JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- FULL TEST
FULL SYLLBUS TEST

ANSWER KEY

PHYSICS

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(B)	**9.**	(B)	**10.**	(A)	**11.**	(B)	**12.**	(A)	**13.**	(C)	**14.**	(C)
15.	(A)	**16.**	(C)	**17.**	(B)	**18.**	(A)	**19.**	(D)	**20.**	(C)	**21.**	2
22.	8.6	**23.**	13.5	**24.**	12	**25.**	5	**26.**	3	**27.**	72	**28.**	5
29.	3	**30.**	5										

CHEMISTRY

31.	(A)	**32.**	(C)	**33.**	(D)	**34.**	(B)	**35.**	(B)	**36.**	(A)	**37.**	(D)
38.	(B)	**39.**	(D)	**40.**	(B)	**41.**	(B)	**42.**	(C)	**43.**	(D)	**44.**	(D)
45.	(D)	**46.**	(A)	**47.**	(A)	**48.**	(D)	**49.**	(B)	**50.**	(B)	**51.**	5
52.	200	**53**	100	**54.**	27	**55.**	57	**56.**	6	**57.**	6	**58.**	592
59.	270	**60.**	8										

MATHEMATICS

61.	(B)	**62.**	(B)	**63.**	(C)	**64.**	(C)	**65.**	(B)	**66.**	(B)	**67.**	(A)
68.	(A)	**69.**	(C)	**70.**	(A)	**71.**	(B)	**72.**	(A)	**73.**	(A)	**74.**	(B)
75.	(C)	**76.**	(B)	**77.**	(B)	**78.**	(C)	**79.**	(A)	**80.**	(C)	**81.**	4
82.	4	**83.**	2	**84.**	2	**85.**	4	**86.**	3	**87.**	2	**88.**	3
89.	4	**90.**	2										

SOLUTIONS

PHYSICS

1. (B)

2. (B)

Sol. $a^2 t$ = constant

$a\sqrt{t}$ = constant

so hyperbola.

3. (D)

Sol. $a \sin\theta = \lambda$

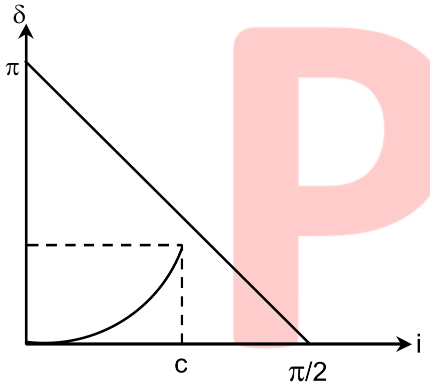
$\therefore \quad \sin\theta \simeq \tan\theta = x/D$

$\lambda = \dfrac{ax}{D} = 5000\ \text{Å}$

4. (D)

Sol. Light is incident from denser to rarer, so there is possibility of TIR

$C = \sin^{-1}\left(\dfrac{4/3}{3/2}\right) = \sin^{-1}\left(\dfrac{8}{9}\right)$



$(\delta_{\text{Reflection}}) = \pi - 2i$

$(\delta_{\text{Reflection}})_{\max} = \pi$, $(\delta_{\text{Reflection}})_{\min} = 0$

$(\delta_{\text{Refraction}})_{\min} = 0$, $\quad (\delta_{\text{Refraction}})_{\max} = \dfrac{\pi}{2} - C$

So for $0 \le \delta \le \dfrac{\pi}{2} - C$, two angle of incidence are possible, one for reflection and another for refraction.

for $\quad \delta > \dfrac{\pi}{2} - C$ only refraction is possible.

here $\delta = \dfrac{\pi}{2} - C = \dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{8}{9}\right) = \sin^{-1}\left(\dfrac{\sqrt{17}}{9}\right)$

so for $\delta > \sin^{-1}\left(\dfrac{\sqrt{17}}{9}\right)$, angle of incidence will be unique.

5. (A)

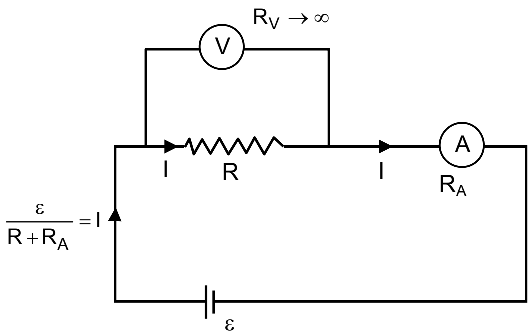
Sol. $\dfrac{C_1}{C_2} = \dfrac{V_2}{V_1} = \dfrac{1}{4}$

$V_1 + V_2 = 100\ V \Rightarrow V_1 = 80V, \quad V_2 = 20V$

$U_1 = \dfrac{1}{2} C_1 V_1^2 = \dfrac{1}{2}\ (6\mu f)(80)^2 = 19200\ \mu J = 19.2\ mJ$

6. (C)

Sol.



Reading of ammeter $I = \frac{\varepsilon}{R+R_A}$

Reading of voltmeter $V = IR$

computed resistance $\frac{V}{I} = R$

7. (C)

Sol. $v \propto \frac{1}{n}$

8. (B)

Sol. $V_T = \frac{2}{9n} r^2 g(\sigma - p)$

$V_1 = \frac{2}{9n} r^2 g(4p - p)$

$V_2 = \frac{2}{9n} r^2 g(8p - p)$

$\frac{V_2}{V_1} = \frac{7}{3}$

9. (B)

Sol. $K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$; $nK = \frac{3hc}{\lambda} - \frac{hc}{\lambda_0}$

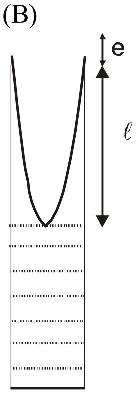
Solving this we get $\lambda_0 = \left(\frac{n-1}{n-3}\right)\lambda = 3\lambda$

10. (A)

Sol. PN Junction is in reverse bias.

11. (B)

Sol.



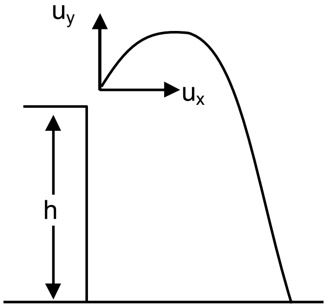
First resonance occurs at fundamental frequency

$f = \frac{V}{4(\ell + e)}$ $\Rightarrow$ $\ell + e = \frac{V}{4f}$ (where e = 0.6 × 2 = 1.2 cm)

$\ell + e = \frac{336}{4 \times 512} = 0.164$ m ; $\ell = 16.4 - 1.2 = 15.2$ cm

12. (A)

Sol.



$t_1 = \frac{2u_y}{g} \Rightarrow u_y = \frac{gt_1}{2}$

$T = \frac{u_y}{g} + \sqrt{\frac{2\left(h + \frac{u_y{}^2}{2g}\right)}{g}}$

$T = \frac{t_1}{2} + \sqrt{\frac{\frac{2\left(h + \frac{gt_1^2}{2g}\right)}{8}}{g}}$

$u_x T \Rightarrow u_x = \frac{d}{\frac{t_1}{2} + \sqrt{\frac{2\left(h + \frac{gt_1^2}{8}\right)}{g}}}$

$\tan\theta = \frac{t_1^2 g\left[1 + \sqrt{1 + \frac{8h}{gt_1^2}}\right]}{4d}$

13. (C)

Sol. Let time taken in boiling the water by the heater is t sec. Then

$Q = ms\Delta T$

$\frac{836}{4.2} t = 1 \times 1000\ (40^\circ - 10^\circ)$

$\frac{836}{4.2} t = 1000 \times 30 \quad \Rightarrow \quad t = \frac{1000 \times 30 \times 4.2}{836} = 150$ sec

14. (C)

Sol. The excess pressure inside the soap bubble in inversely proportional to radius of soap bubble i.e. $P \propto 1/r$, r being the radius of bubble. It follows that pressure inside a smaller bubble is greater than that inside a bigger bubble. Thus, if these two bubbles are connected by a tube, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

15. (A)

Sol. Zero error = -0.007 cm

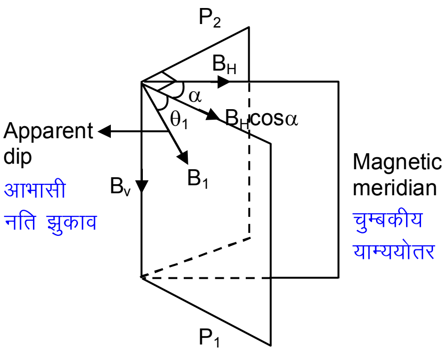
16. (C)

Sol. $M = \frac{2Q\omega}{2\pi}\pi(b/2)^2,\ L = \left(\frac{2mb^2}{4} + \frac{2mb^2}{12}\right)\omega$

$\frac{M}{L} = \frac{3Q}{8m}$

17. (B)

Sol. $\tan\theta_1 = \dfrac{B_v}{B_H\cos\alpha}$



$\tan\theta_1 = \dfrac{\tan\theta}{\cos\alpha} \Rightarrow \cos\alpha = \dfrac{\tan\theta}{\tan\theta_1}$ ....(i)

$\Rightarrow \tan\theta_2 = \dfrac{\tan\theta}{\sin\alpha} \Rightarrow \sin\alpha = \dfrac{\tan\theta}{\tan\theta_2}$ ....(ii)

Squaring (i) & (ii)

$$\left(\frac{\tan\theta}{\tan\theta_1}\right)^2 + \left(\frac{\tan\theta}{\tan\theta_2}\right)^2 = 1$$

If δ is true angle of dip at that location then

$$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$$

$$\cot^2\theta = \cot^2 30^\circ + \cot^2 45^\circ = 4$$

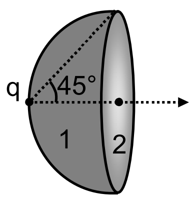
18. (A)

Sol. $y = -2[1 - 2\sin^2\pi t] = -2[1 - 2(x/8)^2] = -2[1 - (x^2/32)]$

$\therefore\ y = -2 + (x^2/16)$

This is equation of parabolic path, which is symmetric about y-axis.

19. (D)



Sol.

From gauss law total flux linked with hemisphere

$\phi_1 + \phi_2 = \dfrac{q}{2\varepsilon_0}$,

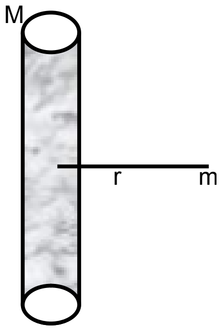
flux linked with circular face.

$\phi_2 = \dfrac{q}{2\varepsilon_0}(1 - \cos 45^\circ)$

$\phi_1 = \dfrac{q}{2\sqrt{2}\varepsilon_0}$

20. (C)

Sol.



$$\frac{2GMm}{L.r} = m\omega^2 r$$

$$\omega^2 \propto \frac{1}{r^2}$$

$$T \propto r$$

21. 2

Sol. When a charged particle of charge q, mass m enters perpendicularly to the magnetic induction $\vec{B}$ of a magnetic field, it will experience a magnetic force

$F = q(\vec{v} \times \vec{B}) = qvB \sin 90° = qvB$ that will provide a centripetal acceleration $\frac{v^2}{r}$

$F = q(\vec{v} \times \vec{B}) = qvB \sin 90° = qvB$

$\Rightarrow \quad qvB = \frac{mv^2}{r}$

$\Rightarrow \quad mv = qBr$

$\Rightarrow$ The de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{qBr}$

$\Rightarrow \quad \frac{\lambda_{\alpha-\text{particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$

Since $\frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \mathbf{2}$

22. 8.6

Sol. $P_{\text{consumed}} = \left(\frac{V_A}{V_R}\right)^2 \times P_R = \left(\frac{110}{115}\right)^2 \times 500 = 457.46\ \text{W}$

23. 13.5

Sol. $y_1 = a\cos\omega_1 t$ and $y_2 = a\cos(\pi + \omega_2 t)$

For same phase $\omega_1 t = \pi + \omega_2 t$

$$t = \frac{\pi}{\omega_1 - \omega_2} = \frac{\pi}{\frac{2\pi}{6} - \frac{2\pi}{18}} = 4.5s$$

24. 12

Sol. $TV^{\gamma-1} = C$

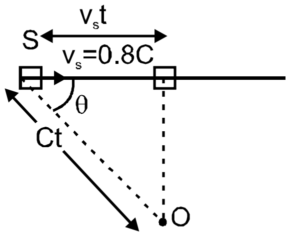
$\gamma - 1 = n$

$n = \frac{6}{13}$

$$\gamma = \frac{4\times\frac{7}{2}R + 2\times\frac{5}{2}R}{4\times\frac{5}{2}R + 2\times\frac{3}{2}R} = \frac{19}{13}$$

25. 5

Sol. $\cos\theta = \frac{v_s t}{Ct} = 0.8$

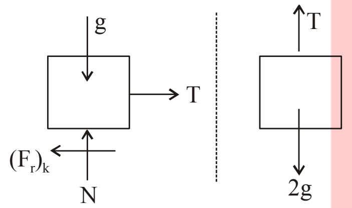


$$f' = f\left[\frac{C}{C - v_s\cos\theta}\right] = 1.8\left[\frac{C}{C - 0.8C\cos\theta}\right]$$

$$= 1.8\frac{1}{1-0.8^2} = 1.8\frac{1}{1-0.64} = \frac{1.8}{0.36} = \frac{180}{36} = 5 \text{ kHz.}$$

26. 3

Sol.



$1g = N$

$(F_r)_k = \mu_k N$

$T - (F_r)_k = 1a$

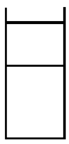
$2g - T = 2a$

$a = 6 \text{ m/s}^2$

$s = \frac{1}{2}\times 6\times 1^2 = 3\text{m}$

27. 72

Sol.



$(p_0 + h_{\rho g})\, v_0 = (p_0 - h_{\rho g})v$

$(H + 8) \times 4 = (H - 8) \times 5$

$4H + 32 = 5H - 40$

$72 = H$

28. 5

Sol. $U = \frac{Ax}{x^2 + B}$

$\therefore \quad [B] = L^2$

$ML^2T^{-2} = \frac{AL}{L^2}$

$\therefore \quad A = ML^3T^{-2}$

$AB = ML^5T^{-2}$

29. 3

Sol. $$\frac{1}{656\times10^{-9}} = 1.097\times10^{7}\left(\frac{1}{z^2}-\frac{1}{n^2}\right)$$

$$\frac{1}{7} \approx \frac{100}{656\times1.097} = \frac{1}{4}-\frac{1}{n^2}$$

$$\Rightarrow \frac{1}{n^2} = \frac{1}{n^2} = \frac{1}{4}-\frac{1}{7} = \frac{3}{28} \approx \frac{1}{9}$$

$$\Rightarrow \quad n = 3$$

30. 5

Sol. $$\frac{1}{2}mv^2 = 35$$

$$\frac{1}{2}mv_1{}^2 + \frac{1}{2}I\omega^2 = 35$$

$$= \frac{1}{2}mv_1{}^2 + \frac{1}{2}\times\frac{2}{5}m\,r^2\times\frac{v_1^2}{r^2} = 35$$

$$= \frac{1}{2}mv_1{}^2\times\frac{7}{5} = 35$$

$$\frac{1}{2}mv_1{}^2 = 25$$

$$v - v_1 = \sqrt{\frac{70}{m}} - \sqrt{\frac{50}{m}}$$

$$x = (v - v_1)t = \left(\sqrt{\frac{70}{m}}-\sqrt{\frac{50}{m}}\right)\left(\sqrt{70}+\sqrt{50}\right) = \frac{20}{\sqrt{m}} = 5\text{ m}$$

31. (A)

Sol. (A) NO^- derivative of O_2 and isoelectronic with O_2.

So $(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2\ (\pi 2p^2_x = \pi 2p^2_y)\ (\pi^*2p_x^{\,1} = \pi^*2p^1_y)$ and 2 unpaired electrons.

(B) O_2^{2-} : $(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p_z)^2\ (\pi 2p^2_x = \pi 2p^2_y)\ (\pi^*2p_x^{\,2} = \pi^*2p^2_y)$ and no unpaired electrons.

(C) CN^- is derivative of and isoelectronic with N_2 : $(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\pi 2p^2_x = \pi 2p^2_y)\ (\sigma 2p_z)^2$ and no unpaired electron.

(D) CO is derivative of and isoelectronic with N_2 : $(\sigma 1s)^2\ (\sigma^*1s)^2\ (\sigma 2s)^2\ (\sigma^*2s)^2\ (\pi 2p^2_x = \pi 2p^2_y)\ (\sigma 2p_z)^2$ and no unpaired electron.

32. (C)

Sol. On account of higher electronegativity of fluorine.

33. (D)

34. (B)

Sol. $[Cr(en)_2Br_2]Br$

dibromidobis(ethylenediamine)chromium(III) Bromide.

35. (B)

Sol.

	L_1	L_2	L_3	L_4
λ absorbed	red	green	yellow	blue

∴ Increasing order of energy of wavelengths absorbed reflect greater extent of crystal-field splitting, hence higher field strength of the ligand.

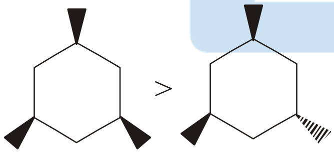
Energy : Blue (L_4) > green (L_2) > yellow (L_3) > red (L_1)

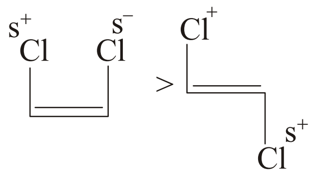
∴ $L_4 > L_2 > L_3 > L_1$ in field strength of ligands.

36. (A)

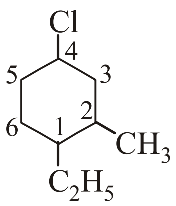
Sol. (A) Cyclobutane exist in nonplanar puckered form.

(B) In cycloalkene after 12 member trans is more stable then cis.

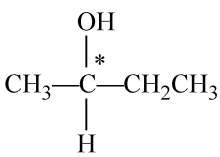
(C) All three are on equitorial position  ⇒ 2 equitorial + 1 axial

(D)  due to resonance]

37. (D)

Sol.  4-chloro-1-ethyl-2-methyl cyclohexane

38. (B)

Sol. 

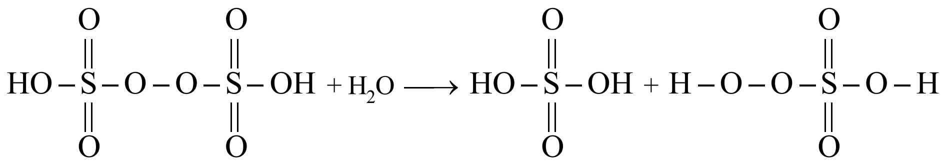
39. (D)

Sol. (4n+2) e^- and complete delocalisation of $\prec$ bond and lone pair.

40. (B)

41. (B)

42. (C)

Sol. 

(Sulphuric Acid) (Peroxomono-sulphuric Acid)]

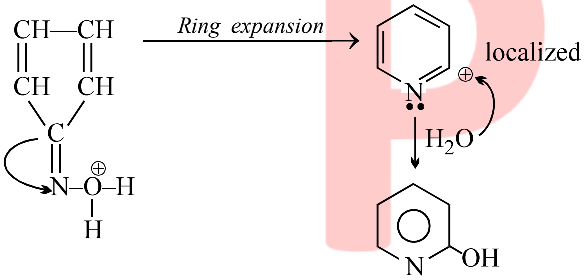
43. (D)

Sol. H_2O_2 is thermally unstable & it decomposes easily.

$$H_2O_2(l) \longrightarrow H_2O(l) + 1/2O_2(g)$$

Its decomposition is catalysed by alkali metals present in traces in the glass of the vessel.]

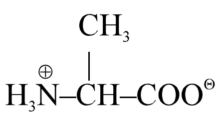
44. (D)

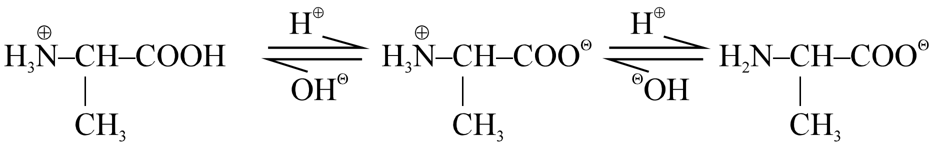
Sol. 

45. (D)

Sol. Peptide chain grow and written from N-terminal to C-terminal so given tripeptide is Ala-Phe-Gly.]

46. (A)

Sol. Alanine zwitter ion is 



Thus Ans is A]

47. (A)

Sol. $[Ni(NH_3)_6]^{2+}$ has sp^3d^2 hybridisation having octahedral geometry as with d^8 configuration no two empty d-orbitals are available for d^2sp^3 hybridisation. As sp^3d^2 hybridisation involves nd orbital (i.e. outer orbitals), so the complex is called as outer orbital complex.

48. (D)

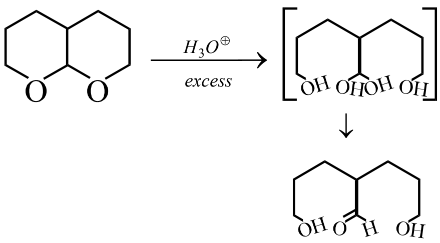
Sol. Chlorophyll a green pigment in plants contains Mg.

49. (B)

Sol. 1°give acid, 3° donot oxidize in such condition

50. (B)

Sol.



51. 5

Sol.

$$\Delta_r G = \Delta_r G^\circ + RT\, ln$$

$$\Rightarrow \quad \Delta_r G = \Delta_r G^\circ + RT\, ln\left(\frac{1}{2}\right) \quad ...(i)$$

At equilibrium $\Delta_r G = \Delta_r G^\circ + RT\, lnK_p$ ...(ii)

From (i) – (ii):

$$\Delta G = RT\, ln\left(\frac{1}{2K_p}\right)$$

On putting the values, we get

$$\frac{5154.3}{8.3 \times 2.3 \times 300} = \log 2\, K_p$$

$\Rightarrow \quad 10^{0.9} = 2 \cdot K_p$

$\Rightarrow \quad \left(10^{0.3}\right)^3 = 2 \cdot K_p$

$\Rightarrow \quad K_p = 4$

Initial	A	B	C
Initial	2	1	1
Initial	3	0	0
Change	– x	x	x
At eq.	3 – x	x	x

$$\frac{x^2}{3 - x} = 4 \quad \Rightarrow \quad x^2 = 12 - 4x \quad \Rightarrow \quad x = 2$$

$\Rightarrow$ Partial pressure at equilibrium 1, 2, 2

Total pressure 5 atm.

52. 200

Sol. $[OH^-] = 10^{-5} \times \frac{0.1}{0.2} = \frac{10^{-5}}{2}$

$$[Mg^{+2}] = \frac{K_{sp}}{(OH^-)^2} = \frac{10^{-11}}{(10^{-5}/2)^2}$$

$= 4 \times 10^{-1} = 0.4$

moles of $MgCl_2 = 0.2$ moles

millimoles $= 0.2 \times 1000 = 200$

53 100

Sol. n = 2, (2bar, 12) to 1 bar

$$W - P(V_2 - V_1) = P_2 \times nRT\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$$

or, $$W = -1 \times P_1V_1\left(\frac{1}{1} - \frac{1}{2}\right) = -2 \times 1\left(\frac{1}{2}\right) = -1 \text{ bar L} = -100 \text{ J }]$$

54. 27

Sol. $\Delta G° = \Delta H° - T\Delta S°$

$\Delta H° = \Delta H_f^o\,(\text{product}) - \Delta H_f^o\,(\text{reactants})$

$= 2 \times 30 = 60$ kJ

$\Delta S° = 204 + 4(42) - 2(121) = +130$

$\Delta G° = \Delta H° - T\Delta S° = 60000 - 300 \times 130$

$\Delta G° = 21000\text{ J} = -RT\,ln\,K$

$$\log K = -\left(\frac{21000}{300 \times 8.3 \times 2.3}\right)$$

$K_p = 2.15 \times 10^{-4}$ atm

The dissociation of Ag_2O is nonspontaneous at 27°C

55. 57

Sol. $A + B \rightarrow P$

$r_1 = \text{Rate} = k_1\,[A]\,[B] \rightarrow \text{at } T_1$

$$r_2 = \frac{k_2[A][B]}{4}$$

$r_2 = 2r_1$

$$\frac{k_2[A][B]}{4} = 2k_1[A][B]$$

$$\frac{k_2}{k_1} = 8 = 2^3 \text{ .... (i)}$$

$T_2 = 25 + 3 \times 10 = 55°C$ **Ans.**

Alternatively

$$\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln 2 = \frac{E_a}{R}\left[\frac{1}{298} - \frac{1}{308}\right] \text{ .... (ii)}$$

when $k_2 = 8\,k_1$, [from equation (i)]

$$\ln 8 = \frac{E_a}{R}\left[\frac{1}{298} - \frac{1}{T}\right] \text{ ...... (iii)}$$

from equation (ii) and (iii)

$$3 = \frac{\frac{1}{298} - \frac{1}{T}}{\frac{1}{298} - \frac{1}{308}} = \frac{\frac{T-298}{T}}{\frac{10}{308}} = \frac{308T - 308 \times 298}{10T}$$

$$T\,(\text{in K}) = \frac{308 \times 298}{(308 - 30)} = 330.16\text{ K}$$

$T = 57.15\,°C$]

56. 6

Sol. In first oxide : $\frac{E_A}{E_O} = \frac{x}{y}$ $\Rightarrow \frac{32/3}{8} = \frac{x}{y}$

i.e. $\frac{x}{y} = \frac{4}{3}$

$\therefore$ In second oxide : $\frac{E_A}{E_O} = \frac{3}{4}$ $\Rightarrow \frac{E_A}{8} = \frac{3}{4}$

$\therefore$ $E_A = 6$ **Ans.**

57. 6

Sol. $\frac{P_o - P_s}{P_s} = i \times m \times \frac{18}{1000}$

$\Rightarrow \frac{21.08 - 20}{20} = 3 \times m \times \frac{18}{1000}$

$\Rightarrow m = 1$

$\Delta T_f = i \times K_f \times m = 3 \times 2 \times 1 = 6$ Ans.

58. 592

Sol. $\Delta T_m = K_m m$

$= 8.5 \times \frac{12/120}{100} \times 1000$; $= 8.5$ K

$T = 273 + 327 + 8,5$; $= 591.5 \approx 592$ K **Ans.**

59. 270

Sol. $\frac{y - 260}{x - \sqrt{0.25}} = \frac{260 - 250}{\sqrt{0.25} - 1}$

$y - 260 = (-20)\left(x - \frac{1}{2}\right)$

$y = 270 - 20x$

Intercept = 270

$\wedge_m^\infty = 270\ \Omega^{-1}\ cm^2\ mol^{-1}$ Ans.

60. 8

Sol. We have $\rightarrow \Delta E = \frac{3}{4} \times 0.85$ eV

as energy = 0.6375 eV the photon will belong to brackett series

(as for brackett series $0.306 \leq E \leq 0.85$)

So $\frac{3}{4} \times 0.85 = 13.6\left[\frac{1}{4^2} - \frac{1}{n^2}\right]$

$\Rightarrow n = 8$; Hence x = 8]

61. (B)

Sol. The equation of given curve is SS … (i)

On differentiating wr.t. x, we get

$$1+\frac{dy}{dx}=e^{xy}\left(y+x\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{ye^{xy}-1}{1-xe^{xy}}$$

Since, tangent is parallel to the y-axis, then

$$\frac{dy}{dx}=\frac{1}{0} \Rightarrow 1-xe^{xy}=0$$

$$\Rightarrow 1-x(x+y)=0$$ [From equ. (i)]

This holds for $x=1, y=0$

62. (B)

Sol. $y^2 = 4ax$ eq^n of normal

$y = mx - 2am - am^3$

$y^2 = 4c(x - b)$ eq^n of normal

$y = m(x - b) - 2cm - cm^3$

It two parabola have common normal then both of eq^n of normal should be identical after comparing the coefficients

$m = \pm\sqrt{\frac{2(a-c)-b}{(c-a)}}$ which is real if

$-2-\frac{b}{c-a}>0 \Rightarrow \frac{b}{a-c}>2$

63. (C)

Sol. The required probability

$y = 1$ – Probability of each receiving atleast one

$$=1-\frac{n(E)}{n(S)}$$

Now, the number of integral solution of

$x_1+x_2+x_3+x_4+x_5+x_6=10$

such that $x_1 \ge 1, x_2 \ge 1, \ldots\ldots x_6 \ge 1$

gives $n(E)$ and the number of integral solutions of

$x_1+x_2+\ldots..+x_5+x_6=10$ such that

$x_1 \ge 0, x_2 \ge 0, \ldots\ldots x_6 \ge 0$ gives $n(S)$

$\therefore$ The required probability

$$=1-\frac{{}^{10-1}C_{6-1}}{{}^{10+6-1}C_{6-1}}$$

$$=1-\frac{{}^{9}C_{5}}{{}^{15}C_{5}}$$

$$=1-\frac{6}{143}=\frac{137}{143}$$

64. (C)

Sol. Since, $\int \frac{dx}{(x+2)\left(x^2+1\right)} = a\log\left(1+x^2\right)+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C$

On differentiating both sides, we get

$$\frac{1}{(x+2)\left(x^2+1\right)} = \frac{d}{dx}\left[a\log\left(1+x^2\right)+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C\right]$$

$$=\frac{2ax}{1+x^2}+\frac{b}{1+x^2}+\frac{1}{5}\cdot\frac{1}{(x+2)}$$

$$=\frac{10ax(x+2)+5b(x+2)+1+x^2}{5(x+2)\left(1+x^2\right)}$$

$$\Rightarrow \frac{1}{(x+2)\left(1+x^2\right)}=\frac{(1+10a)x^2+(5b+20a)x+1+10b}{5(x+2)\left(1+x^2\right)}$$

$$\Rightarrow (1+10a)x^2+(5b+20a)x+1+10b=5$$

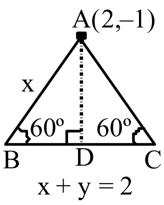
On comparing the coefficient of x^2, x and constant terms on both sides, we get

$1+10a=0, 5b+20a=0, 10b=4$

$$\Rightarrow a=-\frac{1}{10}, b=\frac{2}{5}$$

65. (B)

Sol. Let side AB is x



$\therefore$ length AD = $\frac{|2-1-2|}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

In ΔABD, sin 60° = $\frac{1}{\sqrt{2}x}$

$$\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{2}x}\Rightarrow x=\frac{\sqrt{2}}{\sqrt{3}}$$

$\therefore$ area of equilateral $\Delta = \frac{\sqrt{3}}{4}\frac{2}{6}=\frac{\sqrt{3}}{6}$

66. (B)

Sol. If $a_1, a_2, \ldots.. a_n$ are n positive real numbers and $m_1, m_2, \ldots m_n$ are n positive rational numbers, then

$$\frac{m_1a_1+m_2a_2+\ldots..+m_na_n}{m_1+m_2+\ldots.+m_n}=\left(a_1^{m}\cdot a_2^{m_q}\ldots a_n^{m_n}\right)^{\frac{1}{m_1+m_2-n_n}}$$

Let $P=x^2y^3$

Clearly, P is the product of 5 factors such that two of them are equal to x and the remaining 3 are equal to y.

Now, $3x + 4y = 5$

$$\Rightarrow \quad 2\left(\frac{3x}{2}\right) + 3\left(\frac{4y}{3}\right) = 5$$

$$\Rightarrow \quad \frac{3x}{2} + \frac{3x}{2} + \frac{4y}{3} + \frac{4y}{3} + \frac{4y}{3} - 5$$

Using weighted $\text{AM} \geq \text{GM}$ inequality, we get

$$\frac{2 \cdot \frac{3x}{2} + 3 \cdot \frac{4y}{3}}{5} \geq \left[\left(\frac{3x}{2}\right)^2 \left(\frac{4y}{3}\right)^3\right]^{1/5}$$

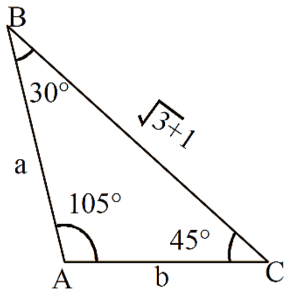
$$\Rightarrow \quad \frac{5}{5} \geq \left(\frac{16x^2y^3}{3}\right)^{1/s} \Rightarrow \left(\frac{16}{3}x^2y^3\right)^{1/5} \leq 1$$

$$\Rightarrow \quad x^2y^3 \leq \frac{3}{16}$$

Hence, greatest value of x^2y^3 is $\frac{3}{16}$.

67. (A)

Sol.



We have, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\Rightarrow \quad \frac{\sin 105°}{\sqrt{3}+1} = \frac{\sin 30°}{b} = \frac{\sin 45°}{c}$$

$$\Rightarrow \quad b = \frac{\sqrt{3}+1}{2\sin 105^\circ} \qquad \dots \text{(i)}$$

and $c = \frac{\sqrt{3}+1}{\sqrt{2}\sin 105^\circ}$ … (ii)

Now, area of ΔABC

$$= \frac{1}{2} b \sin A = \frac{1}{2} bc \sin 105°$$

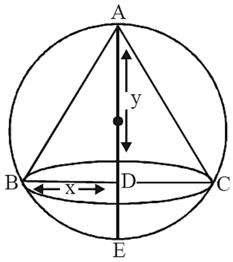
$$= \frac{1}{2} \cdot \frac{(\sqrt{3}+1)^2}{2\sqrt{2}\sin(60° + 45°)} \qquad [From eqs.(i) and (ii)]$$

$$= \frac{(\sqrt{3}+1)^2}{4\sqrt{2}\left(\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)}$$

$$= \frac{(\sqrt{3}+1)^2}{2(\sqrt{3}+1)} = \frac{1}{2}(\sqrt{3}+1)$$

68. (A)

Sol. Let the diameter of the sphere, $AE = 2r$



Let the radius of cone be x and height be y

$\therefore \quad AD = y$

Since, $\quad BD^2 = AD \cdot DE$

$\Rightarrow \quad x^2 = y(2r - y) \quad$ .... (i)

Volume of cone, $V = \frac{1}{3}\pi^2 y$

$= \frac{1}{3}\pi y(2r - y)y$ [from Equ. (i)]

$= \frac{1}{3}\pi\left(2ry^2 - y'\right)$

On differentiating w.r.t. x, we get

$$\frac{dV}{dy} = \frac{1}{3}\pi\left(4ry - 3y'\right)$$

For maxima and minima, put $\frac{dV}{dy} = 0$

$$\Rightarrow \quad \frac{1}{3}\pi\left(4ry - 3y'\right) = 0 \quad \Rightarrow \quad y(4r - 3y) = 0 \quad \Rightarrow \quad y = \frac{4}{3}r, 0$$

Again differentiating wr.t. y, we get $\frac{d^2V}{dy^2} = \frac{1}{3}\pi(4r - 6y)$

At $\quad y = \frac{1}{3}r$

$$\frac{d^2V}{dy^2} = \frac{1}{3}\pi(4r - 8r) = -ve$$

$\therefore$ Volume of cone is maximum at $y = \frac{4}{3}r$.

Now, ratio $= \frac{\text{Height of cone}}{\text{Diameter of sphere}} = \frac{y}{2r} = \frac{\frac{4r}{3}}{2r} = \frac{2}{3}$

69. (C)

Sol. Since, $y^2 = p(x)$

On differentiating w.r.t. x, we get $\therefore \; 2r = \frac{dy}{dx}p'(x)$

Again differentiating, we get $\quad 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = p''(x)$

$$\therefore \quad 2y\frac{d^2y}{dx^2} = p''(x) - 2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad 2y^3\frac{d^2y}{dx^2} = y^2p''(x) - 2y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad 2y^3\frac{d^2y}{dx^2} = p(x)p'(x) - \frac{1}{2}\{p'(x)\}^2$$

Again differentiating, we get

$$2\frac{d}{dx}\left(y'\frac{d^2y}{dx^2}\right) = p'(x)p'(x) + p(x)p''(x) - p'(x)p''(x)$$

$$= p(x)p^{m}(x)$$

70. (A)

Sol. $$\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{1}{5}\right)^2 = \frac{9}{4}\left(\frac{3x+4y-7}{5}\right)^2$$

$\therefore$ focus at $\left(\frac{1}{2},\frac{1}{5}\right)$, directrix is $\frac{3x+4y-7}{5} = 0$

$\therefore$ Equation of latus rectum is $y-\frac{1}{5} = -\frac{3}{4}\left(x-\frac{1}{2}\right)$

71. (B)

Sol. $x^2 + (1-2\lambda)x + \left(\lambda^2-\lambda-2\right) = 0 - - - - - (1)$

$a = 1$ of $\propto, \beta$ are roots of (1)

if $\alpha < 3 < \beta \Rightarrow$ a. $f(3) < 0$

$\Rightarrow f(3) < 0$

$\Rightarrow 9 + (1-2\lambda)3 + \lambda^2 - \lambda - 2 < 0$

$\Rightarrow \lambda \in (2,5)$

72. (A)

Sol. $\int_0^3 dy = \int_0^3 |x-1|\, dx$

$y(3) - y(0) = \int_0^1 (1-x)dx + \int_1^3 (x-1)dx$

$$y(3) = x - \frac{x^2}{2}\Big]_0^1 + \frac{x^2}{2} - x\Big]_1^3 = \frac{1}{2} + 2 = \frac{5}{2}$$

Given $AN = CM = 4 - \frac{c^2}{2} = 9 - \frac{(2-c)^2}{2}$

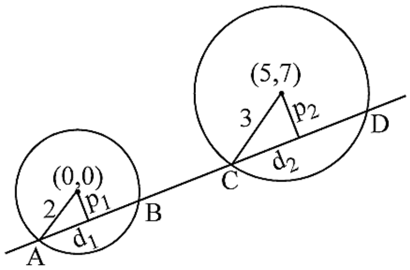
$\Rightarrow c = -\frac{3}{2}$

73. (A)

Sol. Let equation of line be

$y = x + c \qquad y - x = c \qquad \ldots(1)$

perpendicular from $(0,0)$ on (1) is $\left|\frac{-c}{\sqrt{2}}\right| = \frac{c}{\sqrt{2}}$



In ΔAON,

$\sqrt{2^2 - \left(\frac{c}{\sqrt{2}}\right)^2} = AN$

and in $\Delta CPM, \sqrt{3^2 - 2 - \frac{c}{\sqrt{2}}} = CM$

perpendicular from $(5,7)$ on line $y - x = c = \frac{2-c}{\sqrt{2}}$

equation of line $y = x - \frac{3}{2}$ of $2x - 2y - 3 = 0$.

74. (B)

Sol. Solving the equations of the asymptotes the centre is x = 1 and y = 0, since e = $\sqrt{2}$ the equation of the family of the hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y)^2}{a^2} = 1$$

$\Rightarrow 2(x - 1) - 2y\frac{dy}{dx} = 0$

$\Rightarrow (x - 1) = yy'$ is differential equation.

75. (C)

Sol. Put $\tan y = z \quad \therefore \quad \sec^2 y\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{dz}{dx} + xz = x^3$

which is linear in Z

$\therefore \int Pdx = \int xdx = \frac{x^2}{2} \quad \therefore \quad e^{\int Pdx} = e^{x^2/2}$

solution is

$ze^{x^2/2} = \int e^{x^2/2}x^3dx = \int e^{x^2/2}x^2 \cdot xdx$

Put $\frac{x^2}{2} = t \quad \therefore \quad xdx = dt$

$= \int e^t \cdot 2tdt = 2e^t(t-1) + c$

$\Rightarrow \tan y = 2\left(\frac{x^2}{2} - 1\right) + ce^{-x^2/2} = x^2 - 2 + ce^{-x^2/2}$

76. (B)

Sol. $$\lim_{n\to-\infty}\frac{x^2-x^2-3x\cos\frac{1}{|x|}}{x-\sqrt{x^2+3x\cos\frac{1}{|x|}}}\ \text{(Rationalize)}$$

$$\lim_{n\to-\infty}\frac{-3x\cos\frac{1}{|x|}}{x-\sqrt{x^2}\sqrt{1+\frac{3\cos\frac{1}{|x|}}{x}}}$$

$$=\lim_{n\to-\infty}\frac{-3x\cos\frac{1}{|x|}}{x-x\sqrt{1+\frac{3\cos\frac{1}{|x|}}{x}}}$$

$$\left(\sqrt{x^2}=-x,\ \text{as } x\to\infty\right)=\frac{-3}{2}$$

77. (B)

Sol. $f(A) = I + A + A^2 + \dots\dots + A^{16}$

$$A=\begin{bmatrix}0&5\\0&0\end{bmatrix}\Rightarrow A^2=\begin{bmatrix}0&5\\0&0\end{bmatrix}\begin{bmatrix}0&5\\0&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$A^3=A^2.A=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Similarly $A^4 = A^5 = \dots\dots\dots\dots = A^{16} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$

$$f(A)=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}0&5\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}+\dots.+\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}1&5\\0&1\end{bmatrix}$$

78. (C)

Sol. $$f(x)=\begin{cases}(-1)^{m+n}x^n(x-1)^n & \text{if } x<0\\ (-1)^n x^m(x-1)^n & \text{if } 0\le x<1\\ x^m(x-1)^n & \text{if } x\ge 1\end{cases}$$

$g(x) = x^m (x-1)^n$, then

$g'(x) = mx^{m-1}(x-1)^n + nx^m(x-1)^{n-1}$

$= x^{m-1}(x-1)^{n-1}\{mx - m + nx\} = 0$

$f'(x) = 0 \Rightarrow g'(x) = 0 \Rightarrow x = 0, 1$ or $\frac{m}{m+n}$

$f(0) = 0$, $f(1) = 1$ and

$$f\left(\frac{m}{m+n}\right)=(-1)^n\frac{m^m n^n(-1)^n}{(m+n)^{m+n}}$$

$$\Rightarrow \frac{m^m n^n}{(m+n)^{m+n}}>1 \qquad \left[0<\frac{m}{m+n}<1\right]$$

The maximum value $= \frac{m^m n^n}{(m+n)^{m+n}}$

79. (A)

Sol. $|z_1| = |z_2| = |z_3| = 1 \Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}$

$$|\overline{z_1 + z_2 + z_3}| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$$\because |z_1 + z_2 + z_3| = |\overline{z_1 + z_2 + z_3}| = |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$$

$$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1 \text{ (given)}$$

80. (C)

Sol. Ist angle = 60º

2^{nd} angle $= 60^g = 60 \times \frac{90^o}{100} = 54^o$

III angle $= \frac{5\pi}{6} = 150^o$

So fourth angle is $360^o - (60^o + 54^o + 150^o) = 96^o$

81. 4

Sol. Let $f(x) = x^2 - 2(a-1)x + (2a+1)$. Then, $f(x) = 0$ will have both roots positive, if

(1) Discriminant > 0

(2) Sum of the roots > 0

(3) $f(0) > 0$

(1) Discriminant ≥ 0

$\Rightarrow \quad 4(a-1)^2 - 4(2a+1) \geq 0$

$\Rightarrow \quad a^2 - 4a \geq 0$

$\Rightarrow \quad a \leq 0$ or $a \geq 4$ … (i)

(2) Sum of the roots $> 0 \Rightarrow 2(a-1) > 0$

$\Rightarrow \quad a > 1$ … (ii)

(3) $f(0) > 0 \Rightarrow (2a+1) > 0$

$\Rightarrow \quad a > -\frac{1}{2}$ … (iii)

From Eqs.(i), (ii) and (iii), we get $a \geq 4$.

Hence, the least integral value of a is 4.

82. 4

Sol. Let the even number of terms in GP be $2n$, with first term a and common ratio r. Then, Sum of all terms $= 5$ (Sum of odd terms)

$\Rightarrow \quad a_1 + a_2 + \ldots.. + a_{2n} = 5\left(a_1 + a_3 + \ldots.. + a_{2n-1)}\right)$

$\Rightarrow \quad a_1 + ar + ax^2 + \ldots.. + ax^{2n-1}$

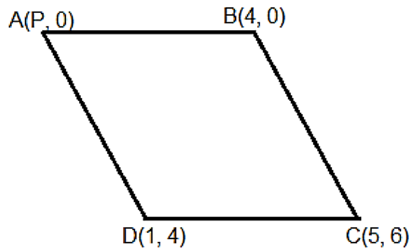
$= 5\left(a + ar + ar^2 + \ldots.. + ar^{2n-2}\right)$

$\Rightarrow \quad \frac{a\left(r^{2n}-1\right)}{(r-1)} = \frac{5a\left(r^{2n}-1\right)}{r^2-1} \Rightarrow \quad r + 1 = 5$

$\Rightarrow \quad r = 4$

83. 2

Sol.



$$\cos\angle ADC = \frac{(AD)^2 + (CD)^2 - (AC)^2}{2AD \cdot CD} < 0$$

$$\Rightarrow (P-1)^2 + 4^2 + (5-1)^2 + (6-4)^2 < (P-5)^2 + 6^2$$

$$\Rightarrow 8P < 24 \quad \Rightarrow P < 3 \quad \Rightarrow P = 2$$

84. 2

Sol. $\frac{dy}{y} = \frac{2xdx}{1+x^2}$

$\Rightarrow y = c\left(1+x^2\right)$

$x = 0, y = 1$

$\Rightarrow c = 1$

$x = 1 \quad \Rightarrow y = 2$

85. 4

Sol. Let A be the origin. $\overline{AB} = \overline{b}$, $\overline{AC} = \overline{c}$

Area of $\Delta ABC = \frac{1}{2}(\overline{b} \times \overline{c})$

$$\overline{AF} = \frac{\overline{b}}{2}, \overline{AE} = \frac{\overline{c}}{2} \Rightarrow \overline{FE} = \frac{\overline{c}}{2} - \frac{\overline{b}}{2}, \quad \overline{FC} = \overline{c} - \frac{\overline{b}}{2}$$

$$\text{area of } \Delta FCE = \frac{1}{2}\left(\frac{\overline{c}}{2} - \frac{\overline{b}}{2}\right) \times \left(\overline{c} - \frac{\overline{b}}{2}\right) = \frac{1}{8}|\overline{b} \times \overline{c}| = \frac{1}{4} \cdot \Delta ABC$$

86. 3

Sol. Any point on $L_1 = (\lambda, \lambda - 1, \lambda)$, any point on $L_2 = (2\mu - 1, \mu, \mu)$

$$\therefore \frac{2\mu - 1 - \lambda}{2} = \frac{\mu - \lambda + 1}{1} = \frac{\mu - \lambda}{2} \Rightarrow \lambda = 3, \mu = 1$$

$$\therefore A = (3,2,3), \quad B = (1,1,1), \quad AB = 3$$

87. 2

Sol. Point of intersection of the given lines are

$\left(\frac{5}{3+4m}, \frac{3+9m}{3+4m}\right)$

x coordinate is an integer

$\therefore \frac{5}{3+4m}$ is an integer

3 + 4m is divisor of 5

3 + 4m = 1, 3 + 4m = – 1

3 + 4m = 5, 3 + 4m = – 5

$\Rightarrow$ m = – 1, – 2

88. 3

Sol. Let $y = 3^{\log_3 \sqrt{9^{|x-2|}}}$ $\quad z = 7^{\frac{1}{5}\log_7[4.3^{|x-2|}-9]}$

$\Rightarrow \log_3 y = \log_3 \sqrt{9^{|x-2|}}$ $\quad \log_7 z = \frac{1}{5}\log_7[4.3^{|x-2|}-9]$

$\Rightarrow y = \sqrt{9^{|x-2|}}$ $\quad z = (4.3^{|x-2|}-9)^{1/5}$

$\Rightarrow y = 3^{|x-2|}$ $\quad z = (4.3^{|x-2|}-9)^{1/5}$

Now $E = (y+z)^7$ and the sixth term is given by

$t_6 = {}^7C_5\, y^2 z^5 \Rightarrow 21.(3^{|x-2|})^2.[(4.3^{|x-2|}-9)^{1/5}]^5$

$= 567$(given)

$\Rightarrow \quad 21.3^{2|x-2|}.[4.3^{|x-2|}-9] = 567$ [put $3^{|x-2|} = k$]

$\Rightarrow \quad 21.k^2.\,[4.k-9] = 567$

$\Rightarrow \quad 4k^3 - 9k^2 - 27 = 0$

$\Rightarrow \quad (k-3)(4k^2+3k+9) = 0$

$\Rightarrow \quad k = 3$

89. 4

Sol. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and $\frac{x^2}{b^2} + \frac{y^2}{c^2} = 1$

Required area $= \pi ab - \pi bc$

$= \pi b\,(a-c)$

$b^2 = a^2(1-e^2)$ and $c^2 = b^2(1-e^2)$

$\Rightarrow c^2 = a^2(1-e^2)^2$

$\Rightarrow c = a(1-e^2)$ $\quad \Rightarrow a - c = ae^2$

so required area $= \pi b\,(ae^2) = \pi abe^2$

$= 9 \times \left(\frac{2}{3}\right)^2$

$= 4$ sq. units.

90. 2

Sol. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$

$\Rightarrow A^3 = 2A^2 = 2^2A$

similarly $A^4 = 2^3A$ and so on

So $A^n = 2^{n-1}A$

$\Rightarrow A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^n - I = \begin{bmatrix} 2^{n-1}-1 & 2^{n-1} \\ 2^{n-1} & 2^{n-1}-1 \end{bmatrix}$

$|A^n - I| = (2^{n-1}-1)^2 - (2^{n-1})^2$

$= 1 - 2^n.$

$\Rightarrow \lambda = 2$