JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- FULL TEST
FULL SYLLBUS TEST

ANSWER KEY

PHYSICS

1.	(D)	**2.**	(A)	**3.**	(D)	**4.**	(B)	**5.**	(B)	**6.**	(D)	**7.**	(A)
8.	(A)	**9.**	(C)	**10.**	(A)	**11.**	(C)	**12.**	(C)	**13.**	(D)	**14.**	(A)
15.	(C)	**16.**	(A)	**17.**	(C)	**18.**	(B)	**19.**	(C)	**20.**	(B)	**21.**	1.25
22.	50	**23.**	4	**24.**	8	**25.**	50	**26.**	4	**27.**	30	**28.**	0
29.	5	**30.**	50										

CHEMISTRY

31.	(A)	**32.**	(B)	**33.**	(C)	**34.**	(C)	**35.**	(B)	**36.**	(B)	**37.**	(B)
38.	(C)	**39.**	(B)	**40.**	(A)	**41.**	(B)	**42.**	(B)	**43.**	(A)	**44.**	(C)
45.	(A)	**46.**	(B)	**47.**	(C)	**48.**	(B)	**49.**	(C)	**50.**	(B)	**51.**	30
52.	11	**53.**	20	**54.**	800	**55.**	0.025	**56.**	16	**57.**	87	**58.**	1350
59.	180	**60.**	4000										

MATHEMATICS

61.	(C)	**62.**	(A)	**63.**	(D)	**64.**	(A)	**65.**	(A)	**66.**	(C)	**67.**	(A)
68.	(B)	**69.**	(C)	**70.**	(D)	**71.**	(B)	**72.**	(B)	**73.**	(C)	**74.**	(C)
75.	(D)	**76.**	(D)	**77.**	(D)	**78.**	(A)	**79.**	(B)	**80.**	(C)	**81.**	5
82.	0	**83.**	50	**84.**	8	**85.**	7	**86.**	4	**87.**	10	**88.**	8
89.	2	**90.**	9										

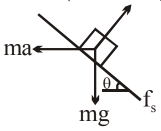
SOLUTIONS

PHYSICS

1. (D)

Sol. $mg\sin\theta + f_s = ma\cos\theta$

$f_s = m(a\cos\theta - g\sin\theta) \le \mu(mg\cos\theta + ma\sin\theta)$



$\therefore \mu \ge \dfrac{a - g\tan\theta}{g + a\tan\theta} = 0.577$

$\dfrac{9}{10} > 0.577$

2. (A)

3. (D)

Sol. $I_1 = I_0\cos^2\theta = \dfrac{I_0}{2}$

$I_2 = I_1\cos^2\theta = \dfrac{I_0}{4}$

$I_3 = I_2\cos^2\theta = \dfrac{I_0}{8}$

4. (B)

Sol. $\lambda_1 T_1 = \lambda_2 T_2$

$\dfrac{T_1}{T_2} = \dfrac{\lambda_2}{\lambda_1} = 2$

Rate of heat loss $\dot{Q} = 4\pi r^2 \sigma e T^4$

$\dfrac{\dot{Q}_1}{\dot{Q}_2} = \left(\dfrac{r_1}{r_2}\right)^2 \left(\dfrac{T_1}{T_2}\right)^4 = 4$

$\dot{Q} = -ms\dfrac{d\theta}{dt} = -\dfrac{4}{3}\pi r^3 \rho s \dfrac{d\theta}{dt}$

$\dfrac{\left(\dfrac{d\theta}{dt}\right)_1}{\left(\dfrac{d\theta}{dt}\right)_2} = \left(\dfrac{\dot{Q}_1}{\dot{Q}_2}\right) \left(\dfrac{r_2}{r_1}\right)^3 = 32$

5. (B)

Sol. Let ϕ be the work function of metal.

$\dfrac{hc}{\lambda_1} = \phi + \dfrac{mv_1^2}{2}$

$\dfrac{hc}{\lambda_2} = \phi + \dfrac{mv_2^2}{2}$

$hc\left[\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right] = \dfrac{m}{2}\left(v_1^2 - v_2^2\right)$

$= \dfrac{m}{2}\left[4v_2^2 - v_2^2\right]$

$= \dfrac{3mv_2^2}{2}$

$$\frac{mv_2^2}{2} = \frac{hc}{3}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$$

$$\phi = \frac{hc}{\lambda_2} - \frac{mv_2^2}{2} = \frac{hc}{\lambda_2} - \frac{hc}{3}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \frac{4hc}{3\lambda_2} - \frac{hc}{3\lambda_1}$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{3}\left[\frac{4}{450\times10^{-9}} - \frac{1}{350\times10^{-9}}\right]$$

$\therefore\ \phi = 3.98 \times 10^{-19}$ J.

6. (D)

Sol. $2\mu_0 t = n \times 10000 = (n + 1) \times 5000$ (where μ_0 is 1.25)

$\therefore\ n = 1$

$2\mu_0 t = 1 \times 10000 \times 10^{-10}$

$t = 0.4\ \mu m$

7. (A)

Sol. $y = f(x \pm c \cdot t)$ is the general wave equation

At $t = 0$, $y = f(x) \Rightarrow y = \frac{1}{\sqrt{1+x^2}}$

$$y = \frac{1}{\sqrt{2-2x+x^2}} = \frac{1}{\sqrt{1+(x-1)^2}} = f(x-1)$$

$\Rightarrow f(x - ct) = f(x - 1)$ at $t = 1$

$\Rightarrow c = 1$ m/s.

8. (A)

Sol. $$E = \frac{Qx}{\frac{4}{3}\pi R^3\, 3\varepsilon_0} + \frac{Q/16}{4\pi\varepsilon_0 x^2}$$

$$\frac{dE}{dx} = 0$$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R^3} - \frac{2}{16x^3}\right) = 0,\quad \text{at } x = \frac{R}{2}\quad \text{E is minimum.}$$

9. (C)

Sol. Radius should be greater than $r_2 - r_1$

$r = \frac{mu}{qB}$; $(r_2 - r_1) < \frac{u}{\sigma B}$

Thus $u > \sigma B\,(r_2 - r_1)$

10. (A)

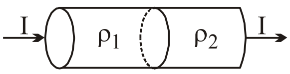
Sol. $$\frac{2c}{mR^2} = \frac{k}{m}$$

$$R = \sqrt{\frac{2c}{k}}$$

11. (C)

Sol. $E_1 = \frac{I}{A}\rho_1$

$E_2 = \frac{I}{A}\rho_2$



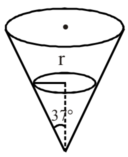
flux through surface dividing medium of ρ_1 and $\rho_2 = E_1A - E_2A = \frac{\sigma}{\varepsilon_0}$

$\frac{\sigma}{\varepsilon_0} = \frac{I}{A}(\rho_1 - \rho_2)$

$\sigma A = I\varepsilon_0(\rho_1 - \rho_2)$

12. (C)

Sol. $\frac{r}{8} = \tan 37° = \frac{3}{4}$



r = 6m

$F = (P_0 + h\rho g)\,\pi r^2$

$= (10^5 + 10 \times 800 \times 10) \times \pi \times 36$ N

$= 1.8 \times 36 \times \pi \times 10^5$ N

$= 2 \times 10^7$ N

13. (D)

Sol.

	x $\longrightarrow$	y
t = 0	16% = N_0	0
t = t	2% = N	14%
$N = \frac{N_0}{8} \Rightarrow$	$\frac{1}{8} = \frac{1}{2^n} \Rightarrow$	n = 3

$t = 3T_{1/2} = 135$ yrs.

14. (A)

Sol. In uniform electric field net force on dipole is zero.

15. (C)

Sol. Loss in P.E. $= 0 - \left(\frac{-G(\lambda 2\pi R)m}{R}\right) = G\lambda 2\pi m$ = gain in K.E.

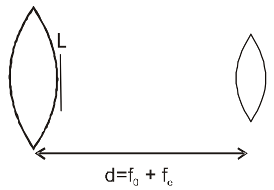
i.e. independent of R

16. (A)

Sol. When Diode get reverse biased no current flows through it and output is similar to input voltage. When diode get forward biased then diode behave as wire and output is 5V.

17. (C)

Sol.



Magnification by eyepiece

$$m = \frac{f}{f+u}$$

$$-\frac{I}{L} = \frac{f_e}{f_e + (-(f_0 + f_e)}$$

$$\Rightarrow \frac{I}{L} = \frac{f_e}{f_0}$$

$$\text{m.p.} = \frac{f_0}{f_e} = \frac{L}{I}$$

18. (B)

Sol. System does not accelerate it $M_2g \le \mu\,(M_1+m)g$

$M_2 \le \mu\,(M_1 + m)$

or $m \ge \frac{M_2 - \mu M_1}{\mu}$ or $m \ge \frac{M_2}{\mu} - M_1$

19. (C)

Sol. $I_0 = \frac{V_0}{R}$ divide the current in L_1 and L_2 like resistors $I_1 = I_0 \frac{L_2}{L_1 + L_2}$

20. (B)

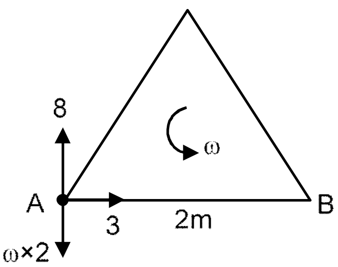
Sol. $4(1 + 11 \times 10^{-6}\,\Delta T) = 3.992\,(1 + 19 \times 10^{-6}\,\Delta T)$

$\Rightarrow 0.008 \quad \Rightarrow 31.848 \times 10^{-6}\,\Delta T$

$\Delta T = 250°C$

common temperature is $30 + 250 = 280°\,C$

21. 1.25



Sol.

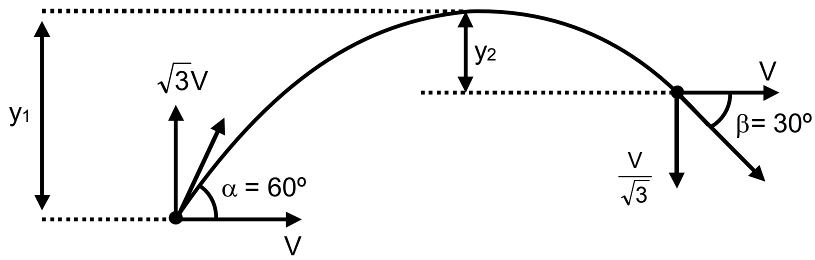
$V_A = \sqrt{3^2 + (8 - \omega \times 2)^2}$

$= \sqrt{3 + (8 - 2 \times 2)^2}$

$= 5$

22. 50

Sol. In Y-direction



$$-\frac{V}{\sqrt{3}} = \sqrt{3}V - 10 \times 4$$

$\therefore$ V = $10\sqrt{3}$

Also : $0 = \left(\sqrt{3}V\right)^2 - 2.10y_1$

$\therefore$ y_1 = 45 m

Again ; $\left(\frac{V}{\sqrt{3}}\right)^2 = 0 + 2.10.y_2$

$\therefore$ y_2 = 5 m

Total distance = $y_1 + y_2$ = 45 + 5 = 50 m

23. 4

Sol. **[x = 4]** $[\overline{C}] = M^{-1}L^{-2}T^{4}A^{+2}$

24. 8

Sol. E_1 = 13.6 eV

$E_n = -\frac{13.6eV}{n^2}$, $KE_n = \frac{13.6eV}{n^2} = \frac{1}{2}I\omega^2$ ....(i)

and $I\omega = \frac{nh}{2\pi}$

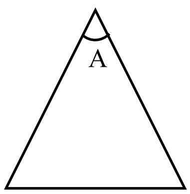
$\therefore$ from (i)

$$\frac{E_1}{n^2} = \frac{1}{2}\left(\frac{nh}{2\pi}\right)\omega$$

$$f = \frac{2E_1}{n^3h}$$

25. 50

Sol.



$\delta = i + e - A$

44° = 42° + 62° – A

44° = 104° – A

A = 104° – 44° = 60º

when deviation is minimum

D = 40°

D = i + e – A

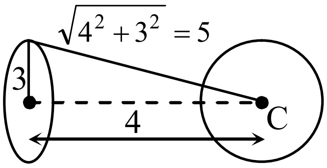
38° = 2i – 60°

100° = 2i

i = 50°

26. 4

Sol.



Potential at C$=\dfrac{kQ}{\sqrt{4^2+3^2}}=\dfrac{kQ}{5}=\dfrac{9\times10^9\times20\times10^{-6}}{5}$

$=\dfrac{18\times10^4}{5}=3.6\times10^4$ volt

27. 30

Sol. As $y = x\tan\theta - \dfrac{gx^2}{2u^2}(1+\tan^2\theta)$

for (a, b), $ga^2\tan^2\theta - 2au^2\tan\theta + (ga^2 + 2bu^2) = 0$

as discriminant must be positive

$4a^2u^2 - 4ga^2(ga^2 + 2bu^2) \geq 0$

Solving, $u \geq \sqrt{bg + g(a^2+b^2)^{1/2}}$

28. 0

Sol. $W_{net} = \Delta K$

$\Rightarrow (F\sin\theta.\ \ell - mg\ell(1-\cos\theta)) = \dfrac{1}{2}mv^2$

where $\theta = 37^\circ$, $F = \dfrac{mg}{3}$ $\qquad \Rightarrow v = \left\{\dfrac{2\ell}{5m}(3F - mg)\right\}^{\frac{1}{2}} = 0$

29. 5

Sol. $\dfrac{q_1}{C_1} = \dfrac{q_2}{C_2}$

$q_1 + q_2 = 2Q_0$

$C_1 = \dfrac{\epsilon_0 A}{d_0 + Vt}$, $C_2 = \dfrac{\epsilon_0 A}{d_0 - Vt}$

$\dfrac{q_1}{q_2} = \dfrac{d_0 - Vt}{d_0 + Vt} \Rightarrow q_2 + q_2\left(\dfrac{d_0 - Vt}{d_0 + Vt}\right) = 2Q_0$

$\Rightarrow q_2\left(\dfrac{2d_0}{d_0 + Vt}\right) = 2Q_0$

$\Rightarrow q_2 = \dfrac{2Q_0}{2d_0}(d_0 + Vt)$

$I = \dfrac{dq_2}{dt} = \dfrac{Q_0V}{d_0} = 20$ amp

30. 50

Sol. $I = \dfrac{20}{10\times10^3} = 20\times10^{-4}$

But $20\times10^{-4} = \dfrac{120}{x + 10^4}$

or $20\times10^{-4}x + 20 = 120$

$\therefore\ x = \left(\dfrac{120-20}{20}\right)\times10^4$

$= 5\times10^4\ \Omega$

$= 50\ k\Omega$

31. (A)

Sol. Atomic size arguments can be used for these species. Larger outer atoms result in larger angles due to steric repulsion.

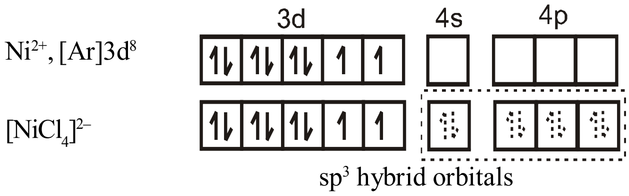
32. (B)

33. (C)

Sol. Among d-block elements
max. M.P. of first transition series = Cr
min. M.P. of second transition series = Cd

34. (C)

Sol. In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure.



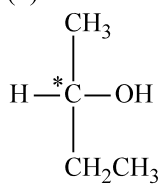
$\mu_{B.M.} = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$ BM

35. (B)

36. (B)

37. (B)

Sol.

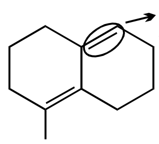


38. (C)

Sol. Due to less distance between two oppositing charge and its aromatic nature and NO_2^- also have –I effect which stabilize it.

39. (B)

Sol.

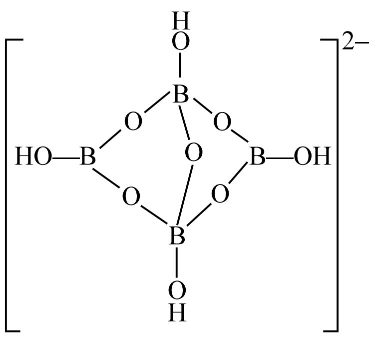


Less substituted bond will be reduced. 1,4-addition not possible in trans dione]

40. (A)

41. (B)

Sol. In Borax $(Na_2B_4O_7 \cdot 10H_2O)$ Among 10 water molecules 2 molecules are part of structure i.e. exists as $Na_2[B_4O_5(OH)_4] \cdot 8H_2O$



$Na_2[B_4O_5(OH)_4] \cdot 8H_2O + 2HCl \longrightarrow 2NaCl + 4H_3BO_3 + 5H_2O$

Methyl orange (pH = 3.7) is used to detect end point. Aq. solution of borax acts as buffer, as borax is salt of strong base NaOH & weak acid H_3BO_3.]

42. (B)

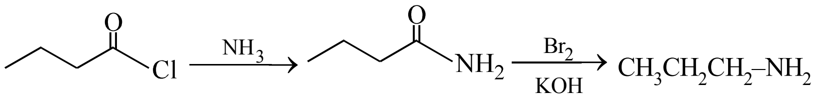
Sol. $XeF_4 + SbF_5 \longrightarrow [XeF_3]^+ [SbF_6]^-$

$[XeF_3]^+ : sp^3d$ Bent-T-shape

$[SbF_6]^- : sp^3d^2$ octahedral

43. (A)

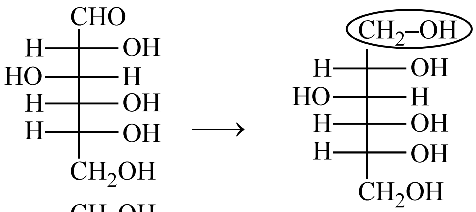
Sol.



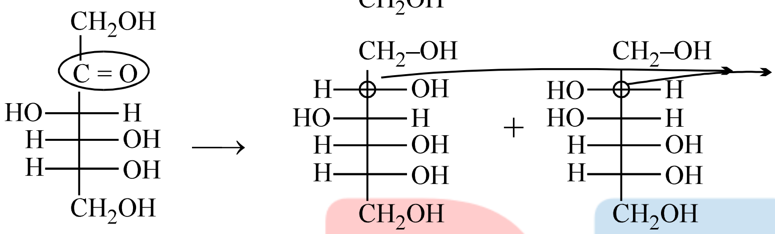
(Hoffmann Bromamide reaction)

44. (C)

Sol.



No New chiral center is created.



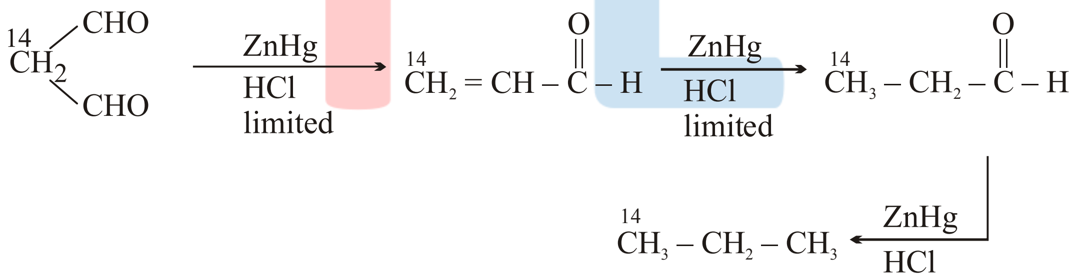
New chiral center is created Hence 2 isomers are possible]

45. (A)

Sol. D-Glucose, D-Mannose and D-Fructose give same osazone so z = 1

46. (B)

Sol.



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47. (C)

48. (B)

Sol. β position of –OH is a 3° so it cannot form a double bond.

49. (C)

50. (B)

Sol. $R–OH \xrightarrow{MeCOCl} R – O – \overset{\overset{O}{||}}{C} – Me$

$C_6H_{14}O_4 \xrightarrow{MeCOCl} Mwt = 234$

Mwt = 150

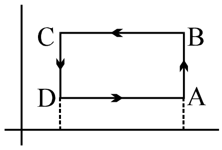
ΔMwt = 234-150 = 84 = 2×(42)

$$= 2 \times (\text{Mwt of } – \overset{\overset{O}{||}}{C} – CH_3 – H)$$

Hence 2 OH groups are present.

51. 30

Sol. $W_{AB} = -nRT\,ln\dfrac{V_2}{V_1}$



$\Delta S = \dfrac{q_{rev}}{T} = -\dfrac{W_{AB}}{T}$

$\Rightarrow$ $-W_{AB} = T\Delta S = 600 \times 100$

$-W_{BC} = -{}^{n}C_V\,(T_1 - T_2)$

$-W_{CD} = T\Delta S = 300 \times (-100)$

$-W_{DA} = -{}^{n}C_V\,(T_2 - T_1)$

net work delivered during one cycle $= -W_{AB} - W_{BC} - W_{CD} - W_{DA} = 300 \times 100 = 30$ kJ

Note : Net work done = area of the rectangle

52. 11

Sol. $\Delta S_{system} = nR\,ln\dfrac{V_2}{V_1} \Rightarrow 2 \times R \times ln\,2 \Rightarrow$ 11.52 J/K

$\Delta S_{surrounding} = -\dfrac{3.3 \times 1000}{300} \Rightarrow -11$ J/K]

53. 20

Sol. $[F^-] = 0.02$ M

$[Mg^{2+}] = \dfrac{K_{sp}}{[F^-]^2} = \dfrac{8 \times 10^{-8}}{(0.02)^2} = 2 \times 10^{-4}$

$\therefore$ $[CaCO_3] = 2 \times 10^{-4}$ mole/L

$= 2 \times 10^{-2}$ g/L

$= 20$ mg/L

$= 20$ ppm ($\because$ 1 mg/L = 1ppm for water)]

54. 800

Sol.

	CH_3COOH	$\rightleftharpoons$	CH_3COO^-	+	H^+
Initial conc.	5×10^{-8} M		0		0
	0		5×10^{-8} M		$(5\times10^{-8} + y)$M
Equ. conc.	xM				

	H_2O	$\rightleftharpoons$	H^+	+	OH^-
Equ. conc.			$(5\times10^{-8} + y)$M		yM

$K_W = [H^+][OH^-]$

or $1.0 \times 10^{-14} = (5\times10^{-8} + y)\cdot y$ $\Rightarrow y = 7.8 \times 10^{-8}$

Now, $K_a = \dfrac{[CH_3COO^-][H^+]}{[CH_3COOH]}$

or $1.6 \times 10^{-5} = \dfrac{5\times10^{-8} \times 12.8\times10^{-8}}{[CH_3COOH]}$

$\therefore$ $[CH_3COOH] = 4 \times 10^{-10}$

$\therefore$ Percent of unionised CH_3COOH molecules $= \dfrac{4\times10^{-10}}{5\times10^{-8}} \times 100 = 0.8\ \%$]

55. 0.025

Sol. Since $t_{1/2} = 0.693/k$ ∴ $t_{1/2} = 500$ sec.

Since volume is changing therefore half life does not mean that concentration will be reduced to half rather half of initial moles of A will be reacted, therfore moles (if initially 1 lit is assumed) remaining of A = .05.

$$A \rightarrow 2B + C$$
$$1-x \quad\quad 2x \quad\quad x$$

$= n_f = 1 + 2x$ $\quad\quad$ ∵ $x = 0.5$

$1 + 2x = 2$

Also final volume of the container will be $V_f = 1$ litre × n_f/n_i

⇒ 2 litre

∴ final concentration of A will be 0.025M

56. 16

Sol. milli equivalent of $K_2Cr_2O_7$(in 50 ml) ⇒ $\frac{1}{10} \times 6 \times 5 = 3$

milli equivalent of Fe^{2+} (in 100 ml) = milli equivalent of $K_2Cr_2O_7$ in 100 ml ⇒ 6

$\frac{w}{56} \times 1000 = 6 \quad\quad ⇒ \quad\quad w = 0.3369$

% of Fe^{2+} = 0.336 × 100 = 33.6

So, % of Fe^{3+} = 50 – 33.6 = 16.4 **Ans.**]

57. 87

Sol.
$$E_{cell} = \frac{0.06}{2} \log \frac{[H^+]^2_{RHS}(P_{H_2})_{LHS}}{[H^+]^2_{LHS}(P_{H_2})_{RHS}}$$

For R NH_3Cl $\quad\quad$ $K_h = \frac{K_w}{K_b} = \frac{10^{-14}}{10^{-5}} = 10^{-9}$

$$\alpha = \sqrt{\frac{K_h}{C}} = \sqrt{\frac{10^{-9}}{0.1}} = 10^{-4}$$

$$RNH_3^+ (aq) + H_2O (l) \rightleftharpoons RNH_2 (aq) + H_3O^+ (aq)$$

At eq^m $\quad$ $C(1-\alpha)$ $\quad$ – $\quad$ $C\alpha$ $\quad$ $C\alpha$

$[H_3O^+]_{LHS} = C\alpha$

$= 0.1 \times 10^{-4} = 10^{-5}$

For HA $\quad \alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{4 \times 10^{-6}}{0.01}} = 2 \times 10^{-2}$

$[H_3O^+]_{RHS} = C\alpha = 0.01 \times 2 \times 10^{-2} ⇒ 2 \times 10^{-4}$

$$E_{cell} = \frac{0.06}{2} \log \frac{(2 \times 10^{-4})}{(10^{-5})^2} \times \frac{1}{0.5} = \frac{0.06}{2} \log (8 \times 10^2)$$

$$= \frac{0.06}{2} \log 8 + \frac{0.06}{2} \times 2$$

$= 0.03 \times 0.9 + 0.06 = 0.087$ V

or $\quad E_{cell} = 87$ mV $\quad$ Ans.

58. 1350

Sol. $i = 0.5$ Amp

Current efficiency $= \frac{1000}{12}\%$ $\quad\quad t = 9.65\text{ hr} = 9.65 \times 3600\text{ sec.}$

$$w = \frac{Eit}{96500} = \frac{9 \times \frac{\frac{1000}{12}}{100} \times 1 \times 0.5 \times 9.65 \times 3600}{96500} \quad \because E_{Al} = 9$$

$$= \frac{9 \times 5 \times 9.65 \times 3}{965} = 1.35\text{ g} = 1350\text{ mg}$$

59. 180

Sol. $\Delta T_f = K_f \times m = 1.9 \times \frac{w_B}{500 \times 180} \times 1000$

$\Delta T_f = \frac{3.8\, w_B}{180}$

$T_f = -\frac{3.8\, w_B}{180}$

$\Delta T_b = 0.6 \times \frac{w_B}{180} \times 2 = \frac{1.2\, w_B}{180}$

$T_b = 100 + \frac{1.2\, w_B}{180}$

$T_b - T_f = 105 = 100 + \frac{1.2\, w_B}{180} - \frac{(3.8\, w_B)}{180}$

$100 + \frac{5\, w_B}{180} = 105$

$\frac{w_B}{180} = 1\text{ gm}$

$w_B = 180\text{ gm}$ Ans.]

60. 4000

Sol. $\Rightarrow$ 3 Radial and 2 angular node means

$l = 2$

$n - l - 1 = 3$ $\quad\quad \therefore n = 6$ i.e. 6d

$\Rightarrow$ one radial and 1 angular node means

$l = 1$

$n - l - 1 \;\; n = 3$ $\quad\quad$ i.e.3p

$\frac{1}{\lambda} = R \times 3^2 \left[\frac{1}{3^2} - \frac{1}{6^2}\right] = \frac{3R}{4}$; $\quad \lambda = \frac{4}{3R} = \frac{X}{3000\,R}$

$X = 4000$ Ans.

61. (C)

Sol: $\int_0^3 dy = \int_0^3 |x-1|\,dx$

$y(3)-y(0)=\int_0^1 (1-x)\,dx+\int_1^3 (x-1)\,dx$

$y(3)=x-\frac{x^2}{2}\Big]_0^1+\frac{x^2}{2}-x\Big]_1^3=\frac{1}{2}+2=\frac{5}{2}$

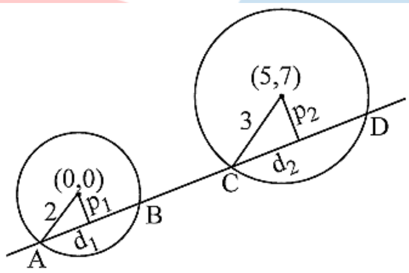
Given $AN = CM = 4-\frac{c^2}{2}=9-\frac{(2-c)^2}{2}$

$\Rightarrow c=-\frac{3}{2}$

62. (A)

Sol: Let equation of line be $y=x+c$ $\qquad$ $y-x=c$ ….(1)

perpendicular from $(0,0)$ on (1) is $\left|\frac{-c}{\sqrt{2}}\right|=\frac{c}{\sqrt{2}}$



In ΔAON, $\sqrt{2^2-\left(\frac{c}{\sqrt{2}}\right)^2}=AN$ and in ΔCPM, $\sqrt{3^2-2-\frac{c}{\sqrt{2}}}=CM$

perpendicular from $(5,7)$ on line $y-x=c=\frac{2-c}{\sqrt{2}}$

$\therefore$ equation of line $y=x-\frac{3}{2}$ of $2x-2y-3=0$

63. (D)

Sol: Given, $y\cos\alpha - x\sin\alpha = p$

and $y\sin\left(30^\circ-\alpha\right)-x\cos(30^\circ-\alpha)=p$ are inclined at 60° so line ax + by $=1$ can be acute angle bisector … (i)

i.e., $y\cos\alpha - x\sin\alpha - p$

$=-\left(y\sin(30^\circ-\alpha)-x\cos\left(30^\circ-\alpha\right)-p\right)$

$\Rightarrow y\left[\cos\alpha+\sin\left(30^\circ-\alpha\right)\right]$

$-x\left[\sin\alpha+\cos\left(30^\circ-\alpha\right)\right]=2p$ … (ii)

From Eqs.(i) and (ii), we get

$$\frac{b}{\cos\alpha+\sin\left(30^\circ-\alpha\right)}=\frac{a}{\left(\sin\alpha+\cos\left(30^\circ-\alpha\right)\right)}$$

$$=\frac{1}{2p}$$

$$\Rightarrow \frac{\sqrt{a^2+b^2}}{\sqrt{2+1}}=\frac{1}{2p}$$

$$\Rightarrow a^2+b^2=\frac{3}{4p^2}$$

64. (A)

Sol. Let the equation $\frac{x}{a}+\frac{y}{b}=1$

given b = 2a

$\therefore \frac{x}{a}+\frac{y}{2a}=1$ … (1)

it passes through (1, 2)

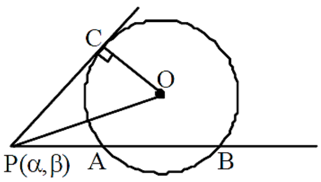
$\therefore \frac{1}{a}+\frac{2}{2a}=1$

a = 2

$\frac{x}{2}+\frac{y}{4}=1$ $\Rightarrow$ 2x + y = 4

65. (A)

Sol: From the figure, $PA.PB=$ constant



Also $PA_* PB=PC^2$

But $PC^2=OP^2-OC^2$

$=\alpha^2+\beta^2-a^2$

$\Rightarrow PA.PB=\alpha^2+\beta^2-a^2$

66. (C)

Sol: $|x|=2\quad ;|y|=-3$

$\left|(\text{adj}(\text{adj}(\text{adj}\,x)))\left(\text{adj}\,y^{-1}\right)\right|=|\mathbf{x}|^{2^3}\left|y^{-1}\right|^2$

$=2^8\times\frac{1}{9}=\frac{256}{9}$

67. (A)

Sol: Let $S\equiv x^2+y^2-15x+5y=0$ ... (i)

Any point circle through $(1,2)$ is given as $S_1\equiv(x-1)^2+(y-2)^2=0$

Family of circles passing through $S_1=0$ and $S_2=0$ is $S_1+\lambda S_2=0$ … (iii), where, λ is a parameter $(\lambda\neq-1)$.

Now, circle (iii) passes through $(0,2)\Rightarrow\lambda=-14$

Putting, $\lambda=-14$ in (iii), we get the required equation.

68. (B)

Sol:

$$\lim_{n\to-\infty}\frac{x^2-x^2-3x\cos\frac{1}{|x|}}{x-\sqrt{x^2+3x\cos\frac{1}{|x|}}}\ \text{(Rationalize)}$$

$$=\lim_{n\to-\infty}\frac{-3x\cos\frac{1}{|x|}}{x-\sqrt{x^2}\sqrt{1+\frac{3\cos\frac{1}{|x|}}{x}}}$$

$$=\lim_{n\to-\infty}\frac{-3x\cos\frac{1}{|x|}}{x-x\sqrt{1+\frac{3\cos\frac{1}{|x|}}{x}}}$$

$$\left(\sqrt{x^2}=-x,\ \text{as } x\to\infty\right)=\frac{-3}{2}$$

69. (C)

Sol: $f'(x)=\frac{1}{2}\left[\frac{(1-\cos x)2x-x^2\sin x}{(1-\cos x)^2}\right]=\frac{x}{2}\left[\frac{2(1-\cos x)-x\sin x}{(1-\cos)^2}\right]$

$$=4\sin^2\frac{x}{2}-2x\sin\frac{x}{2}\cos\frac{x}{2}=2x\sin\frac{x}{2}\cos\frac{x}{2}\left[\frac{\tan\frac{x}{2}}{\frac{x}{2}}-1\right]>0\Rightarrow f\ \text{is increasing}$$

$$g'(x)=\frac{1}{6}\left[\frac{(x-\sin x)2x-x^2(1-\cos x)}{(x-\sin x)^2}\right]=2x\cos^2 x-4\sin x\cos x$$

$$=2x\cos^2 x\left[1-\frac{\tan\frac{x}{2}}{\frac{x}{2}}\right]<0\Rightarrow g\ \text{is decreasing}$$

70. (D)

$$\frac{1}{a}\int_0^a(\sqrt{x+a}-\sqrt{x})dx=\int_0^{\pi/8}\frac{2\tan\theta}{2\tan\theta}\cdot\sec^2\theta d\theta$$

$$\left(\sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

Sol:

$$=\frac{1}{a}\cdot\frac{2}{3}\left[(x+a)^{3/2}x^{3/2}\right]_0^a=\tan\theta\Big|_0^{\pi/8}$$

$$=\frac{2}{3a}\left[(2a)^{3/2}-a^{3/2}-a^{3/2}\right]=(\sqrt{2}-1)$$

$$=\frac{2}{3a}\cdot 2a^{3/2}[\sqrt{2}-1]=\sqrt{2}-1\quad\Rightarrow\frac{4}{3}\sqrt{a}=1\quad\Rightarrow\quad a=\frac{9}{16}$$

71. (B)

Sol: $\frac{dy}{dx}=1-\frac{1}{x^2}$

$\int dy=\int\left(1-\frac{1}{x^2}\right)dx; y=x+\frac{1}{x}+c$

at $\left(2,\frac{7}{2}\right);\frac{7}{2}=2+\frac{1}{2}+c$ or $c=1$

$\therefore\quad y=x+\frac{1}{x}+1$ or $xy=x^2+1+x$

72. (B)

Sol: Put $x+y=v$

$\frac{dy}{dx}=\frac{dv}{dx}-1$

Then, the given equation can be written as $\frac{dv}{dx}-1=\frac{v+7}{2v+3}$

or $\frac{dv}{dx}=\frac{3v+10}{2v+3}$

or $\frac{(2v+3)}{(3v+10)}dv=dx$

or $\frac{9(2v+3)}{(3v+10)}dv=9dx$

or $\frac{(6(3v+10)-33)}{(3v+10)}dv=9dx$

$\Rightarrow\left\{6-\frac{33}{3v+10}\right\}dv=9dx$

On integrating, we get $6v-11\ln 3v+10=9x+c$

or $6(x+y)-11\ln(3(x+y)+10)=9x+c$

73. (C)

Sol: $A(2,1);\quad B(1,4);\quad C(4,5);\quad D(5,2)$

$m_{AB}=\frac{3}{-1}=-3;\quad m_{BC}=\frac{1}{3};\quad m_{CD}=\frac{-3}{1}=-3$

$AB=\sqrt{9+1}-\sqrt{10};\quad BC=\sqrt{1+9}-\sqrt{10}$

Hence, a square

74. (C)

Sol: Put $1+\sin 2x\sin\theta=t^2\Rightarrow\cos\theta d\theta=\frac{2tdt}{\sin 2x}$

$\therefore\quad f(x)=\int_{\sqrt{1-\sin 2x}}^{\sqrt{1+\sin 2x}}\frac{2\left(t^2-1\right)}{\sin 2x}\cdot\frac{1}{t}\cdot\frac{2tdt}{\sin 2x}$

$\Rightarrow\quad f(x)=\int_{-(\cos x-\sin x)}^{\cos x+\sin x}\frac{4\left(t^2-1\right)}{\sin^2 2x}dt$

$\Rightarrow\quad f(x)=-\frac{4}{3}\cot x\operatorname{cosec} x\quad\int f(x)dx=\frac{4}{3}\operatorname{cosec} x+c$

75. (D)

Sol: The given system of equations will have a non-trivial solution if $\begin{vmatrix} \alpha+a & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

we get $\begin{vmatrix} \alpha+a & \alpha & \alpha \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$

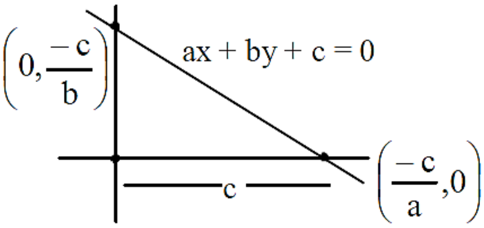
$\Rightarrow \alpha ab + c(\alpha b + ab + a\alpha) = 0$

$\Rightarrow \alpha(bc + ca + ab) + abc = 0$

$\Rightarrow \dfrac{1}{\alpha} = -\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right) \quad (\because a, b, c \neq 0)$

76. (D)

Sol: $abc \neq 0$



$\dfrac{-c}{a} > 0, \quad \dfrac{-c}{b} > 0$

$\dfrac{c}{a} < 0, \quad \dfrac{c}{b} < 0$

77. (D)

Sol. $\int_0^a f(x)\, g(x)\, h(x)\, dx$

$= \int_0^a f(a-x)\, g(a-x)\, h(a-x)\, dx$

$= -\int_0^a f(x)\, g(x) \left[\dfrac{3h(x)-5}{4}\right] dx$

$= -\dfrac{3}{4}\int_0^a f(x)\, g(x)\, h(x)\, dx + \dfrac{5}{4}\int_0^a f(x)\, g(x)\, dx$

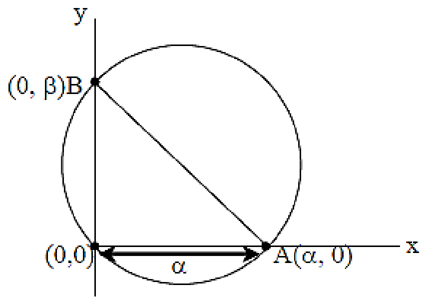
$\dfrac{7}{4}\int_0^a f(x)\, g(x)\, h(x)\, dx = \dfrac{5}{4}\int_0^a f(x)\, g(x)\, dx = 0$

$\{f(a-x)\, g(a-x) = -f(x)\, g(x)\}$

So $\int_0^a f(x)\, g(x)\, h(x)\, dx = 0$

78. (A)

Sol: Equation of circle $(x-\alpha)(x-0)+(y-0)(y-\beta)=0$



$$x^2+y^2-\alpha x-\beta y=0$$

$$a=\sqrt{\frac{\alpha^2}{4}+\frac{\beta^2}{4}}$$

$$\alpha^2\beta+^2=4a^2$$

Let centroid be (x,y)

$$x=\alpha/3, y=\beta/3$$

$$\alpha=3x, \beta=3y$$

$$9\left(x^2+y^2\right)=4a^2$$

79. (B)

Sol: Put $y=vx$

$$v+\frac{xdv}{dx}=\frac{4v^2+4v+1}{4}\Rightarrow\frac{xdv}{dx}=\frac{4v^2+1}{4}$$

$$\int\frac{dv}{v^2+\frac{1}{4}}=\ln\ x+c$$

$$2\tan^{-1}(2v)=\ln\ x+C$$

$$x=1,\quad y=0$$

$$\therefore\quad C=0$$

$$2v=\tan\left(\frac{1}{2}\ln x\right).$$

80. (C)

Sol: $m_1+m_2=-\frac{2h}{b}$ .....(1)

$m_1m_2=\frac{a}{b}$ .....(2)

Solove (1) & (2) $\Rightarrow\ \frac{h^2}{ab}=\frac{4}{3}$

81. 5

Sol: $2\vec{a}[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]+\vec{b}[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$

$=2[\vec{a}\times(\vec{a}\times\vec{b})\cdot(\vec{a}-2\vec{b})]+[\vec{b}\times(\vec{a}\times\vec{b})\cdot(\vec{a}-2\vec{b})]$

$=-2\vec{b}\ (\vec{a}-2\vec{b})+(\vec{a})(\vec{a}-2\vec{b})=5$

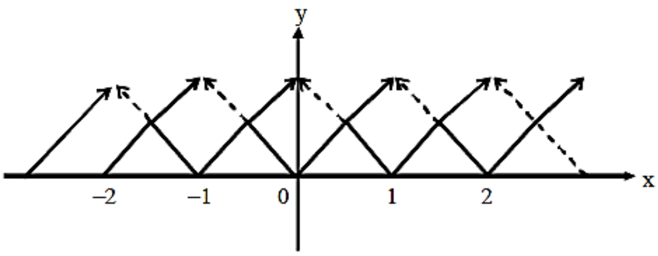
82. 0

Sol: $a^3+b^3+c^3=3abc$ [By sine rule]

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a^3+b^3+c^3-3abc=0$$

83. 50

Sol:



The graph with solid line is the graph of f(x) = {x} and the graph with dotted lines is the graph of $f(x)=\{-x\}$. Now the graph of $\min(\{x\},\{-x\})$ is the graph with dark solid lines. $\int_{-\infty}^{100} f(x)dx=$ area of 200 triangles shown as solid dark lines in the diagram $=200,1/2\ (1)\left(\frac{1}{2}\right)=50$.

84. 8

Sol: $I-\int \frac{\left(f'g-g'f\right)/g^2}{(f/g+1)\sqrt{f/g-1}}dx$

Let $f/g=t \Rightarrow \frac{f'g-g'f}{g^2}dx-dt$

$=\int \frac{dt}{(t+1)\sqrt{t-1}}$

Let $t-1=z^2 \Rightarrow dt=2zdz$

$-\int \frac{2zdz}{\left(z^2+2\right)z}-2\int \frac{dz}{z^2+2}$

$=\sqrt{2}\tan^{-1}\frac{z}{\sqrt{2}}+c$

$=\sqrt{2}\tan^{-1}\sqrt{\frac{f-g}{2g}}+c$

85. 7

Sol. $\sum x_i = 14$

$\sum x_i^2 = \left(\sum x_i\right)^2 - 2\sum x_1x_2$

$= 196 - 2 \cdot 70 = 56$

$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2\right) - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2$

$\Rightarrow 4n^2 - 56n + (14)^2 = 0$

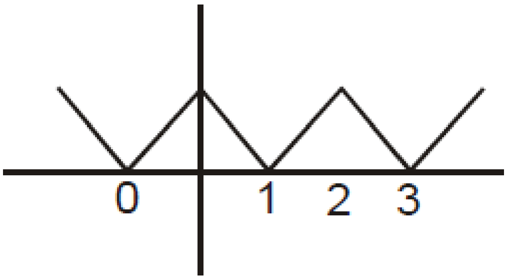
$\Rightarrow 4(n-7)^2 = 0$

$\Rightarrow n = 7$ **Ans.**]

86. 4

Sol: $f(x)-\begin{cases} x-[x] & , \quad \text{if } [x] \text{ is odd} \\ 1+[x]-x, & \quad \text{if } [x] \text{ is even} \end{cases}$; $[x - x] = \{x\}$, $1 + [x] - x = 1 - \{x\}$

so, $f(x)=\begin{cases} \{x\} & , \quad f(x) \text{ is odd} \\ 1+\{x\}, & \quad f(x) \text{ is even} \end{cases}$



$f(x)$ is a function with period '2' its curve is shown below

so, $\dfrac{\pi^2}{10}\displaystyle\int_{-10}^{10} f(x)\cos \pi x \, dx$

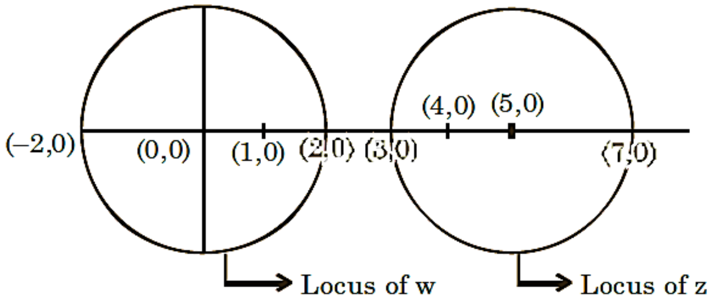
$=\dfrac{\pi^2}{10}\times 2\displaystyle\int_0^{10} f(x)\cos \pi x \ dx = \dfrac{\pi^2}{5}\times 5\int_0^2 f(x)\cos \pi x \ dx$

$=\pi^2\left[\displaystyle\int_0^1 (1-x)\cos \pi x dx + \int_1^2 (x-1)\cos \pi x \, dx\right]$

$=\pi^2\left(\dfrac{4}{\pi^2}\right)=4$

87. 10

Sol: $\left|\dfrac{Z-1}{Z-4}\right| = 2$ and $\left|\dfrac{w-4}{w-1}\right| = 2$



$\therefore |z - w|_{max} = 9$, $|z - w|_{min} = 1$

88. 8

Sol: $A = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

$|\text{adj (adj A)}| = |A|^4 (4a^2b^2c^2) = \boxed{\lambda = 8}$

89. 2

Sol: For some $\lambda \in R - \{0, 1\}$

$$\lim_{x \to 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L$$

$$\Rightarrow \quad L = \lim_{x \to 0^-} \left| \frac{1 - x - x}{\lambda - 1} \right| = \lim_{x \to 0^+} \left| \frac{1 - x + x}{\lambda - 0} \right|$$

$$\Rightarrow \quad L = \frac{1}{|\lambda - 1|} = \frac{1}{|\lambda|}$$

$$\because \quad |\lambda - 1| = |\lambda| \Rightarrow = \frac{1}{2}$$

$$\therefore \quad L = 2$$

90. 9

Sol: Since, surface area of cube, $A = 6a^2$cmit is given, $\frac{dA}{dt} = 3.6cm^2/sec$

$$\Rightarrow \quad 12a \frac{da}{dt} = 3.6 \text{ cm}^2/\text{sec} \qquad \text{....(i)}$$

Now, as volume of cube, $v = a^2$cm $\therefore \; \frac{dv}{dt} = 3a^2 \frac{da}{dt} = 3a^2 \frac{3.6}{12a}$ [from Eq.(i)]

So, at a = 10cm, $\frac{dA}{dt} = 0.9 \times 10 = 9cm^3/sec$