin 2x \cos y = 1$ and $\cos 2x \sin y = 0$

on solving we get 2 ordered pair.

$\left(\dfrac{\pi}{4}, 0\right) \& \left(\dfrac{3\pi}{4}, \pi\right)$

**82.** 500

**Sol.** $y = \operatorname{cosec}x - \cot x$

$\dfrac{dy}{dx} = -\operatorname{cosec}x\cot x + \operatorname{cosec}^2 x$

$\dfrac{dy}{dx} = \operatorname{cosec}x(\operatorname{cosec}x - \cot x)$

$y_1 = y\operatorname{cosec}x \Rightarrow y_1 \sin x = y$

$y_2 \sin x + y_1 \cos x = y_1 \Rightarrow y_2 \sin x = y_1(1-\cos x)$

$y_2 \sin x = \dfrac{y}{\sin x}(1-\cos x)$

$y_2 = \dfrac{y\left(2\sin^2\dfrac{x}{2}\right)}{4\sin^2\dfrac{x}{2}\cos^2\dfrac{x}{2}} \Rightarrow y_2 = \dfrac{1}{2}y\sec^2\dfrac{x}{2}$

$\dfrac{y_2}{y} = \dfrac{1}{2}\sec^2\dfrac{x}{2} \Rightarrow \lambda = \dfrac{1}{2} \Rightarrow 1000\lambda = 500$

**83.** 2

**Sol.** $2(g_1g_2 + f_1f_2) = c_1 + c_2$

$$\Rightarrow 2\left(n_1\left(\frac{n_2}{2}\right)+(1)\left(\frac{n_2}{2}\right)\right)=n_1$$

$\Rightarrow n_1n_2 + n_2 = n_1$

$$\Rightarrow n_2=\frac{n_1}{(1+n_1)} \Rightarrow n_2 = 1-\frac{1}{(1+n_1)}$$

$1 + n_1 = 1,\ 1 + n_1 = -1$

$n_1 = 0$ & $n_1 = -2$

$\Rightarrow n_2 = 0$ & $n_2 = 2$

**84.** 3

**Sol.** $T_{r+1} = {}^{40}C_r\, x^{\frac{40-r}{3}} \cdot x^{\frac{2r}{5}}$

$= {}^{40}C_r\, x^{\frac{200+r}{15}}$

$r = 10, 25, 40$

$\Rightarrow$ Number of such terms = 3

**85.** 60

**Sol.** From skew symmetric matrix

$a\alpha^2 + b\alpha + c = 0,\ a\beta^2 + b\beta + c = 0$

$a\gamma^2 + b\gamma + c = 0$

$\Rightarrow ax^2 + bx + c = 0$ has three roots

$a = b = c = 0$

$$\lambda = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 4 & 1 \\ 9 & 0 & 3 \end{vmatrix} = 12 + 2(-36) = 60$$

**86.** 192

**Sol.** Required value is $2\times \lfloor 4 \times \frac{\lfloor 4}{\lfloor 3} = 192$

**87.** 3

**Sol.** $\cos(2\sin^{-1}(\cot(\tan^{-1}(\sec(6\operatorname{cosec}^{-1}x))))) = -1$

$\sin^{-1}(\cot(\tan^{-1}(\sec(6\operatorname{cosec}^{-1}x)))) = \pm\frac{\pi}{2}$

$\cot(\tan^{-1}(\sec(6\operatorname{cosec}^{-1}x))) = \pm 1$

$\tan^{-1}(\sec(6\operatorname{cosec}^{-1}x)) = \pm\frac{\pi}{4}$

$\sec(6\operatorname{cosec}^{-1}x) = \pm 1$

$6\operatorname{cosec}^{-1}x = \pm 3\pi, \pm 2\pi, \pm \pi$

$\operatorname{cosec}^{-1}x = \pm\frac{\pi}{2}, \pm\frac{\pi}{3}, \pm\frac{\pi}{6}$

$\Rightarrow x = 1, \frac{2}{\sqrt{3}}, 2 \qquad (\because x > 0)$

**88.** 3

**Sol.** Let $X = x^{\frac{1}{\ell ny}+\frac{1}{\ell nz}} \cdot y^{\frac{1}{\ell nz}+\frac{1}{\ell nx}} \cdot z^{\frac{1}{\ell nx}+\frac{1}{\ell ny}}$

$$\Rightarrow \ell n\, x = \ell n\, x\left(\frac{1}{\ell n\, y}+\frac{1}{\ell n\, z}\right) + (\ell n\, y)\left(\frac{1}{\ell n\, z}+\frac{1}{\ell n\, x}\right) + \left(\frac{1}{\ell n\, x}+\frac{1}{\ell n\, y}\right)(\ell n\, z)$$

Now given $\ell n\, x + \ell n\, y + \ell n\, z = 0$

$$\therefore \frac{\ell n\, x}{\ell n\, y}+\frac{\ell n\, z}{\ell n\, y} = -1$$

similarly $\frac{\ell n\, y}{\ell n\, x}+\frac{\ell n\, z}{\ell n\, x} = -1$ &

$$\frac{\ell n\, x}{\ell n\, z}+\frac{\ell n\, y}{\ell n\, z} = -1$$

$\therefore$ R.H.S. $= -3$

$\therefore \ell n\, X = -3$

$X = e^{-3}$

**89.** 2401

**Sol.** $2x^2 + 6xy + 5y^2 + 8x + 12y + 1 = t$

$\Rightarrow$ $x(2x + 3y + 4)+y(3x + 5y + 6)+4x + 6y+1-t = 0$

So we have equation as

$2x + 3y + 4 = 0$ … (1)

$3x + 5y + 6 = 0$ … (2)

and $4x + 6y + 1 - t = 0$ …(3)

for existance of solution $\begin{vmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 1-t \end{vmatrix} = 0$

$\Rightarrow t = -7$

$t^4 = 2401$

**90.** 50

**Sol.** $p = \frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{8} = \frac{37}{56}$