JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- FULL TEST
FULL SYLLBUS TEST

ANSWER KEY

PHYSICS

1.	(C)	**2.**	(A)	**3.**	(D)	**4.**	(A)	**5.**	(A)	**6.**	(B)	**7.**	(C)
8.	(B)	**9.**	(D)	**10.**	(A)	**11.**	(D)	**12.**	(B)	**13.**	(A)	**14.**	(A)
15.	(D)	**16.**	(C)	**17.**	(B)	**18.**	(C)	**19.**	(B)	**20.**	(C)	**21.**	(32.5)
22.	22.5	**23.**	16	**24.**	10	**25.**	31.5	**26.**	2	**27.**	125	**28.**	8
29.	2	**30.**	1										

CHEMISTRY

31.	(B)	**32.**	(C)	**33.**	(B)	**34.**	(B)	**35.**	(D)	**36.**	(B)	**37.**	(C)
38.	(B)	**39.**	(C)	**40.**	(B)	**41.**	(B)	**42.**	(D)	**43.**	(C)	**44.**	(B)
45.	(C)	**46.**	(A)	**47.**	(C)	**48.**	(A)	**49.**	(C)	**50.**	(D)	**51.**	32
52.	40	**53.**	46	**54.**	240.40	**55.**	3	**56.**	1	**57.**	2	**58.**	5
59.	110	**60.**	100										

MATHEMATICS

61.	(A)	**62.**	(B)	**63.**	(D)	**64.**	(C)	**65.**	(B)	**66.**	(B)	**67.**	(A)
68.	(C)	**69.**	(B)	**70.**	(B)	**71.**	(B)	**72.**	(C)	**73.**	(D)	**74.**	(A)
75.	(D)	**76.**	(B)	**77.**	(A)	**78.**	(B)	**79.**	(C)	**80.**	(B)	**81.**	5
82.	82	**83.**	2	**84.**	0	**85.**	7	**86.**	2	**87.**	3	**88.**	4
89.	42	**90.**	178										

SOLUTIONS

PHYSICS

1. (C)

Sol. COM of triangular portion is at $\frac{h}{3}$ from base

COM of square portion is at $\frac{\ell}{2}$ from base

$$y_{cm} = \frac{M\left(\frac{\ell}{2}\right) - m\left(\frac{h}{3}\right)}{M - m} = h$$

$$\Rightarrow h = \left(\frac{3-\sqrt{3}}{2}\right)\ell$$

2. (A)

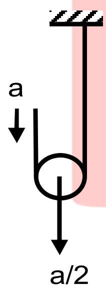
Sol. $L = I\omega$

$$\therefore \frac{L_2}{L_1} = \frac{I_2}{I_1} \quad [\because \omega = \text{const.}]$$

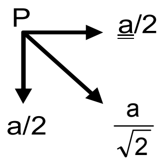
$$\text{or } \frac{L_2}{L_1} = \frac{m\left(\frac{r}{2}\right)^2}{mr^2} = \frac{1}{4}$$

$$\Rightarrow L_2 = \frac{L_1}{4}$$

3. (D)



Sol.



$$\alpha = \frac{a}{2R}$$

4. (A)

Sol. $T.2\pi r \cos\theta = mg$

5. (A)

Sol. For second lens

$u = F - d$

$f = F$

$$v = \frac{F(F-d)}{2F-d}$$

6. (B)

Sol. $\Delta V = \Delta A \times \ell$

$A\ell\gamma\Delta T = A\beta\Delta T\ell$

$\gamma = \beta = 2\alpha$

7. (C)

8. (B)

Sol. $v = \omega\sqrt{(A^2 - x^2)}$

$\Rightarrow$ Amplitude $A = 2x_0$

9. (D)

Sol. $f \propto \sqrt{T}$

$\rho_0 = \frac{4}{3}\rho_w$

$\rho_L = \frac{32}{27}\rho_w$

10. (A)

Sol. $e = \int_a^b \frac{F}{\pi KY}\frac{dr}{r^2} = \frac{F\ell}{\pi Yab}$

$K = \frac{b-a}{\ell}$

11. (D)

Sol. $P = \int (3R\cos\theta d\theta)(2R\cos\theta)$

$= 48\pi \times 10^{-4}$ cm

12. (B)

Sol. $\Delta x = 2\sqrt{D^2 + d^2} - 2D = \frac{d^2}{D}$

$\frac{d^2}{D} = n\lambda$

$d = \sqrt{D\lambda}$, when $n = 1$

13. (A)

Sol. $\rho = \frac{\pi D^2 V}{4Li} = \frac{(3.14)\,(2.00 \times 10^{-3})^2}{4 \times (0.314)} \times \frac{100.0}{10.0}$

$= 1.00 \times 10^{-4}$

14. (A)

Sol. $E = 3.1$ eV

$K.E._{max} = 0.8$ eV

$E_{He^+} = -3.4$ eV

$\Delta E = K.E._{max} - E_4$

$= 4.2$ eV

15. (D)

Sol. $Y = \overline{AB}$ (NAND gate)

16. (C)

17. (B)

Sol. $d\sin\theta = \lambda$

18. (C)

Sol. Both the energy densities are equal.

19. (B)

Sol. $a = v\dfrac{dv}{dx} = cx + d$ Let at $x = 0$ $v = u$

$\therefore \int_u^v v\,dv = \int_0^x (cx + d)dx$

or $v^2 = cx^2 + 2dx + u^2$

v shall be linear function of x if $cx^2 + 2dx + u^2$ is perfect square

20. (C)

Sol. Total power (P) = $(15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000) = 2500W$

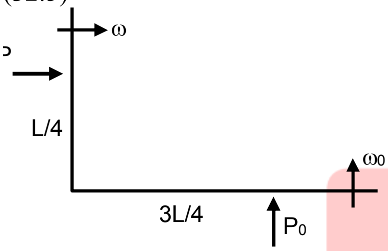
$P = VI$

$\Rightarrow I = \dfrac{2500}{220} A = \dfrac{125}{11} = 11.3\ A$

Minimum capacity should be 12 A

21. (32.5)

Sol.



$P_0 \dfrac{3L}{4} = \dfrac{M\ell^2}{3}\omega_0$

$t_1 = \dfrac{\pi}{2\omega_0}$

$t_2 = \dfrac{8\pi ML}{27P_0} - t_1$

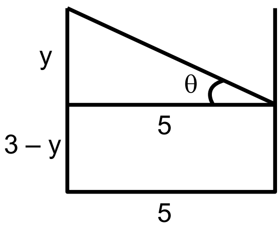
$\omega = \dfrac{\pi}{2t_2}$

$P\dfrac{L}{4} = \dfrac{M\ell^2}{3}(\omega_0 + \omega)$

$P = 12P_0$

22. 22.5

Sol.



$\dfrac{1}{2} \times 5 \times y \times 3 + (3 - y) \times 5 \times 3 = 5 \times 2 \times 3 \times \dfrac{90}{100}$

$\Rightarrow y = \dfrac{12}{5}$

$\tan\theta = \dfrac{y}{5} = \dfrac{a}{g}$

$\Rightarrow a = \dfrac{24}{5}$

23. 16

Sol. For sonometer wire

$$\nu_n = \frac{n\sqrt{\frac{F}{\mu}}}{2\ell}$$

$\Rightarrow n\sqrt{F} = \text{constant}$

[$\therefore \nu, \mu, \ell$ are constant for two cases of comparison]

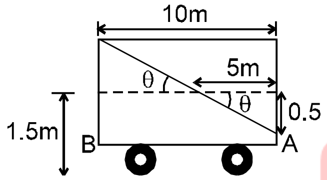
$$\Rightarrow F_2 = \frac{n_1^2}{n_2^2} . F_1$$

$\Rightarrow m_2 = 25$ kg

$\therefore$ Additional mass = 16 kg

24. 10

Sol. $\tan\theta = \frac{0.5}{5} = \frac{1}{10}$



$$\frac{a}{g} = \frac{1}{10}$$

$a = 1$ m/s^2

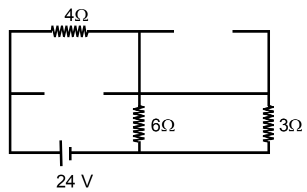
$v = u + at$

$10 = 0 + 1 \times t$

$t = 10$ second.

25. 31.5

Sol. In steady state

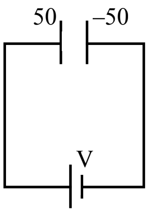


$$R_{eq} = 4 + \frac{6\times3}{6+3} = 6$$

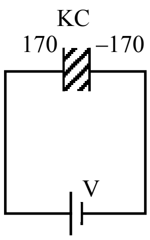
$$I = \frac{\varepsilon}{R_{eq}} = 4A$$

26. 2

Sol.



$V = \frac{50}{C} \Rightarrow$



$V = \frac{120}{KC}$

$$\frac{50}{C} = \frac{120}{KC} \Rightarrow K = \frac{120}{50} = 2.4$$

27. 125

Sol. $qvB \sin\theta = F$

$1.6 \times 10^{-19} \times 4 \times 10^{6} \times 0.2 \sin\theta = 8 \times 10^{-13}$

$$R = \frac{mv\sin\theta}{qB} = \frac{1.6\times10^{-27}}{1.6\times10^{-19}\times0.2} \times \left(\frac{8\times10^{-13}}{0.2\times1.6\times10^{-19}}\right)$$

$= 1.25$ m $= 125$ cm

28. 8

Sol. $Q = CV = 10^{-5} \times 1 = 10^{-5}$

$$\frac{1}{2}Li^2 + \frac{1}{2}CV^2 = \frac{1}{2}Li_{max}^2$$

$$i_{max} = \sqrt{i^2 + \frac{CV^2}{L}} = \sqrt{6.4^2 + \frac{300\times16\times10^{-3}}{10^{-3}}} = \sqrt{6.4^2 + 4.8^2} = 1.6\sqrt{4^2+3^2} = 8 \text{ A}$$

29. 2

Sol. Given $\frac{dp}{dt} = cv^n$

$\therefore \quad \frac{mv^2}{r} = cv^n$

On comparing $n = 2$ Ans.

30. 1

Sol. $K_{max} = \frac{hc}{\lambda} - \phi = \frac{1240}{500} = 1.48$

$\frac{p^2}{2m} = 1$ eV

$\Rightarrow p_{max} = \sqrt{2\times9\times10^{-31}\times1.6\times10^{-19}}$

$$\lambda_{min} = \frac{h}{p_{max}} = \frac{6.63\times10^{-34}}{\sqrt{3.2\times9\times10^{-25}}} = 1 \text{ nm}$$

31. (B)

Sol. Li_2 $\sigma 1s^2$ $\sigma^* 1s^2$ $\sigma 2s^2$ Bond order = 1

Li_2^+ $\sigma 1s^2$ $\sigma^* 1s^2$ $\sigma 2s^1$ Bond order = 0.5

Li_2^- $\sigma 1s^2$ $\sigma^* 1s^2$ $\sigma 2s^2$ $\sigma^* 2s^1$ Bond order = 0.5

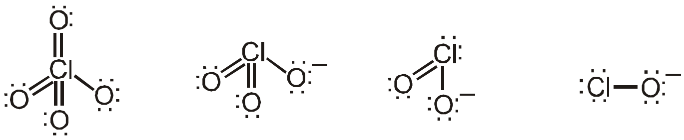
Stability order $Li_2 > Li_2^+ > Li_2^-$

32. (C)

Sol. Decreasing order of strength of oxoacids :

$HClO_4 > HClO_3 > HClO_2 > HOCl$

Reason : Consider the structures of conjugate bases



Negative charge is more delocalized on ClO_4^- due to resonance,

hence ClO_4^- is more stable (& less basic).

33. (B)

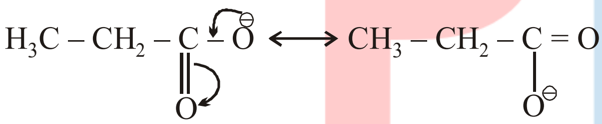
34. (B)

35. (D)

Sol. Same stereoisomers have same configuration [R or S]. All have 'S' configuration.

36. (B)

Sol.

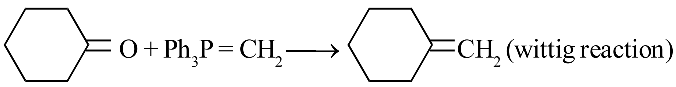


$$\text{Bond order} = \frac{\text{Number of bonds [between atoms } (-O)]}{\text{Resonating structure}} = \frac{3}{2} = 1.5$$

37. (C)

38. (B)

Sol.



39. (C)

Sol. $I_2 + 10HNO_3 \longrightarrow 2HIO_3 + 10NO_2 + 4H_2O$

40. (B)

41. (B)

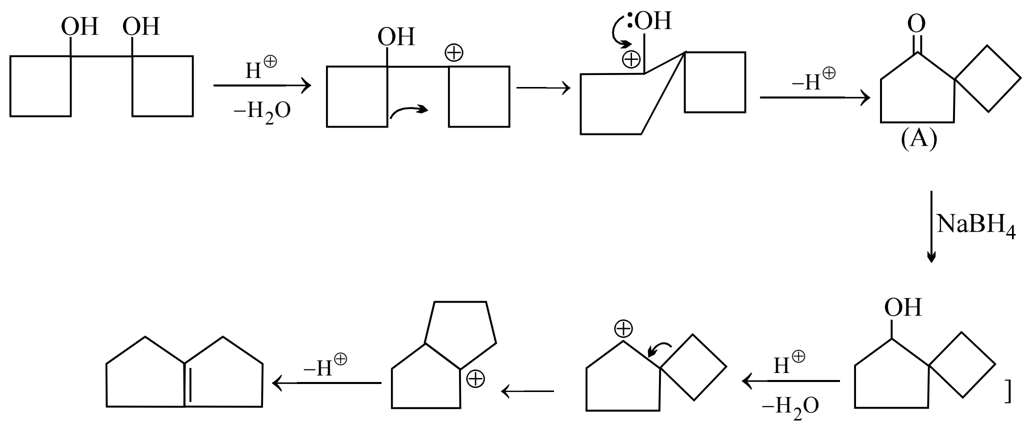
42. (D)

43. (C)

44. (B)

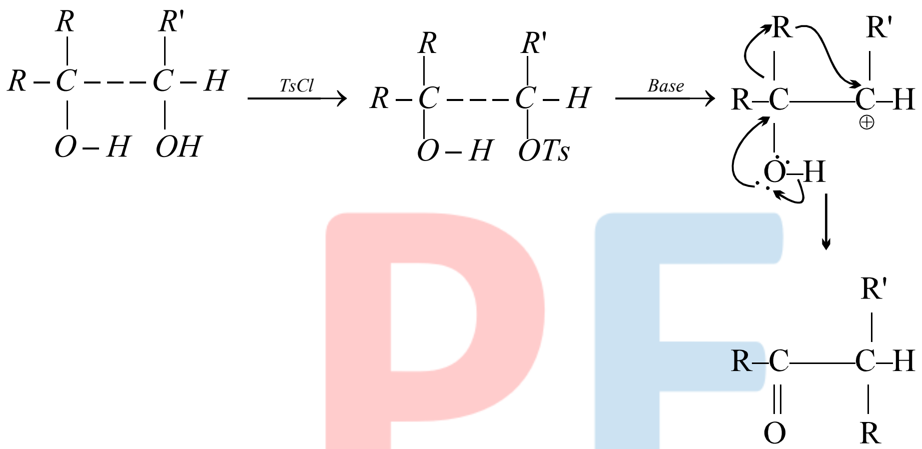
45. (C)

Sol.



46. (A)

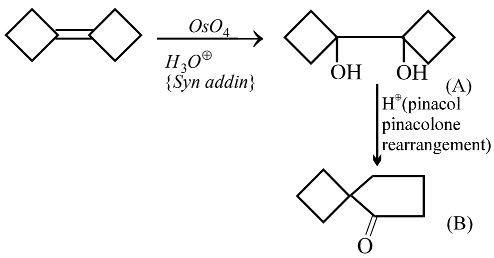
Sol.



Ans is A]

47. (C)

Sol.



Ans: is (C)]

48. (A)

Sol. Correct option (A) as all other Nitrogen lonepairs in the given structure are participating in resonance so their Nucleophilic power is decreased.

49. (C)

Sol. $Cu + 4HNO_3\,(conc.) \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$

50. (D)

Sol. (A) $NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$ (B) $(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

(C) $2Zn(NO_3)_2 \xrightarrow{\Delta} 2ZnO + 4NO_2 + O_2$ (D) $Ba(N_3)_2 \xrightarrow{\Delta} Ba + N_2$

51. 32

Sol. $P_{total} = P_{HNO_3} + P_{NO_2} + P_{H_2O} + P_{O_2}$

$\because \quad P_{NO_2} \Rightarrow 4P_{O_2}$ & $P_{H_2O} = 2P_{O_2}$

$\therefore P_{total} = P_{HNO_3} + 7P_{O_2} \Rightarrow 30 - 2 = P_{O_2} \times 7$

$$P_{O_2} \Rightarrow \frac{28}{7} \Rightarrow 4$$

$$K_P \Rightarrow \frac{P_{NO_2}^{4} . P_{H_2O}\ P_{O_2}}{P_{HNO_3}^{4}} \Rightarrow \frac{(4\times4)^4 \times (2\times4)^2 \times 4}{2^4} = 2^{20}$$

$$K_P = K_C\ (RT)^{\Delta n_g} \Rightarrow K_C (0.08 \times 400)^3$$

$$K_C = \frac{2^{20}}{(32)^3} \Rightarrow \frac{2^{20}}{2^{15}} \Rightarrow 32 \textbf{ Ans.}$$

52. 40

Sol. $\Delta T_b = i K_b m$

$0.0015 = 3 \times 0.5 \times m$

$m = 5 = 10^{-3}$ mol / L

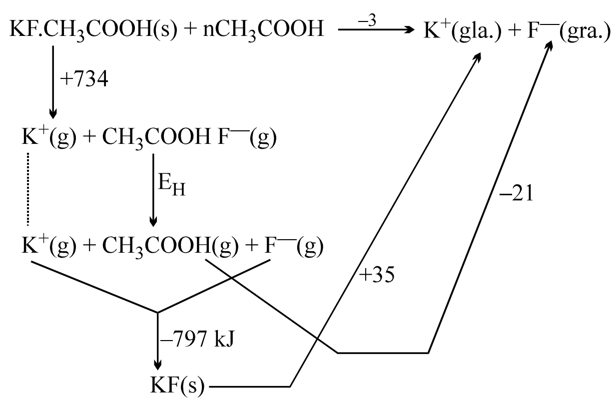
$K_{sp} = 45^3 = 4 \times (10^{-3})^3$

$= 4 \times 10^{-9}$

Answer $4 \times 10^{-9} \times 10^{10} = 40$]

53. 46

Sol.



$-3 = +734 + E_H - 797 + 35 - 21$

$E_H = -3 - 734 - 35 + 797 + 21 = +46$ kJ/mole

54. 240.40

Sol. moles of $CH_3COONa \cdot 3H_2O = \dfrac{1.36}{1.36} = 0.01$ mole

$q_P = 0.4\,(90 + 400)$

$\Delta_r H = \dfrac{q_p}{0.01} = 19.60$ kJ/mole

$\Delta_r H = \Delta H_f^o$ (Products) – ΔH_f^o (Reactants)

$+19.60 = \Delta H_f^o [Na^+(aq)] + \Delta H_f^o [CH_3COOH] + 3\,\Delta H_f^o [H_2O\,(l)] - \Delta H_f^o [H^+(aq)] -$

$\Delta H_f^o [CH_3COONa \cdot 3H_2O\,(s)]$

$+19.60 = \Delta H_f^o [Na^+(aq)] - 485 + 3 \times (-285) - 0 + 1600$

$\Delta H_f^o [Na^+(aq)] = +19.60 + 485 + 3(285) - 1600 = -240.40$ kJ/mole

55. 3

Sol.

	$3A\,(g)$	$\rightarrow$	$2B(g) +$	$2C(g)$	
$t = 0$	P				
$t = 9$	$P - 3x$		$2x$	$2x$	$P_{Total} = P + x$
$t = \alpha$	0		2/3P	2/3P	$P_{Total} = 4P/3$

$$\frac{4P}{3} = 400$$

$P = 300$ mm of Hg

Also $t = 9$ min

$P + x = 387.5$

$\Rightarrow \quad x = 87.5$

at $t = 0$, $P_A = 300$, and at $t = 9$ $\quad P_A = 300 - 3 \times 87.5 = 37.5$

As after nth half life

$$[A] = \frac{[A]_0}{2^n}$$

$\Rightarrow 37.5 = \dfrac{300}{2^n} \Rightarrow 2^n = 8$

$n = 3$

$\therefore$ 9 min = 3 half life

$\therefore t_{1/2} = 3$ min

56. 1

Sol. $2MnO_4^- + H_2O + I^- \rightarrow 2MnO_2 + 2OH^- + IO_3^-$

Ans. 1

57. 2

Sol. $t_{x\%} \propto (a)^{1-n}$

$\dfrac{160}{40} = \left(\dfrac{0.1}{0.4}\right)^{1-n} \Rightarrow 2^2 = 2^{2(n-1)}$

$\Rightarrow 2 = 2\,(n + 1)$

n = 2 **Ans.**

58. 5

Sol. (a) Ethylalcohol + Methyl alcohol $\longrightarrow$ Ideal

(b) Acetone + Ethanol $\longrightarrow$ (+ve)

(c) Acetone + Aniline $\longrightarrow$ (–ve)

(d) $CCl_4 + CHCl_3 \longrightarrow$ (+ve)

(e) Acetone + $CS_2 \longrightarrow$ (+ve)

(f) $CH_3OH + CH_3COOH \longrightarrow$ (–ve)

(g) $H_2O + HNO_3 \longrightarrow$ (–ve)

(h) Water + Ethanol $\longrightarrow$ (+ve)

(i) Chloroform + Benzene $\longrightarrow$ (–ve)

(j) Water + HCl $\longrightarrow$ (–ve)

59. 110

Sol. $0.8 = 1.1 - \frac{0.06}{2} \log \frac{[Zn^{+2}]}{Cu^{+2}}$

$$\frac{[Zn^{+2}]}{[Cu^{+2}]} = 10^{10}$$

To get above ratio almost all the Cu^{+2} will have to be consumed

$[Zn^{+2}] = [Zn^{+2}]_{original} + [Zn^{+2}]_{formed}$

$= 0.1 + 1 = 1.1$

$Cu^{+2} = 1.1 \times 10^{-10}$

60. 100

Sol. Electron transit from higher level n to lower level 2.

$$6 = \frac{(n-2)(n-2+1)}{2}$$

$n = 5$

$$\frac{1}{\lambda} = R_H . z^2 \left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right)$$

$n_1 = 1$ and $n_2 = 5$

$$\therefore \frac{1}{\lambda} = \left(\frac{1}{96} \times 10^9\right) \times 1^2 x \left(\frac{1}{1^2} - \frac{1}{5^2}\right)$$

$\therefore \lambda = 10^{-7}\, m = 100\, nm$

61. (A)

Sol. m and n are the roots of $a(\ell+x)^2+2b\ell x+c=0$

$\Rightarrow \quad ax^2+(2a+2b)\ell x+\left(a\ell^2+c\right)=0$

$\Rightarrow \quad m\cdot n=\frac{a\ell^2+c}{a}=\ell^2+\frac{c}{a}$

62. (B)

Sol. Total 5 digits no. = x x x x x

↓ ↓ ↓ ↓ ↓

4 × 4 × 3 × 2 × 1 = 96

% by 4 = – – – $\underline{04}$ = 6

– – – $\underline{40}$ = 6

– – – $\underline{20}$ = 6 $\therefore$ P = 30 $\frac{30}{96}=\frac{5}{16}$

– – – $\underline{12}$ = 4

– – –.$\underline{32}$ = 4

– – –.$\underline{24}$ = 4

63. (D)

Sol. $f(x)+f\left(\frac{1}{x}\right)=1$

$\Rightarrow \quad x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$\Rightarrow\left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$

$x+\frac{1}{x}=[x]+\left[\frac{1}{x}\right]+1=$ integer ....(1)

$\Rightarrow \quad x+\frac{1}{x}=n$

$x^2-nx+1=0\Rightarrow x=\frac{n\pm\sqrt{n^2-4}}{2}$

But $n\neq 2,-2$ as it does not satisfy (i)

$\Rightarrow$ n can be any integer in $(-\infty,-2)\cup(2,\infty)$

So infinite solutions.

64. (C)

Sol. $D_1=b^2-4ac<0$, $D_2=b^2-4ac<0$ as the root is non- real $\Rightarrow$ both roots will be common

$\Rightarrow \frac{a}{c}=\frac{b}{b}=\frac{c}{a}=1\Rightarrow a=c$

65. (B)

Sol. Let nth term be the first negative term.

$T_n<0$

$40+(n-1)-2<0$ or $42-2n<0$

$2n>42$ or $n>21$

Hence first 21 terms of A.P are non -negative

sum will be maximum if no negative terms is taken.

$S_{max}=\frac{21}{2}[2\times 40+20(-2)]=420$

66. (B)

Sol. (i) is false,

If $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Thus, AB = 0 $\Rightarrow$ A = O or B = O

(iii) is false since matrix multiplication is not commutative.

(ii) is true as product AB is an identity matrix, if B is inverse of the matrix A.

67. (A)

Sol. Let $a-3=b$ so that $a-2=1+b$

The given equation is

$$\sum_{i=0}^{2n} \lambda_i (1+b)^i = \sum_{i=0}^{2n} \mu_i b^i$$

$$\Rightarrow \lambda_0 + \lambda_1(1+b) + \ldots + (1+b)^n + \ldots + (1+b)^{2n}$$

$$= \mu_0 + \mu_1 b + \ldots + \mu_n b^n + \ldots + \mu_{2n} b^{2n},$$

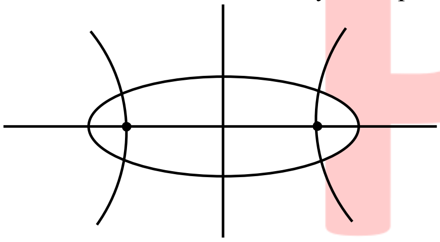
$$(\because \lambda_1 = 1 \forall i \geq n)$$

Comparing coefficient of b^n we get

$${}^{n}C_n + {}^{n+1}C_n + \ldots + {}^{2n}C_n = \mu_n = {}^{2n+1}C_n .$$

68. (C)

Sol. Let e_1 & e_2 are the eccentricity of ellipse & hyperbola



$\because b = ae_1$ …(1)

$b^2 = a^2(1 - e_1^2)$ …(2)

$a^2 = b^2(e_2^2 - 1)$ …(3)

From (1) & (2) $2e_1^2 = 1 \quad \Rightarrow e_1 = 1/\sqrt{2}$

From (1) & (3) $2 = e_2^2 - 1 \Rightarrow e_2 = \sqrt{3}$

69. (B)

Sol. Degree of LHS = 3n + 5

Degree of RHS = 2

$\Rightarrow 3n+5=2 \quad \Rightarrow \quad n=-1$

70. (B)

Sol. $\sqrt{\sin x} = -\cos x \quad (\cos x \leq 0)$

$$\Rightarrow \sin x = \cos^2 x = 1 - \sin^2 x$$

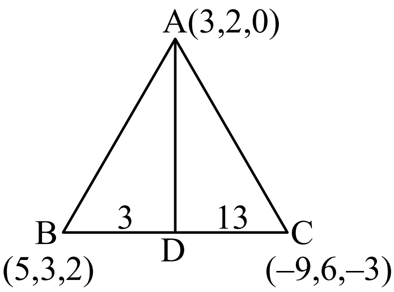
$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$

$$\sin x = \frac{\sqrt{5}-1}{2}, k \in \text{II}^{\text{nd}} \text{ quadrant}$$

$$x = \pi - \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$$

71. (B)

Sol.



$AB = \sqrt{4+1+4} = 3$

$AC = \sqrt{144+16+9} = 13$

$$D\left(\frac{-27+65}{3+13}, \frac{18+39}{3+13}, \frac{-9+26}{3+13}\right) = \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

72. (C)

Sol. $\cos\left(\frac{\theta}{2}\right) = \frac{x}{3}\cdot\frac{y}{2} - \sqrt{\left(1-\frac{x^2}{9}\right)\left(1-\frac{y^2}{4}\right)}$

$$4x^2 + 9y^2 + 36\cos^2\frac{\theta}{2} - 12xy\cos\frac{\theta}{2} = 36$$

$$4x^2 - 12xy\cos\frac{\theta}{2} + 9y^2 = 36\sin^2\frac{\theta}{2} = 18(1-\cos\theta)$$

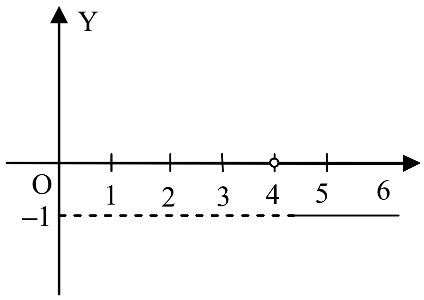
73. (D)

Sol. For $0 \le x < 1$, $f(x) = [\sin x] = 0$

$1 \le x < 2$, $f(x) = [\sin 1] = 0$

$2 \le x < 3$, $f(x) = [\sin 2] = 0$

$3 \le x < 4$, $f(x) = [\sin 3] = 0$



Hence there is discontinuity at point (4, –1).

74. (A)

Sol. $a\sin^2\frac{B}{2} + b\sin^2\frac{A}{2} + C$

$$= a\left(\frac{(s-c)(s-a)}{ac}\right) + \frac{b(s-b)(s-c)}{bc} + C$$

$$= \left(\frac{s-c}{c}\right)(s-a+s-b) + c$$

$$= \left(\frac{s-c}{c}\right)(c) + c = s$$

75. (D)

Sol. $\int (\sin 2x - \cos 2x)\,dx = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow -\frac{1}{2}(\sin 2x + \cos 2x) = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$\Rightarrow -\left[\frac{1}{\sqrt{2}}\sin 2x + \frac{1}{\sqrt{2}}\cos 2x\right] = \sin(2x-a)+b\sqrt{2}$

$\Rightarrow \sin\left(2x+\frac{5\pi}{4}\right) = \sin(2x-a)+b\sqrt{2}$

$\Rightarrow b$ is any constant and $a = \frac{-5\pi}{4}$.

76. (B)

Sol. $12x-5y+7=0$

$4x-3y+1=0$

$a_1a_2+b_1b_2>0$

$\Rightarrow$ obtuse angle bisector.

$\frac{12x-5y+7}{13} = \frac{4x-3y+1}{5}$

$\Rightarrow 4x+7y+11=0$

77. (A)

Sol. Let $\log_5 x = t$

$t^2+t-2<0$

$t \in (-2,1)$

$x \in \left(\frac{1}{25},5\right)$

78. (B)

Sol. $f(2x+3y, 2x-7y) = 20x$

$2x+3y=u$

$2x-7y=v$

$10y=u-v$

$y=\frac{u-v}{10}$

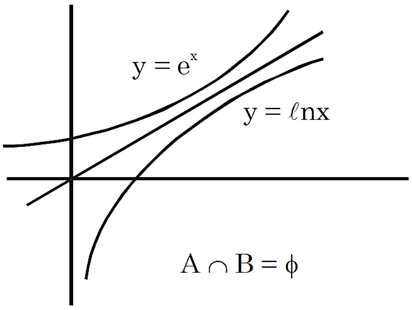
$2x = u-3y = u-\frac{(3u-3v)}{10} = \frac{7u+3v}{10}$

$f(u,v) = \frac{20(7u+3v)}{20} = 7u+3v$

$f(x,y) = 7x+3y$

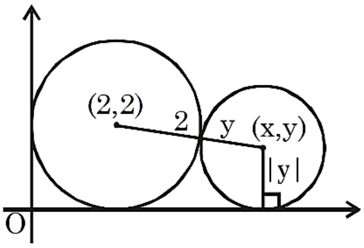
79. (C)

Sol.



80. (B)

Sol.



$$(2-x)^2+(2-y)^2=(y+2)^2$$

$$x^2-4x-8y+4=0$$

81. 5

Sol. Equation of tangent at P is $ty = x + t^2$ it intersects the line $x = 0$ at Q

$\therefore$ coordinates of Q are (0, t)

$$\therefore \text{ area of } \Delta PQS = \frac{1}{2}\begin{vmatrix} 0 & t & 1 \\ 1 & 0 & 1 \\ t^2 & 2t & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,[-t(1-t^2)+2t] = \frac{1}{2}\,(t+t^3)$$

$$\frac{dA}{dt} = \frac{1}{2}\,(3t^2+1) > 0 \;\forall\; t$$

$\therefore$ area is maximum for $t = 2$

Max. area $= \frac{1}{2}\,[2+8] = 5$

82. 82

Sol. no. of numbers = 5 ! = 120

E = Event that the number is divisible by 4.

So last two digits to see **12**, 13, 14, 15
21, 23, **24**, 25
31, **32**, 34, 35
41, 42, 43, 45
51, **52**, 53, 54

of the above 4 nos are divisible by 4

So n(E) = no. of numbers divisible by 4
= 3!.4 = 24

Probability $= \frac{n(E)}{n(S)} = \frac{24}{120} = \frac{1}{5} = 0.2$

$\Rightarrow$ integral part of $(\sqrt{2}+1)^5$ will be 82

83. 2

Sol. $f(x)=\frac{1}{2}(1-\cos 2x)+\frac{1}{2}\left(1-\cos\left(2x+\frac{2\pi}{3}\right)\right)$

$+\frac{1}{2}\left(\cos\left(2x+\frac{\pi}{3}\right)+\cos\frac{\pi}{3}\right)$

$=\frac{5}{4}-\frac{1}{2}\left\{2\cos\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}-\cos\left(2x+\frac{\pi}{3}\right)\right\}=\frac{5}{4}$

$\Rightarrow g\left(f\left(\frac{\pi}{8}\right)\right)=g\left(\frac{5}{4}\right)=1$

84. 0

Sol. $=\frac{1}{a}\begin{vmatrix}-a & a\cos C & a\cos B\\ \cos C & -1 & \cos A\\ \cos B & \cos A & -1\end{vmatrix}$

$=R_1\to R_1+bR_2+cR_3$

$=\frac{1}{a}\begin{vmatrix}0 & 0 & 0\\ \cos C & -1 & \cos A\\ \cos B & \cos A & -1\end{vmatrix}=0$

85. 7

Sol. $y=Ax^m+Bx^{-n}$

$\Rightarrow \frac{dy}{dx}=Amx^{m-1}-nBx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2}=Am(m-1)\,x^{m-2}+n(n+1)\,Bx^{-n-2}$

Putting these values in $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=12y$

We have $=m(m+1)\,Ax^m+n(n-1)\,Bx^{-n}=12\left(Ax^m+Bx^{-n}\right)$

$\Rightarrow m(m+1)=12$ or $n(n-1)=12$

$\Rightarrow m=3,-4$ or $n=4,-3$

86. 2

Sol. Let $\sin x=t$

if $x\to\frac{\pi}{2},\ t\to 1$

so $\lim_{t\to1}\frac{t-(t)^t}{1-t+\ell nt}$

using L Hospitals Rule

$=\lim_{t\to1}\frac{1-t^t(1+\log t)}{-1+\frac{1}{t}}$

$=\lim_{t\to1}\frac{-t^t(1+\log t)^2-t^t\left(\frac{1}{t}\right)}{-\frac{1}{t^2}}=+2.$

87. 3

Sol. Area of quadrilateral OABC = ΔOAC + ΔABC

$= \frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{AC}\right| + \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right|$

$= \frac{1}{2}|\vec{a}\times(\vec{b}-\vec{a})| + \frac{1}{2}|(2\vec{a}+10\vec{b}-\vec{a})$

$\times(\vec{b}-2\vec{a}-10\vec{b})|$

$= \frac{1}{2}|\vec{a}\times\vec{b}| + \frac{1}{2}|(\vec{a}+10\vec{b})\times(2\vec{a}+9\vec{b})|$

$= \frac{1}{2}|\vec{a}\times\vec{b}| + \frac{11}{2}|\vec{a}\times\vec{b}| = 6|\vec{a}\times\vec{b}|$

$|\vec{a}\times\vec{b}| = m$

$\ell = 2\lambda m$

$\Rightarrow 6|\vec{a}\times\vec{b}| = 2\lambda|\vec{a}\times\vec{b}|$

$\Rightarrow \lambda = 3.$

88. 4

Sol. $\int_{\log 2}^{x}\frac{du}{(e^u-1)^{1/2}} = \frac{\pi}{6}$

$\Rightarrow \int_{1}^{\sqrt{e^x-1}}\frac{2t}{1+t^2}\,dt = \frac{\pi}{6}$ as $e^u - 1 = t^2$

$\Rightarrow 2(\tan^{-1}t)_1^{\sqrt{e^x-1}} = \frac{\pi}{6} \Rightarrow \tan^{-1}\sqrt{e^x-1} - \frac{\pi}{4} = \frac{\pi}{12}$

$\Rightarrow \sqrt{e^x-1} = \tan\frac{\pi}{3} \Rightarrow \sqrt{e^x-1} = \sqrt{3} \Rightarrow e^x = 4.$

89. 42

Sol. If $z^2 + z + 1 = 0$

$\Rightarrow \quad (z-\omega)(z-\omega^2) = 0$

$\Rightarrow \quad z = \omega, \omega^2$

If $z = \omega$, then $\frac{1}{2} = \omega^2$

To find the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots + \left(z^{21}+\frac{1}{z^{21}}\right)^2$

Now, $z+\frac{1}{z} = \omega+\frac{1}{\omega} = -1,\ z^2+\frac{1}{z^2} = \omega^2+\frac{1}{\omega^2} = -1,\ z^3+\frac{1}{z^3} = 2$

$z^4+\frac{1}{z^4} = \omega^4+\frac{1}{\omega^4} = \omega+\frac{1}{\omega} = -1,\ z^5+\frac{1}{z^5} = \omega^2+\frac{1}{\omega^2} = -1$ and $z^6+\frac{1}{z^6} = 2$..... and so on

Therefore,

$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots + \left(z^{21}+\frac{1}{z^{21}}\right)^3$

$= \{(1-1)^2 + (-1)^2 + (2)^2\} + \{(-1)^2 + (-1)^2\}\,(2)^2 \times$ ……. 7 times

$= (1+1+4) + (1+1+4) \times$ …...7 times

$= 6 + 6 \times$ ….. 7 times

$= 6 \times 7 = 42$

90. 178

Sol. $\overline{x} = \frac{1357+1090+1666+1494+1623}{5} = \frac{7230}{5} = 1446$

x_i	$\lvert x_i - \overline{x} \rvert$
1357	89
1090	356
1666	220
1494	48
1623	177
Total	890

M.D. $(\overline{x}) = \frac{\Sigma \lvert d_i \rvert}{N} = \frac{890}{5} = 178$