ANSWERS WITH EXPLANATION

Physics

1. (2) Given that :

$x = at^3 \rightarrow v_x = \frac{dx}{dt} = 3at^2 \rightarrow a_x = \frac{dv_x}{dt} = 6at$

and $y = 2at \rightarrow v_y = \frac{dy}{dt} = 2a \rightarrow a_y = \frac{dv_y}{dt} = 0.$

$\therefore \quad a^2 = a_x^2 + a_y^2$

$a^2 = ax^2 + 0$

$\therefore \quad a = ax$

$a = 6at$

$at\ t = 1$

$a = 6a$

2. (4) We define the mean free path as the average distance a gas particle travels before colliding.

$\therefore \quad n = [M^0 L^{-3} T^0]\ d = [M^0 L^1 T^0]$

$\therefore \frac{1}{\sqrt{2}nd^2} = \frac{1}{[M^0L^{-3}T^0]}[M^0L^1T^0]^2 = [M^0L^1T^0]$

3. (3) **Hm^{-1}**

Magnetic Permeability formula is given as–

Magnetic Permeability $(\mu) = \frac{B}{H}$

Where, B = Magnetic Intensity

H = Magnetising field

S.I. Unit → Henry Per meter $\left(H/_m\right)$ or Newton per ampere square.

So, Answer is Hm^{-1} or NA^{-2}

4. (2) Given : T – time period, H – maximum height

$$\frac{H}{T} = \frac{\frac{u^2\sin^2\theta}{2g}}{\frac{2u\sin\theta}{g}} = \frac{1}{4}u\sin\theta.$$

5. (3) The two projected particle will collide if the the particle will be at the same position in the same time.

$$t = \frac{x}{u_1\cos\theta_1 - u_2\cos\theta_2} \quad ...(1)$$

And $u_1\sin\theta_1 \cdot t - \frac{1}{2}gt^2 = u_2\sin\theta_2 \cdot t - \frac{1}{2}gt^2$

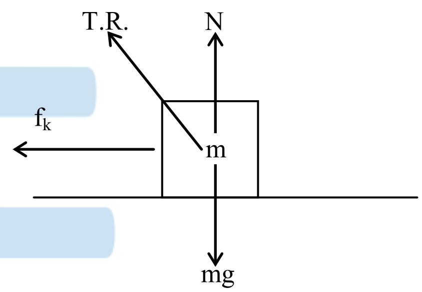
$\therefore \quad u_1\sin\theta_1 = u_2\sin\theta_2 \quad ...(2)$

from equation no. (1) & (2)

$$t = \frac{x}{u_1\cos\theta_1 - \frac{u_1\sin\theta_1\cdot\cos\theta_2}{\sin\theta_2}}$$

$$\therefore \quad t = \frac{x\sin\theta_2}{u_1\sin(\theta_2 - \theta_1)}$$

6. (1)



The only electromagnetic forces acting on the box are the normal force and the friction.

$f_k = \mu mg$

$T.R. = \sqrt{N^2 + f^2}$

$= \sqrt{(mg)^2 + (\mu mg)}$

$= mg\sqrt{1+\mu^2}$

7. (2) Given, $f = 1200$ rpm

Now, converting in seconds

and $f = \frac{1200}{60} = 20$ rps

Angular velocity of particle.

$\omega = 2\pi f$

$= 20 \times 2\pi = 40\pi$ rod/s

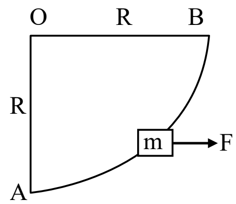
Now, acceleration $a = \omega^2 r$

$= (40\pi)^2 \times \frac{30}{100}$

$= 4737\ m/s^2$

$\sim 4740\ m/s^2$

8. (1)



A block of mass m is taken from A to B under constant force F.

$\therefore$ work done $W = \vec{F}\cdot\vec{d}$

$$W_{AB} = \vec{F}x\cdot\vec{R}x + \vec{F}x\cdot\vec{R}y$$

$$= FR\cos 0^\circ + FR\cos 90^\circ$$

$$\Rightarrow W_{AB} = FR$$

9. (4) Let the final speed of the trolley becomes V

Initial momentum of the system in x direction = (20 + 40) × 8 when the monkey jumped off from the trolley, then momentum along x direction =(40) V

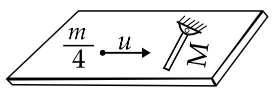
Now, applying the conservation of momentum along the x axis,

60 × 8 = 40V

V = 12 m/s

Hence, speed of the trolley will become 12 m/s.

10. (1)



Angular momentum of bullet with respect to the poivot :

$$L_B = \frac{m}{4}u\left(\frac{L}{2}\right) = \frac{muL}{8}$$

Angular momentum of system when the bullet hits rod :

$$L_S = [I_B + I_R]\,\omega = \left[\frac{m}{4}\left(\frac{L}{2}\right)^2 + \frac{1}{3}mL^2\right]\omega$$

$$= \frac{19}{48}mL^2\omega$$

By conservation of angular momentum

$$\frac{mul}{8} = \frac{19}{48}ml^2\omega$$

$$\omega = \frac{6}{19}\frac{u}{l}$$

11. (3)

$$g_e = \frac{Gm_e}{R^2}$$

(Given R → Radius of earth,

ρ → mean density of earth)

$$g_e = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2}\ ;\ g_e = \frac{4}{3}\pi R\rho G$$

$$\rho = \frac{3}{4}\cdot\frac{g_e}{\pi RG}$$

12. (3) $X = 7\cos 0.5\,\pi t$ general form of equation.

$X = A\cos\omega t$

Given equation compare with standard form

$$\omega = \frac{2\pi}{T} = 0.5\,\pi;$$

$$T = 4\text{ s}$$

$$T' = \frac{T}{4} = \frac{4}{4} = 1\text{ s}$$

13. (1)

$$Y = \frac{\frac{F}{A}}{\frac{\ell}{L}} = \frac{FL}{A\ell} \text{ on comparing}$$

$$\Rightarrow F = \frac{YA}{L}\ell$$

$$\Rightarrow F = kx$$

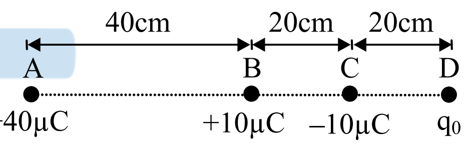
$$k = \frac{YA}{L}$$

14. (2) Liquid pressure is always normal to the surface

Hence thrust per unit area at point

P = (H – h) ρg.

15. (3)



Force on B due to A (+x) $f_{AB} = \frac{Kq_Aq_B}{r_{AB}^2}$

$$= \frac{9\times10^9\times40\times10^{-6}\times10\times10^{-6}}{\left(40\times10^{-2}\right)^2} = \frac{90}{4}$$

$$= 22.5\text{ N}$$

Force B due to C (+x)$f_{CB} = \frac{Kq_Cq_B}{r_{BC}^2}$

$$= \frac{9\times10^9\times10\times10^{-6}\times10\times10^{-6}}{\left(20\times10^{-2}\right)^2} = \frac{90}{4} = 22.5\text{ N}$$

Force on B due to D (–x) $f_{BD} = \frac{Kq_Bq_D}{r_{BD}^2}$

$$= \frac{9\times10^9\times10\times10^{-6}\times q_0}{\left(40\times10^{-2}\right)^2} = \frac{9}{16}\times10^6\,q_0$$

For the equilibrium,

$$22.5 + 22.5 = \frac{9}{16}\times10^6\times q_0$$

$$= \frac{45\times16}{9\times10^6} = q_0$$

$$q_0 = +80\times10^{-6}$$

$$q_0 = +80\mu C$$

16. (3) Hence, the correct option is (3).

$$E = \frac{d\phi}{dt} \Rightarrow \int_{\phi_1}^{\phi_2} d\phi = \Delta\phi = \int E dt$$

The total charge flown in the loop $q = \int I dt$

or $q = \int \frac{E}{R} dt = \frac{\Delta\phi}{R} = \frac{B\pi r^2}{R}$

$q \propto B$, $q \propto r^2$, and $q \propto \frac{1}{R}$

17. (2) Given $I = 1.1$ A

$e = 1.6 \times 10^{-19}\ C$

$A = \pi r^2 = 3.14 \times (0.05)^2$

$= 78.5 \times 10^{-4}\ cm^2$

$n = \frac{6 \times 10^{23}}{7\ cm^3} = 0.86 \times 10^{23}$

$v_d = \frac{I}{neA}$

$= 0.86 \times 10^{23} / m^3$

$v_d = \frac{1.1}{0.86 \times 1.6 \times 10^{-19} \times 78.5 \times 10^{-4}}$

(volume of 63g Cu)

$v_d = 0.01$ cm/s

$= 0.1$ mm/s

18. (2) Refer to figure given in question
For magnetic field at centre to be zero

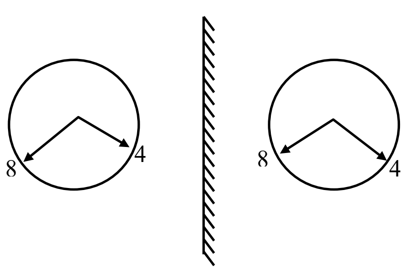
$$\frac{\mu_0 I dl}{4\pi R^2} = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R}$$

$$\frac{\mu_0 I R\theta}{4\pi R^2} = \frac{\mu_0 I}{4\pi}\left[\frac{1}{R} + \frac{1}{R}\right]$$

$\because$ $dl = R\theta$

or $\theta = 2$ rad

19. (2)



Mirror turns left into right and right into left. Hence actual time is 3 : 40

20. (1) $L = \frac{nh}{2\pi}$; for $n = 1$, $L = \frac{h}{2\pi}$

21. [4.00] ${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + e^- + \overline{v}$

$Q = [m(^{23}NF) - m(^{23}Na)] \times 931.5$ MeV

$Q = 4.375$ MeV $= 4.4$ MeV

$Q = 4$ MeV

$Q = KE_y + KE_e + E\overline{v}$

KE_y is very very small

$Q \approx KE_e + E\overline{v}$

when KE_e is maximum $E\overline{v}$ is negligible

$KE_e = Q = 4$ MeV

22. [8.00] $eV_s = h\nu - W$

$= 12eV - 4eV$

$eV_s = 8eV$ or $V_s = 8V$

23. [7.00] As per Snell's rule,

$\sin(i)/\sin(r) = n_2/n_1$

Here, $i = 30°$

$r = \sin^{-1}(5/6)$

$\sin r = 5/6$

$n_2 = ?$

$n_1 = 5/3$

$$\frac{5}{3}\sin 30° = n_2 . \frac{5}{6} \Rightarrow n_2 = 1$$

$$\frac{n_2}{n_1} = \sin c$$

$$\frac{1}{5/3} = \sin c$$

$$\frac{3}{5} = \sin c$$

$c = 37°$

So, required difference in angle of incidence be $37° - 30° = 7$

24. [5.00] $10^{-10} - x$ | x $\quad -x$ | $2 \times 10^{-10} + x$

A B

Let the induced charge be x,
At the steady state, the potential will be equal, we know that potential due to charged plate $V = [\frac{Q}{2}(\varepsilon \times S)] \times d$

$10^{-10} - x = 2 \times 10^{-10} + x$

$2x = -10^{-10}$

$x = -5 \times 10^{-11}$ C

25. [1.00] Instantaneous flux

$= \pi a^2 B \cos 0° + \pi b^2 B \cos 180°$

$= \pi(a^2 - b^2) B$

$\phi = \pi(a^2 - b^2) B_0 \sin \omega t$

$l = \frac{d\phi}{dt}$

$i = \frac{\ell}{R}$

$i = \frac{\pi(a^2 - b^2) B_0 \omega \cos \omega t}{R}$

$R = \rho \times 2\pi(a + b)$

$$\therefore \quad i_{max} = \frac{1}{2\ell}(a-b)B_0\omega = 1 \text{ Amp}$$

26. [8.00] At resonance reactance = 0

$$I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} \text{ Amp.}$$

$$V_L = I \times X_L = I \times \omega L$$

$$\therefore \quad L = \frac{V_L}{I\omega} \quad \ldots(1)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{L\omega_0^2} \quad \ldots(2)$$

Calculate L and C from (1) and (2) current will lag the applied voltage by 45°

$$\text{if } \tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Solve for ω

$$\omega = 8 \times 10^5 \text{ rad/s}$$

27. [1375] Relation between temperature on the unknown X scale to unknown scale Y,

$(X - T_2)/(T_2 - T_1) = (Y - T_3)/(T_4 - T_3)$

Here, $T_1 = 375X$

$T_2 = -125X$

$T_3 = -70Y$

$T_4 = -30Y$

$Y = 50$

$X = ?$

Now, substituting the values in the equation

$$\frac{X-(-125)}{500} = \frac{Y-(-70)}{40}$$

if $Y = 50$

$X = 1375°X$

28. [140]

$$\frac{\Delta Q}{W} = \frac{nC_p\Delta T}{nR\Delta T}$$

$$\Rightarrow \quad \Delta Q = \frac{C_p}{R} \cdot W$$

$$= \frac{7}{2} \times 20$$

$$= 140 \text{ J}$$

29. [1.00] Work done by gas

$$= \pi \cdot \frac{\left(\frac{400}{\pi} \times 10^3\right)}{2} \times \frac{\left(20 \times 10^{-6}\right)}{2} \text{J} = 1 \text{ J}$$

30. [2.00]

As $I = neAv_d$

$$v_d = \frac{I}{neA}$$

$$= \frac{1.5}{\left(9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}\right)}$$

$= 0.02 \times 10^{-3}$ m/s.

$= 2.0 \times 10^{-5}$ m/s.

Chemistry

31. (3)

Given:

Enthalpy of combustion of benzene

$= -3268 \text{ kJ mol}^{-1}$

Enthalpy of combustion of acetylene

$= -1300 \text{ kJ mol}^{-1}$

The change in enthalpy for the reaction

$$3C_2H_{2\,(g)} \rightarrow C_6H_6(l)$$

$\Delta H = \Delta H_{reactant} - \Delta H_{product}$

$= 3 \times (-1300 \text{ kJ mol}^{-1}) - (-3268 \text{ kJ mol}^{-1})$

$= -3900 \text{ kJ mol}^{-1} + 3268 \text{ kJ mol}^{-1}$

$= -632$ kJ/mole

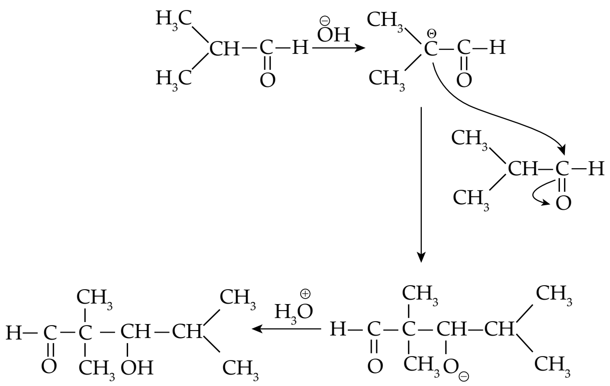
The change in enthalpy for the reaction

$3C_2H_{2\,(g)} \rightarrow C_6H_6(l)$ is -632 kJ/mole.

32. (1) Molecules that have identical hybridisation would have identical shapes.

	Type of hybridization	Geometry
$BeCl_2$	Cl –(s)– Be –(p)– Cl $H = \frac{1}{2}(2 + 2 - 0 + 0) = 2$ $H = 2 = sp$	Linear
XeF_2	F –(d)– Xe –(p)– F (lone pairs: s, p, p) $H = \frac{1}{2}(8 + 2 - 0 + 0) = 5$ $H = 5 = sp^3d$	Linear
CO_2	O =(s)= C =(p)= O $H = \frac{1}{2}(4 + 0 - 0 + 0) = 2$ $H = 2 = sp$	Linear

33. (2)



Aldol formation takes place.

An aldol condensation is a reaction in which an enol or an enolate ion reacts with a carbonyl compound to form a β–hydroxyaldehyde or β–hydroxyketone,

34. (3) According to Nernst equation

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{n}\log\frac{[P]}{[R]}$$

For the given cell

$$E_{cell} = 0.576V,\ E^{\circ}_{cell} = 0.34V$$

$$n = 2$$

$$0.576 = 0.34 - \frac{0.0591}{2}\log\frac{[H^+]^2}{[Cu^{2+}]}$$

$$0.236 = \frac{0.0591}{2} \times 2\log\frac{[H^+]}{0.01}$$

$$3.993 = [\log H^+ - \log 0.01]$$

$$3.993 = \log 10^{-2} - \log H^+$$

$$3.993 = -2 - \log H^+$$

$$5.993 = -\log [H^+]$$

Also $pH = -\log [H^+]$

$$= 5.993 \approx 6.$$

35. (1) Thin-layer chromatography (TLC) is an adsorption chromatography technique used to separate non-volatile mixtures. Thin-layer chromatography is performed on a sheet of glass, plastic, or aluminium foil, which is coated with a thin layer of adsorbent material, usually silica gel, aluminium oxide(alumina) or cellulose. This layer of adsorbent is known as the stationary phase.

36. (1) Here, $n = 2$

$V_1 = 15\ l$

$V_2 = 50\ l$

Temperature, T = 25° = 298 K

Pressure, P = 1 atm

Work done $= -P(V_2 - V_1)$

$= -1\,(50 - 15)$

$= -1 \times 35$

$= -35\ l$/atm

As 1 l/atm = 101.3 J

Therefore, $-35\ l$ atm $= -35 \times 101.3$

$= -3545.5$ J

As 1 calorie = 4.184 J

So, $$-3545.5J = \frac{-3545.5}{4.184}\text{ cal}$$

$= -848.2$ cals

37. (4) N_2O

$2x - 2 = 0$

$\Rightarrow \quad x = +1$

$H_3\,\underline{P}O_2$

$3\,(1) + x + 2\,(-2) = 0$

$x = 4 - 3 = +1$

38. (4) $N_2O_4 \rightleftharpoons 2NO_2$

At equilibrium $\frac{0.2}{2}$ $\qquad \frac{2\times10^{-3}}{2}$ (2ℓ)

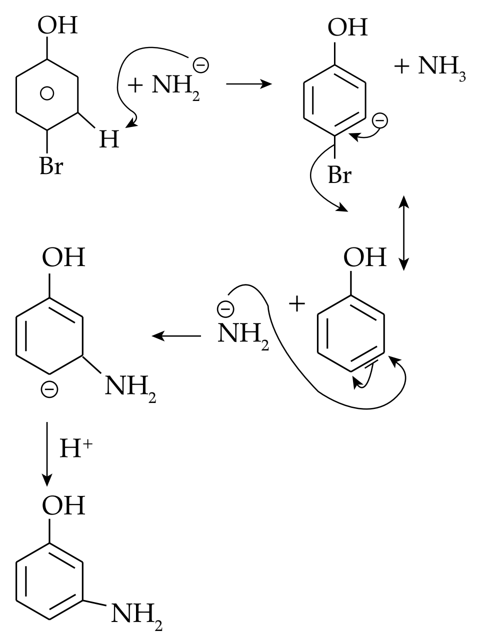
$0.1 \qquad 1 \times 10^{-3}$

$$Kc = \frac{(1\times10^{-3})^2}{0.1} = 10^{-5}$$

39. (1) Boric acid H_3BO_3, is monobasic and works as Lewis acid according to the following reaction.

$$H_3BO_3 + H_2O \rightarrow B(OH)_4^- + H^-$$

40. (4)



41. (2) $CH_3-\overset{Cl}{\overset{|}{C}H}-CH_3 \xrightarrow{\text{Alco. KOH/Heat}} CH_3CH{=}CH_2$

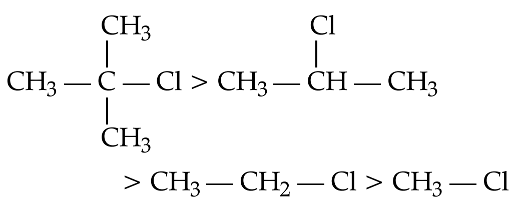
2-chloropropane

$\xrightarrow[\text{Peroxide}]{\text{HBr}} CH_3-CH_2-CH_2-Br \xrightarrow{\text{aq. KOH}}$

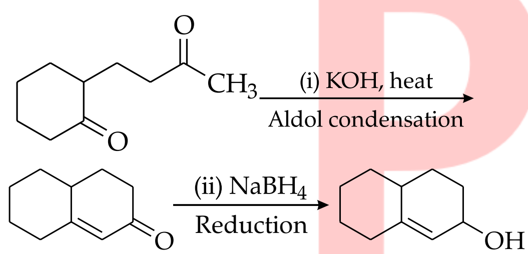
$CH_3-CH_2-CH_2-OH$

propan-1-ol

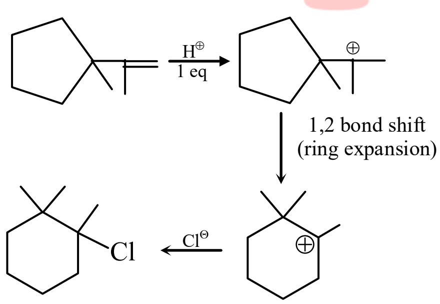
42. (2) The order of reactivity in S_N1 reaction is mainly dependant on stability of carbocation, formed thus the order of reactivity of the given compounds are as follows.



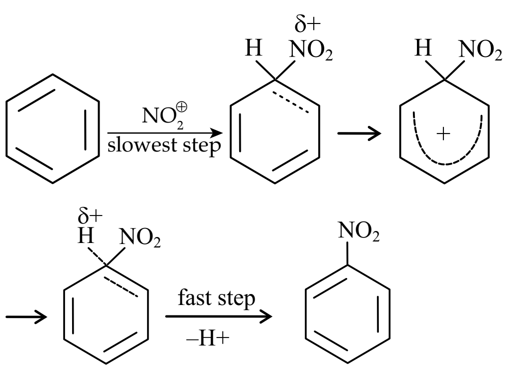
43. (4)



44. (3)

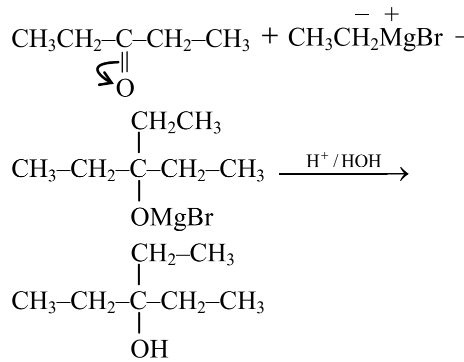


45. (3)



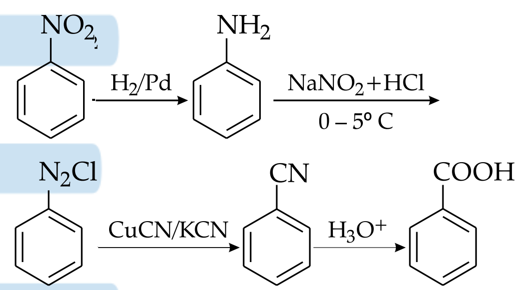
The rate determining step in electrophilic substitution reaction, is the bonding of the electrophile to the aromatic ring without cleavage of C – H or C – D bond.
This bond is broken in the fast step (second step) that restore the stable aromatic system. Also the bond strength of C-H and C-D bonds are equal. Hence the rate of Nitration of benzene is almost the same as that of Hexa deuterobenzene.

46. (3)

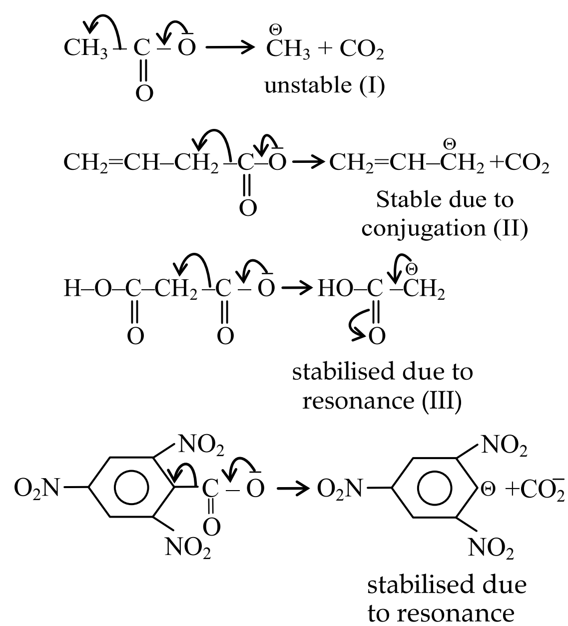


3-ethylpentan-3-ol

47. (1)

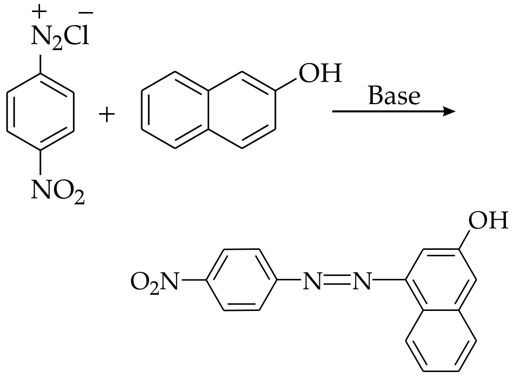


48. (3) The reactivity of decarboxylation depends upon the stability of the conjugate base .The conjugate bases of the given compounds are as follows:

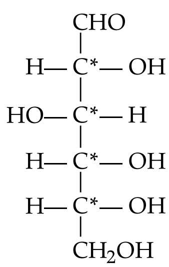


(IV) conjugate base is more stable than (III) as it has more resonating structures. Therefore, stability of carbanion α-decarboxylation is IV > III > II > I.

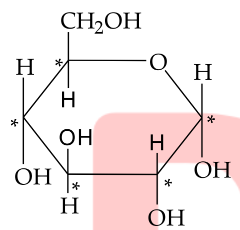
49. (2)



50. (1)



D–(+)–Glucose



α–D–(+)–Glucopyranose
(Haworth structure)

There are four asymmetric carbon in open structure of glucose,whereas there are five asymmetric carbon in cyclic form.

51. [17.49] Given $k = 0.008\ \text{min}^{-1}$

From unit of k, the reaction is a first order reaction.

From ∴ $k = \dfrac{2.303}{t} \log \dfrac{V_\infty}{V_\infty - V}$

$$\Rightarrow \quad 0.008 = \frac{2.303}{20} \log \frac{V_\infty}{V_\infty - 16}$$

$$\Rightarrow \quad 0.0695 = \log \frac{V_\infty}{V_\infty - 16}$$

$$\Rightarrow \quad V_\infty = 17.49\ \text{mL}$$

52. [13.842] According to Gibb's Helmholtz equation,

heat of reaction ΔH, given as

$$\Delta H = nF\left[T\left(\frac{\delta E}{\delta T}\right)_P - E\right]$$

$$T = (273 + 25)\ K$$
$$= 298\ K,\ n = 2,$$

$$F = 96500\ C,\ E = 0.03\ CV$$

$$\left(\frac{\delta E}{\delta T}\right)_P = -1.4 \times 10^{-4}\ V/K$$

$$\Delta H = 2 \times 96500\ [298 \times (-1.4 \times 10^{-4})] - 0.03$$
$$= -13842\ J = -13.842\ kJ/mole$$

53. [40]

The given reaction is of the first order with respect to A and of zero order with respect to B. Therefore, the rate of the reaction is given by,

rate $(r) = K[x]^a[y]^b$

k = rate constant

Given that a = 1; b = 0

For experiment I :

$r_I = k[0.1]^a[0.1]^b = 2 \times 10^{-3}$

$k[0.1]^1[0.1]^0 = 2 \times 10^{-3}$ …(i)

For experiment II :

$r_{II} = k[L]^1[0.2]^0 = 4 \times 10^{-3}$ …(ii)

equation (ii) ÷ equation (i)

$$\frac{L}{0.1} = \frac{4 \times 10^{-3}}{2 \times 10^{-3}}$$

L = 0.2

For experiment III:

$r_{III} = k[0.4]^1[0.4]^0 = M \times 10^{-3}$ …(iii)

For experiment IV: -

$r_{IV} = k[0.1]^1[0.2]^0 = 2 \times 10^{-3}$ …(iv)

Divide equation (iii) by equation (iv) :-

$$\frac{0.4}{0.1} = \frac{M \times 10^{-3}}{2 \times 10^{-3}} = M = 8$$

$$\frac{M}{L} = \frac{8}{0.2} = 40$$

Ratio of M and L = 40

54. [200].

$$C_{(s)} + O_{2(g)} \longrightarrow CO_{2(g)};\ \Delta H = -kJ/mol$$

From

$$Q = C\Delta T$$
$$Q = 20\ kJ \times 2$$

40 kJ of heat is released from 2.4 gm of C – atom

For 1 mole of C – atom

$$Q = \frac{40}{2.4} \times 12$$

$$Q = \frac{40}{2.4} \times 12 = 200\ kJ/mol$$

From

$$\Delta H = \Delta E + \Delta n_g RT$$

$\Delta n_g = 0,\ \Delta H = \Delta E$

$Q = \Delta H = \Delta E$

$\Delta H = 200$

55. [111.10] Assume the vapoar pressure of water = 100

$\therefore$ Vapour pressure of urea solution = 75

Weight of urea = w_1

Molecular weight of urea = Mw_1

Weight of water = w_2

Molecular weight of water = Mw_2

By Raoult's law

$$= \frac{\frac{w_1}{Mw_1}}{\frac{w_2}{Mw_2} \quad \frac{w_1}{Mw_1}}$$

So, $$\frac{100-75}{100} = \frac{\frac{w_1}{60}}{\frac{100}{18}+\frac{w_1}{60}}$$

$\Rightarrow$ Weight of urea w_1 = 111.1 g

56. [54].

Isotonic solutions are the solutions that have the same osmotic pressure.

The osmotic pressure of blood is 7.47 bar

T = 300 K

R = 0.083 L bar mol^{-1} K^{-1}

Molar mass of glucose = 180 g/mol

π = CRT

$$7.47\text{bar} = C \times 0.083 \ \text{L bar K}^{-1} \ \text{mol}^{-1} \times 300 \ \text{K}$$

$$C = \frac{7.47 \text{ bar}}{0.083 \text{ L bar K}^{-1}\text{mol}^{-1} \times 300 \text{ K}} = 0.3\text{M}$$

= 0.3 moles glucose in 1 L

Strength of Glucose $(g/L) = 0.3 \times 180 = 54 \ \ g/L$

57. [3].

Dipole moment is the measure of the polarity between two atoms in a molecule.

BeF_2 - zero Dipole moment

BF_3 - zero Dipole moment

H_2O - Non - zero Dipole moment

NH_3 - Non - zero Dipole moment

CCl_4 - zero Dipole moment

HCl - Non - zero Dipole moment

58. [5.00] EAN = 36 = 26 + 2 × x

or $2x$ = 10

x = 5.00

59. [0.00] Co = 27 = [Ar] $3d^7$ $4s^2$

Co oxidation state : +3

Co^{3+} = 24 [Ar] $3d^6$ $4s^0$

Orbitals of Co^{3+} ion	3d: ↑↓ ↑ ↑ ↑ ↑; 4s: □; 4p: □ □ □
$[Co(NH_3)_6]^{3+}$	3d: ↑↓ ↑↓ ↑↓ ↑↓ ↑↓; 4s: ↑↓; 4p: ↑↓ ↑↓ ↑↓ (d^2sp^3)
Geometry shape	Octahedral
Type of hybridization	d^2sp^3
No. of unpaired electrons (n)	0
Magnetic nature	Diamagnetic
Nature of complex	Low spin complex, inner orbital complexes
Magnetic moment calculation $\mu = \sqrt{n(n+2)}$	0

60. Correct answer is [1107].

2 mole of N_2 gas was present as inert gas.

Equilibrium pressure = 2.46 atm

PCl_5 (g) ↔ PCl_3(g) + Cl_2(g)

Initial moles

5 0 0

Equilibrium moles

$5-x$ x x

P = 2.46 atm

V = 200 Ltr

R = 0.082 L atm K^{-1}

T = 600 K

PV = nRT

$$P_{\text{equilibrium}} = \frac{(5+x)\times 0.082 \times 600}{200} = 2.46$$

x = 3

n_{total} = 10

$$K_p = \frac{P_{PCl_3} \times P_{PCl_2}}{P_{PCl_5}}$$

$$P_{PCl_3} = \frac{3}{10} \times 2.46$$

$$P_{PCl_2} = \frac{3}{10} \times 2.46$$

$$P_{PCl_5} = \frac{2}{10} \times 2.46$$

$$K_P = \frac{(0.3 \times 2.46)(0.3 \times 2.46)}{0.2 \times 2.46}$$

$$K_P = 1.107 \text{ atm} = 1107 \times 10^{-3} \text{ atm}$$

Mathematics

61. (3)

Given, $|\vec{a}| = |\vec{b}| = 1$, $\vec{b} = \vec{c} + 2(\vec{c} \times \vec{a})$

Angle between $\hat{a}$ and $\hat{c} = \frac{\pi}{12}$

So, $\vec{b} - \vec{c} = 2(\vec{c} \times \vec{a})$

$$\Rightarrow |\vec{b} - \vec{c}|^2 = 4(\vec{c} \times \vec{a})^2$$

$$\Rightarrow |\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c} = 4\left\{|\vec{c}|^2 |\vec{a}|^2 \sin^2 \frac{\pi}{12}\right\}$$

$$\Rightarrow 1 + |\vec{c}|^2 - 2\left(\vec{c} + 2(\vec{c} \times \vec{a})\right).\vec{c} = 4\left\{|\vec{c}|^2 (1)^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2\right\}$$

$$\Rightarrow 1 + |\vec{c}|^2 - 2\left(|\vec{c}|^2 + 0\right) = 4|\vec{c}|^2 \frac{(\sqrt{3}-1)^2}{8}$$

$$\Rightarrow 1 - |\vec{c}|^2 = \frac{(\sqrt{3}-1)^2 |\vec{c}|^2}{2}$$

$$\Rightarrow 2 - 2|\vec{c}|^2 = 4|\vec{c}|^2 - 2\sqrt{3}|\vec{c}|^2$$

$$\Rightarrow |\vec{c}|^2 (6 - 2\sqrt{3}) = 2$$

$$\Rightarrow |\vec{c}|^2 = \frac{2}{6-2\sqrt{3}} = \frac{1}{3-\sqrt{3}}$$

Now, $|6\vec{c}|^2 = 36 \times \frac{1}{3-\sqrt{3}}$

$$= \frac{36(3+\sqrt{3})}{6}$$

$$= 6(3+\sqrt{3})$$

Hint :

(i) Use $(\vec{p} - \vec{q})^2 = |\vec{p}|^2 + |\vec{q}|^2 - 2\vec{p}.\vec{q}$

(ii) $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}| \sin\theta$, where θ is the angle between $\vec{p}$ and $\vec{q}$.

Shortcut method:

$$\vec{b} = \vec{c} + 2(\vec{c} \times \vec{a})$$

$$\Rightarrow |\vec{b} - \vec{c}|^2 = 4(\vec{c} \times \vec{a})^2$$

$$|\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c} = 4\left\{|\vec{c}|^2 |\vec{a}|^2 \sin^2 \frac{\pi}{12}\right\}$$

$$\Rightarrow |\vec{c}|^2 = \frac{1}{3-\sqrt{3}}$$

Now, $|6\vec{c}|^2 = 36 \times \frac{1}{3-\sqrt{3}}$

$$= 6(3+\sqrt{3})$$

62. (2)

Let $S = 2\cos\frac{5\pi}{11} . \cos\frac{4\pi}{11} \cos\frac{3\pi}{11} . \cos\frac{2\pi}{11} \cos\frac{\pi}{11}$

Now $\cos\frac{3\pi}{11} = -\cos\frac{8\pi}{11}$

And $\cos\frac{5\pi}{11} = -\cos\frac{16\pi}{11}$

$$\therefore S = 2\cos\frac{\pi}{11} . \cos\frac{2\pi}{11} . \cos\frac{4\pi}{11} . \cos\frac{8\pi}{11} . \cos\frac{16\pi}{11}$$

$$= \frac{2\sin\left(32\frac{\pi}{11}\right)}{2^5 \sin\left(\frac{\pi}{11}\right)} = \left(\frac{1}{16}\right)$$

63. (1) Correct mean $= \frac{20 \times 40 - 33 + 53}{20} = 41$

64. (3) $\because \quad (b + c)^2 - a^2 = \lambda bc$

$$\Rightarrow b^2 + c^2 - a^2 + 2bc = \lambda bc$$

$$\Rightarrow 2bc \cos A + 2bc = \lambda bc$$

(from cosine rule)

$$\Rightarrow 2(\cos A + 1) = \lambda$$

$$\Rightarrow \cos A = \frac{\lambda}{2} - 1$$

But $-1 < \cos A < 1$

$\Rightarrow \quad -1 < \frac{\lambda}{2} - 1 < 1$

$\Rightarrow \quad 0 < \frac{\lambda}{2} < 2$

$\Rightarrow \quad 0 < \lambda < 4$

65. (3) $\because$ Triangle is equilateral, so

$$\Delta = \frac{\sqrt{3}}{4}a^2 \text{ and } R = \frac{a^3}{4\Delta}$$

$$\Rightarrow \quad R = \frac{a^3}{4\frac{\sqrt{3}}{4}a^2} = \frac{a}{\sqrt{3}}$$

$\because \quad a = 2\sqrt{3}$

$\Rightarrow \quad R = 2$

Shortcut Method:

We have $a = 2R \sin 60°$

$\Rightarrow \quad 2\sqrt{3} = 2R \cdot \frac{\sqrt{3}}{2}$

$\Rightarrow \quad R = 2$

66. (1) $\log_p \log_p p^{\frac{1}{p^n}} = \log_p \log_p p^{p^{-n}}$

$= \log_p p^{-n} \log_p p$

$= -n \log_p p = -n$

67. (3) $\left(\frac{9}{10}\right)^x = -(x^2 - x + 3)$

$$\Rightarrow \quad \left(\frac{9}{10}\right)^x = -\left\{\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}\right\}$$

LHS is always positive while RHS is always negative. Hence, LHS $\neq$ RHS

$\therefore$ No solution

68. (4) We know that in an A.P. $a_1 + a_{24} = a_5 + a_{20}$

$= a_{10} + a_{15} = a_{12} + a_{13}$

So, $3(a_{12} + a_{13}) = 225$

$\Rightarrow \quad a_{12} + a_{13} = 75$

Therefore,

$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_{23} + a_{24}$

$= 12\,(a_{12} + a_{13})$

$= 12 \times 75 = 900$

69. (2) $(1 - 3x + 3x^2 - x^3)^6 = (1 - x)^{18}$

If in the expansion of $(1 - x)^n$, is even, then the middle term is $\left(\frac{n+2}{2}\right)$

So, the middle term is $\frac{18+2}{2} = 10^{\text{th}}$ term

$T_{10} = {}^{18}C_9(-x)^9$

70. (4) $\Rightarrow (m + n)(m + n - 1) = 90 = 10 \times 9$

$\Rightarrow m + n = 10 \quad \ldots(i)$

and $(m - n)(m - n - 1) = 30 = 6 \times 5$

$\Rightarrow m - n = 6 \quad \ldots(ii)$

Solving eq.(i) and (ii) we get

$m = 8, \quad n = 2$

71. (4) For $x^2 + 2x + 8 > 0$ here, $D = 4 - 8(4) < 0$

$\therefore \quad x^2 + 2x + 8 > 0 \qquad \forall\, x \in R$

$-\log_{0.3}(x - 1) \geq 0$

$\Rightarrow \quad \log_{0.3}(x - 1) \leq 0$

$\Rightarrow \quad (x - 1) \geq 1$

$\Rightarrow \quad x \geq 2$

Also, $\quad x - 1 \neq 1$

$\Rightarrow \quad x \neq 2$

$\therefore$ Domain is $R \cap (2, \infty) = (2, \infty)$

72. (4) $\lim\limits_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$;

$$\lim_{x\to 0} \frac{\sqrt{2\sin^2 x}}{\sqrt{2}x} = \lim_{x\to 0} \frac{|\sin x|}{x} \left[\because \lim_{x\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

The above limit does not exist as

LHL $= -1 \neq$ RHL $= 1$

73. (2) $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ -1 \text{ if } x \text{ is irrational} \end{cases}$

Let 'a' is any rational number

$\Rightarrow \quad f(a) = 1$

Then, $\lim\limits_{\substack{x\to a \\ x\in Q}} f(x) = 1 = f(a)$

and $\lim\limits_{\substack{x\to a \\ x\to Q^C}} f(x) = -1 \neq f(a)$

$\Rightarrow f(x)$ is not continuous at any rational number.

Now, Let $a \in Q^C \Rightarrow f(a) = -1$

Then, $\lim\limits_{\substack{x\to a \\ x\in Q}} f(x) = 1 \neq f(a)$

and $\lim\limits_{\substack{x\to a \\ x\to Q^C}} f(x) = -1 = f(a)$

$\Rightarrow f(x)$ is not continuous at any irrational number.

$\therefore$ The set of points of continuity $= \phi$

74. (3) $2^x + 2^y = 2^{x+y}$

Differentiating both the sides of above equation w.r.t. x, we get

$$\Rightarrow 2^x \ln 2 + 2^y \ln 2 \frac{dy}{dx} = 2^{x+y} \ln 2\left(1+\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{\left(2^x \ln 2 - 2^y 2^x \ln 2\right)}{\left(2^y \ln 2 - 2^y 2^x \ln 2\right)}$$

$$= -2^{x-y}\left[\frac{1-2^y}{1-2^x}\right]$$

$$\Rightarrow \frac{dy}{dx} = 2^{x-y}\left[\frac{2^y-1}{1-2^x}\right]$$

75. (2) $e^{2y} = 1 + 4x^2$

Taking logartihm on both sides of the above equation

$$2y = \log_e (1+4x^2)$$

$$y = \frac{1}{2}\log_e (1+4x^2)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{1+4x^2} \times 4 \times 2x = \frac{4x}{1+4x^2}$$

$$\frac{dy}{dx} = \frac{4x}{1+4x^2} = m$$

$\Rightarrow$ $4mx^2 - 4x + m = 0$

for $x \in R$,

Discriminant ≥ 0

$\Rightarrow$ $16 - 16\, m^2 \geq 0$

$\Rightarrow$ $|m| \leq 1$

76. (4) Let $I = \int \frac{a^{2x} + b^{2x} - 2a^x b^x}{a^x b^x}\, dx$

$$= \int\left[\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x - 2\right] dx$$

$$= \left(\frac{a}{b}\right)^x / \ln\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)^x / \ln\left(\frac{b}{a}\right) - 2x + c$$

$$= \frac{\left(\frac{a}{b}\right)^x}{\ln\left(\frac{a}{b}\right)} + \frac{\left(\frac{b}{a}\right)^x}{-\ln\left(\frac{a}{b}\right)} - 2x + c$$

$$= \frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\log\left(\frac{a}{b}\right)} - 2x + c$$

77. (4) Let $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$$I = \int_0^1 \frac{dt}{(1+t)(2+t)} = \int_0^1 \left(\frac{1}{1+t} - \frac{1}{2+t}\right) dt$$

$$= \left[\ln(1+t) - \ln(2+t)\right]_0^1$$

$$= \ln 2 - \ln 3 + \ln 2 = \ln\frac{4}{3}$$

$$= \log_e \frac{4}{3}$$

78. (1) $$A = \int_{\pi/6}^{\pi/3} \sec^2 x \; dx = \left[\tan x\right]_{\pi/6}^{\pi/3}$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

79. (3) $(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{(1+y^2)}$$

It is form of linear differential equation.

$$I.F = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1+y^2} e^{\tan^{-1} y} dy$$

$$x(e^{\tan^{-1} y}) = \frac{e^{\tan^{-1} y}}{2} + c$$

$$\left[\because \int e^{2x} dx = \frac{e^{2x}}{2}\right]$$

$\therefore$ $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$ $\quad [k = 2c]$

80. (2) $$\left(\frac{1+i}{1-i}\right)^n = \left[\frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)}\right]^n$$

$$= \left[\frac{(1+i)^2}{1+1}\right]^n$$

$$= \left[\frac{1-1+2i}{2}\right]^n$$

$$= (i)^n$$

The Smallest positive integer must be 2 so that $\left(\frac{1+i}{1-i}\right)^n = -1$

81. [2.00] Let first box has exactly a and the other has exactly b white balls.

$\Rightarrow$ Probability that both balls are white

$= \dfrac{a}{20}\cdot\dfrac{b}{20} = \dfrac{21}{100}$

$\Rightarrow \quad ab = 84$

$\Rightarrow$ (a, b) is either (6, 14) or (7, 12), (14, 6), (12, 7)

But (6, 14) & (14, 6) is not possible

$\because\ a + b = 20$

$\Rightarrow$ (a, b) is (7, 12) or (12, 7)

$\Rightarrow P$(both drawn balls are black)

$= \dfrac{13}{20}\times\dfrac{8}{20}$

$= 0.26 = k$

Now, $\dfrac{100k}{13} = \dfrac{100\times 0.26}{13} = 2.00$

82. [24].

Let $B = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix}$

So, $A = I_3 + B$ & $B^2 = \begin{vmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

$B^3 = 0$ (Null matrix)

$A^n = (I + B)^n$

$A^n = I + nB + n\dfrac{(n-1)}{2}B^2 + \ldots\ldots + B^n$ **(1)**

$\Rightarrow A^n = \begin{vmatrix} 1 & na & na + \dfrac{n(n-1)ab}{2} \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix} = \begin{vmatrix} 1 & na & na + \dfrac{(n-1)n}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow na = 48, nb = 96, \dfrac{n(n-1)}{2}ab = 2160 - na$

$48\dfrac{(96-b)}{2} = 2112$

$96 - b = 88$

$\therefore\ b = 8, a = 4,$ So, $n = 12$

$n + a + b = 12 + 4 + 8 = 24$

83. [9.00] $A(\text{adj}\,A) = |A|\,I_3 = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad |A| = 3$

$|\text{adj}(\text{adj}\,A)| = 3^{(3-1)^2} = 3^4$

$|\text{adj}\,A| = 3^{(3-1)} = 3^2$

$\Rightarrow \quad \dfrac{|\text{adj}(\text{adj}\,A)|}{|\text{adj}\,A|} = \dfrac{3^4}{3^2} = 3^2 = 9$

84. [0.00] $\vec{a} + \vec{b} = \vec{c}$

$((\lambda x)\hat{i} + y\hat{j} + 4z\hat{k}) + (y\hat{i} + x\hat{j} + 3y\hat{k})$

$= -z\hat{i} - 2z\hat{j} - (\lambda + 1)x\hat{k}$

$\Rightarrow \lambda x + y + z = 0;\ x + y + 2z = 0$

and $(\lambda + 1)x + 3y + 4z = 0$

$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ 1 & 1 & 2 \\ (\lambda+1) & 3 & 4 \end{vmatrix} = 0$

$\Rightarrow \lambda(4 - 6) - (4 - 2(\lambda + 1)) + (3 - (\lambda+1)) = 0$

$\Rightarrow \quad -2\lambda - 4 + 2\lambda + 2 + 3 - \lambda - 1 = 0$

$\Rightarrow \quad -\lambda = 0$

$\Rightarrow \quad \lambda = 0$

85. [2].

Given lines $L_1 : \vec{r} = \left(-\hat{i} + 3\hat{k}\right) + \lambda\left(\hat{i} - a\hat{j}\right)$

$L_2 : \vec{r} = \left(-\hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

As we know shortest distance between two skew lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$d = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

Here, $\vec{a_1} = -\hat{i} + 3\hat{k}$

$\vec{a_2} = -\hat{j} + 2\hat{k}$

$\vec{b_1} = \hat{i} - a\hat{j}$

$\vec{b_2} = \hat{i} - \hat{j} + \hat{k}$

Now, $\vec{a_2} - \vec{a_1} = \hat{i} - \hat{j} - \hat{k}$

And $\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$= -a\hat{i} - \hat{j} + (a-1)\hat{k}$

So, $\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right) = -a + 1 - a + 1 = 2(1 - a)$

$\because$ Shortest distance between line L_1 and L_2 is $\sqrt{\frac{2}{3}}$

$$\therefore \quad \sqrt{\frac{2}{3}} = \left|\frac{2(1-a)}{\sqrt{a^2+1+(a-1)^2}}\right|$$

$\Rightarrow$ $2(2a^2 - 2a + 2) = (3)(4)(a^2 + 1 - 2a)$

$\Rightarrow$ $2a^2 - 5a + 2 = 0$

$\Rightarrow$ $(2a - 1)(a - 2) = 0$

$\Rightarrow$ $a = \frac{1}{2}, 2$

$\therefore$ The integral value of a is 2.

Hint :

Shortest distance between two skew lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ is given by

$$d = \left|\frac{\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

Shortcut method:

$$L_1 : \vec{r} = \left(-\hat{i}+3\hat{k}\right)+\lambda\left(\hat{i}-a\hat{j}\right)$$

$$L_2 : \vec{r} = \left(-\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$$

Now, $\vec{a_2}-\vec{a_1} = \hat{i}-\hat{j}-\hat{k}$

$\vec{b_1}\times\vec{b_2} = -a\hat{i}-\hat{j}+(a-1)\hat{k}$

$$\therefore \quad \sqrt{\frac{2}{3}} = \left|\frac{\left(\hat{i}-\hat{j}-\hat{k}\right).\left(-a\hat{i}-\hat{j}+(a-1)\hat{k}\right)}{\sqrt{a^2+1+(a-1)^2}}\right|$$

$\Rightarrow 2a^2 - 5a + 2 = 0$

$\Rightarrow$ $a = \frac{1}{2}, 2$

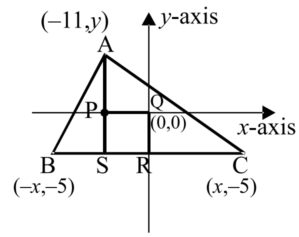
$\therefore$ The integral value of a is 2.

86. [7.00] Let Q(0, 0), P(–11, 0), R(0, –5) and S(–11, –5)

Now BP is an altitude, therefore BP. is perpendicular to AC

$\Rightarrow m_{BP} \cdot m_{AC} = -1$

or $\left(\frac{5}{x-11}\right)\left(\frac{y+5}{-11-x}\right) = -1$



or $5(y + 5) = (x + 11)(x - 11)$ ...(i)

Also Q is equidistant from A and C,

so $y^2 + 121 = x^2 + 25$ ...(ii)

From (i) and (ii), we get

$5y + 25 = (y^2 + 96) - 121$

or $y^2 - 5y - 50 = 0$ gives $y = 10, -5$

But $y = -5$ is not possible

Hence $y = 10$

$\Rightarrow$ $x = 14$

$\therefore$ $BC = 2x = 28$

$\frac{k}{4} = \frac{2B}{4} = 7$

87. [2.00] $2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\Rightarrow$ $2\left(n_1\left(\frac{n_2}{2}\right)+(1)\left(\frac{n_2}{2}\right)\right) = n_1$

$\Rightarrow$ $n_1n_2 + n_2 = n_1$

$\Rightarrow$ $n_2 = \frac{n_1}{(1+n_1)}$

$\Rightarrow$ $n_2 = 1 - \frac{1}{(1+n_1)}$

$1 + n_1 = 1$ or $1 + n_1 = -1$

$n_1 = 0$ or $n_1 = -2$

$\Rightarrow$ $n_2 = 0$ or $n_2 = 2$

The number of ordered pairs (n_1, n_2) is 2 i.e., (0, 0) and (–2, 2)

88. [3.00] Let

$A\left(-a, a\left(t_1+t_2\right)\right)$, $B(at_1^2, 2at_1)$, $C(at_2^2, 2at_2)$

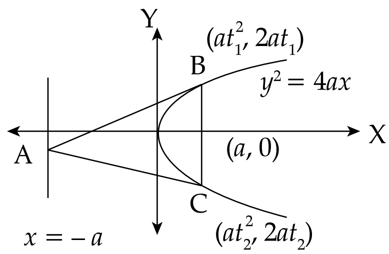
Let (h, k) is centroid of ΔABC

$\Rightarrow$ $h = \frac{a(t_1^2+t_2^2)-1}{3}$

and $k = a\left(t_1+t_2\right)$

$\because$ B and C are the end points of the chord of parabola

$\therefore t_1 t_2 = -1$



$\Rightarrow \quad 3h = a((t_1 + t_2)^2 + 2) - a$

$\Rightarrow \quad 3h = a\left(\frac{k^2}{a^2}+1\right)$

$\Rightarrow \quad 3h = \frac{k^2}{a} + a$

$\Rightarrow \quad k^2 = 3a\left(h - \frac{a}{3}\right)$

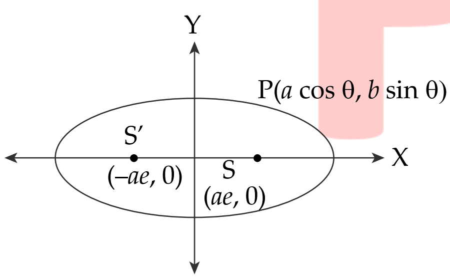
So, the locus of centroid of Δ ABC is

$$y^2 = 3a\left(x - \frac{a}{3}\right)$$

⇒ The length of latus rectum is $\lambda = 3a$

$\Rightarrow \quad \frac{\lambda}{a} = 3$

89. [4.00] Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



be an ellipse

Area of ellipse = $A_1 = \pi ab$

Let (h, k) be the mid-point of PS

$\Rightarrow \quad 2h = a\cos\theta + ae$ and $2k = b\sin\theta$

Eliminating θ, we get

$$\frac{\left(x-\frac{ae}{2}\right)^2}{\left(\frac{a}{2}\right)^2}+\frac{y^2}{\left(\frac{b}{2}\right)^2}=1$$

The area enclosed by the locus of mid-point of PS is $A_2 = \pi\frac{a}{2}\cdot\frac{b}{2} = \frac{\pi ab}{4}$

$\Rightarrow A_1 : A_2 = 4:1$

90. [4.00] $a^2 + b^2 = r^2$

$a^2 - b^2 = \frac{r^2}{4}$

$a^2 = \frac{5r^2}{8}$ and $b^2 = \frac{3r^2}{8}$

$b^2 = a^2\left(1-e_1^2\right)$ if $\frac{b^2}{a^2} = \left(e2^2 - 1\right)$

$\Rightarrow e_2^2 = \frac{8}{5}$ and $e_1^2 = \frac{2}{5}$

Now, $\frac{e_2^2}{e_1^2} = \frac{8}{2} = 4$

Shortcut Method:

$a^2 + b^2 = a^2e_2^2 = r^2$

$a^2 - b^2 = a^2e_1^2 = \frac{r^2}{4}$

$\Rightarrow \quad \frac{e_2^2}{e_1^2} = 4$