JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-9**
CHAPTER :- CIRCLE

ANSWER KEY

1.	(C)	**2.**	(A)	**3.**	(B)	**4.**	(A)	**5.**	(A)	**6.**	(A)	**7.**	(B)
8.	(A)	**9.**	(B)	**10.**	(D)	**11.**	(B)	**12.**	(B)	**13.**	(D)	**14.**	(A)
15.	(D)	**16.**	(C)	**17.**	(A)	**18.**	(D)	**19.**	(A)	**20.**	(C)	**21.**	32
22.	4	**23.**	0	**24.**	4	**25.**	62	**26.**	6	**27.**	4	**28.**	4
29.	4	**30.**	5										

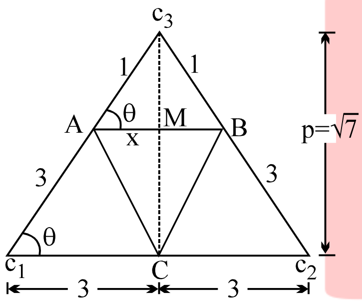
SOLUTIONS

1. (C)

Sol. $\cos\theta = \frac{3}{4} = \frac{x}{1} \Rightarrow x = \frac{3}{4} \Rightarrow AB = \frac{3}{2}$

$p = \sqrt{4^2 - 3^2} = \sqrt{7}$ and $c_3M = \frac{\sqrt{7}}{4}$

$\Rightarrow CM = \sqrt{7} - \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{4}$



$\Rightarrow A = \frac{3\sqrt{7}}{4} \cdot \frac{3}{2} \cdot \frac{1}{2} = \frac{9\sqrt{7}}{16}$

2. (A)

Sol. Let radius of the circle is (r,r)

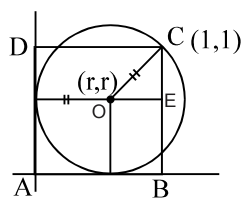
OE = DE = 1 – r

applying pythagoras theorem in ΔOEC

$2(1 - r)^2 = r^2$

$\sqrt{2}\,(1 - r) = r$

$r\left(\sqrt{2} + 1\right) = \sqrt{2}$



$r = \frac{\sqrt{2}}{\sqrt{2}+1} = \sqrt{2}\left(\sqrt{2} - 1\right) = 2 - \sqrt{2}$]

3. (B)

Sol. $\angle OAP = \angle OBP = 90°$

(Line From centre to tangent at point of contact is a $\perp$)

PA = PB (tangents from common external point to circle)

OA = OB (radius of same circle)

Hence, $\angle AOP = \angle BOP = 60°$

Area of sector AOB = $\frac{\pi}{360} \times 120° = \frac{\pi}{3}$

Area of quadrilateral BPOA = $2 \times \text{area}(\Delta POA)$

$= 2 \times \frac{1}{2} \times 1 \times \sqrt{3} = \sqrt{3}$

Hence, required area = area (PBOA) – area (sector AOB) = $\sqrt{3} - \frac{\pi}{3}$

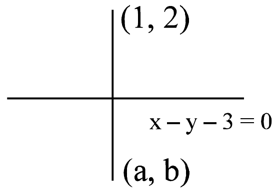
4. (A)

Sol. Centre of the circle is (1, 2) and r = 1

image of (1, 2) in x – y – 3 = 0

$\frac{b-2}{a-1} = -1 \Rightarrow a + b = 3$ ....(1)

also $\frac{a+1}{2}, \frac{b+2}{2}$ lies on x – y = 3



(a + 1) – (b + 2) = 6 $\Rightarrow$ a – b = 7 ....(2)

from (1) and (2) a = 5 and b = – 2

Hence the required circle has the centre (5, –2) and r = 1

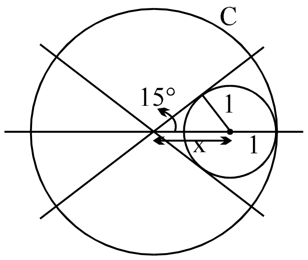
$(x - 5)^2 + (y + 2)^2 = 1 \Rightarrow x^2 + y^2 - 10x + 4y + 28 = 0 \Rightarrow$ (A)]

5. (A)

Sol. $\text{cosec}15° = \dfrac{x}{1}$

$x = \text{cosec}15°$

$R = x + 1 = 1 + \text{cosec } 15°$



$= 1 + \dfrac{2\sqrt{2}}{\sqrt{3}-1} = 1 + \dfrac{4}{\sqrt{6}-\sqrt{2}} = 1 + \sqrt{6}+\sqrt{2}$]

6. (A)

Sol. Hypotenuse AB = $\sqrt{a^2+b^2}$

hence D = $\sqrt{b^2+a^2}$ ....(1)

Now $\dfrac{\Delta}{s} = \dfrac{ab}{2s}$

$\therefore \dfrac{d}{2} = \dfrac{ab}{a+b+\sqrt{a^2+b^2}}$ or d

$= \dfrac{2ab}{a+b+\sqrt{a^2+b^2}}$ ....(2)

from (1) and (2)

d + D

$= \dfrac{\sqrt{a^2+b^2}\left[(a+b)+\sqrt{a^2+b^2}\right]+2ab}{a+b+\sqrt{a^2+b^2}}$

$= \dfrac{(a+b)^2+(a+b)\sqrt{a^2+b^2}}{a+b+\sqrt{a^2+b^2}}$

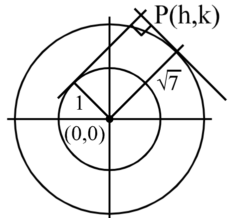
$= \dfrac{(a+b)\left((a+b)+\sqrt{a^2+b^2}\right)}{a+b+\sqrt{a^2+b^2}} = a+b$

7. (B)

Sol. $h^2 + k^2 = 1 + 7$

$\therefore$ locus of the point P is

$x^2 + y^2 = 8$



This is the director circle of circle $x^2 + y^2 = 4$

$\therefore$ $x^2 + y^2 = 8$ is director circle of a circle with radius = 2.]

8. (A)

Sol. $m_{AB} = -1$

$m_{CM} = 1$

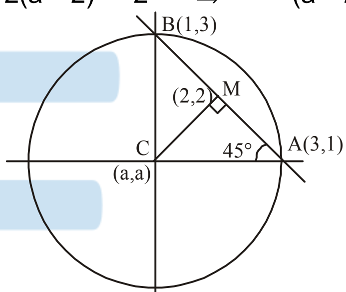
equation of CM is

$y = x$

Let C(a, a)

Hence $(CM)^2 = (AM)^2$

$2(a-2)^2 = 2 \quad \Rightarrow \quad (a-2)^2 = 1$



a – 2 = 1 or – 1

a = 3 or 1

but centre can not be (3, 3)]

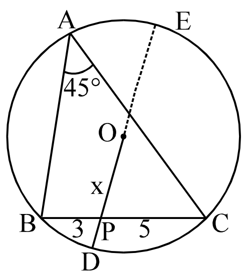
cente (1, 1)

9. (B)

Sol. Using sine law

$\dfrac{a}{\sin A} = 2R$

$8\sqrt{2} = 2R \quad \Rightarrow \quad R = 4\sqrt{2}$



using power of a point

(PB)(PC) = (PD)(PE)

15 = (R – x)(R + x)

$15 = R^2 - x^2 \quad \Rightarrow x^2 = R^2 - 15 = 32 - 15 = 17$

$\therefore \quad x = \sqrt{17}$ **Ans.**]

10. (D)

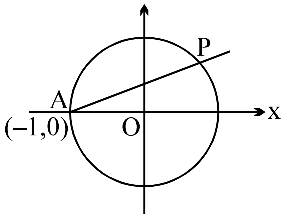
Sol. Equation of the line l is

$y - 0 = m(x + 1)$ ....(1)

solving it with $x^2 + y^2 = 1$

$x^2 + m^2(x + 1)^2 = 1$

$(m^2 + 1)x^2 + 2m^2x + (m^2 - 1) = 0,\ m \in Q$



$$x = \frac{-2m^2 \pm \sqrt{4m^4 - 4(m^4 - 1)}}{2(m^2+1)}$$

$$= \frac{-2m^2 \pm 2}{2(m^2+1)}$$

taking – ve sign $x = -1$ (corresponding to A)

with + ve sign $x = \frac{1-m^2}{1+m^2}$

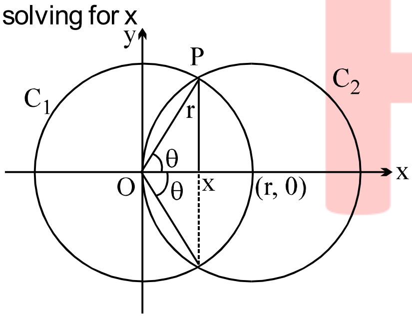
since $m \in Q$ hence x will be rational.

If x is rational then y is also rational from (1)]

11. (B)

Sol. $C_1 : x^2 + y^2 = r^2$; $C_2 : (x - r)^2 + y^2 = r^2$

solving for x



$(x - r)^2 + r^2 - x^2 = r^2$

$x - r = x$ or $-x$

$\therefore \quad x = \frac{r}{2}$

$\therefore \quad r\cos\theta = \frac{r}{2} \qquad \theta = \frac{\pi}{3}$

$2\theta = \frac{2\pi}{3}$; length of C_1 inside $C_2 = r(2\theta) = r \cdot \frac{2\pi}{3}$

$= \frac{2\pi r}{3}$ Ans.]

12. (B)

Sol. Let the equation of the requred circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(i)

It passes through (1,–2) and (4,–3)

$5 + 2g - 4f + c = 0$ ....(ii)

$25 + 8g - 6f + c = 0$ ....(iii)

The centre (–g,–f) of (i) lies on $3x + 4y + 7$

$-3g - 4f = 7$ ....(iv)

Subtracting (ii) From (iii)

$20 + 6g - 2f = 0$

$10 + 3g - f = 0$ ....(v)

Soluing (iv) and (v) as simultaneous equations,we get

$$g = \frac{-47}{15} \text{ and } f = \frac{3}{5}$$

Substituting the value of g and f in (ii)

$$5 - \frac{94}{15} - \frac{12}{5} + c = 0$$

$$c = \frac{55}{15} = \frac{11}{3}$$

Substituting the value of g , f & c in (i)

$$x^2 + y^2 - \frac{94}{15}x + \frac{6}{5}y + \frac{11}{3} = 0$$

or $15(x^2 + y^2) - 94x + 18y + 33 = 0$

13. (D)

Sol. Given points P (2, 1) ; Q (0, 0) ; R (4, –3)

$S : x^2 + y^2 - 5x + 2y - 5 = 0$

$C = \left(\frac{5}{2}, -1\right)$

distance PC = $\sqrt{\left(\frac{5}{2} - 2\right)^2 + (-1-1)^2}$

$r = \sqrt{\left(\frac{5}{2}\right)^2 + 1 + 5} = \sqrt{\frac{25}{4} + 1 + 5} = \sqrt{\frac{49}{4}} = \frac{7}{2}$

$= \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2} < r$

distance CA = $\sqrt{\left(\frac{5}{2}\right)^2 + (1)^2} = \sqrt{\frac{25}{4} + 1} = \frac{\sqrt{29}}{2}$

$< r$

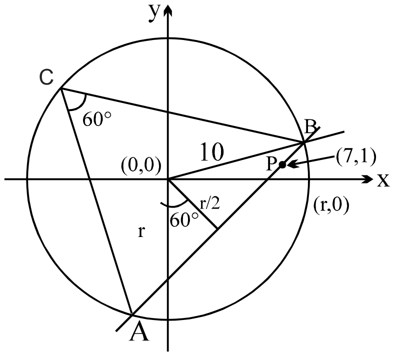
distance RC = $\sqrt{\left(4 - \frac{5}{2}\right)^2 + (-3+1)^2} = \sqrt{\frac{9}{4} + 4}$

$= \frac{\sqrt{25}}{2} < r$

$\therefore$ all points line inside circle]

14. (A)

Sol. Sin 30° = $\frac{d}{10} = \frac{1}{2}$



d = 5

$x^2 + y^2 = 100$

$y - 1 = \mu(n - 7)$

mn –y + 1 = 0

$\therefore \perp$ distance from (0,0) to line AB

$5 = \left|\frac{1-7nm}{\sqrt{m^2+1}}\right|$

$d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$25 = \frac{(1-7m)^2}{m^2+1}$

$25m^2 + 25 = 1 + 49m^2 - 14m$

$24m^2 - 14m - 24 = 0$

or $12m^2 - 7m - 12 = 0$

using relation b/w coefficent

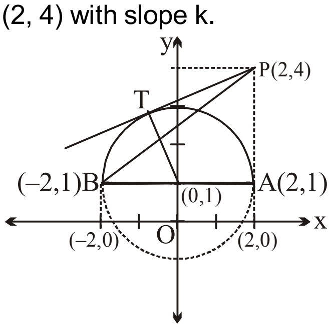
$m_1 \cdot m_2 = \frac{-12}{12} = -1$

15. (D)

Sol. Given $y = 1 + \sqrt{4 - x^2}$

Now curve is $(y-1)^2 = 4 - x^2 \Rightarrow x^2 + (y-1)^2 = 4$

Also line is $y - 4 = k(x - 2)$, passing through (2, 4) with slope k.



Now $m_{PB} = \frac{4-1}{2+2} = \frac{3}{4}$

Again for slope of PT, using condition of tangency we get

$\left|\frac{-1+4-2k}{\sqrt{1+k^2}}\right| = 2 \Rightarrow (3-2k)^2 = 4(1+k^2) \Rightarrow 9 - 12k = 4 \Rightarrow k = \frac{5}{12}$

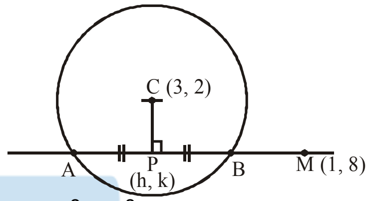
$\therefore$ For two distinct points of intersection, we must have $\frac{5}{12} < k \le \frac{3}{4}$. **Ans.**

16. (C)

Sol. Clearly, $m_{CP} \times m_{AB} = -1$

$\Rightarrow \left(\frac{k-2}{h-3}\right)\times\left(\frac{k-8}{h-1}\right) = -1$

$\therefore$ Locus of (h, k) is $(x-1)(x-3) + (y-2)(y-8) = 0$



i.e., $x^2 + y^2 - 4x - 10y + 19 = 0$.

Point P moves on circle with CM as diameter

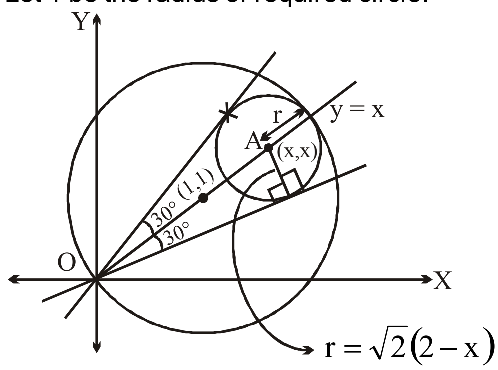
$\therefore$ Radius = $\frac{CM}{2} = \frac{\sqrt{2^2+6^2}}{2} = \sqrt{1+9} = \sqrt{10}$. **Ans.**]

17. (A)

Sol. Clearly, centre and radius of circle $x^2 + y^2 - 2x - 2y = 0$

are (1, 1) and $\sqrt{2}$.

Let r be the radius of required circle.



As $\sqrt{(x-1)^2 + (x-1)^2} = \sqrt{2} - r$

$\Rightarrow \quad r = \sqrt{2} - \sqrt{2}\,|x-1|$

As $x > 1$, so $r = \sqrt{2}(1 - x + 1) \Rightarrow r = \sqrt{2}(2 - x)$]

Also, $\sin 30° = \dfrac{\sqrt{2}(2-x)}{\sqrt{2}\,x} \Rightarrow \dfrac{1}{2} = \dfrac{2-x}{x} \Rightarrow$

$x = \dfrac{4}{3}$

$\therefore$ Centre $\left(\dfrac{4}{3}, \dfrac{4}{3}\right)$

Also, radius = r = $\sqrt{2}\,(2-x) = \sqrt{2}\left(2-\dfrac{4}{3}\right)$

$= \sqrt{2}\left(\dfrac{2}{3}\right) = \dfrac{2\sqrt{2}}{3}$

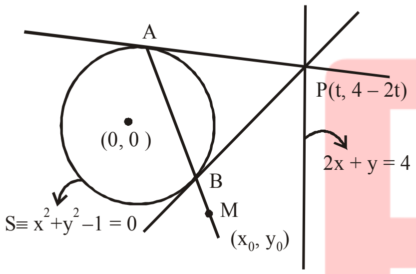
18. (D)

Sol. Clearly, equation of chord of contact is

$(4y - 1) + t\,(x - 2y) = 0,\ t \in R$

$\Rightarrow L_1 + t L_2 = 0,\ t \in R$

$\therefore$ M (a, b) = $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$



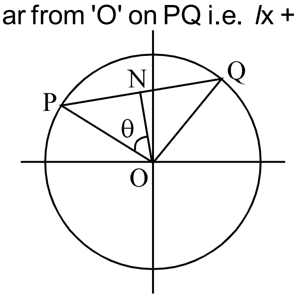
Hence, $\left(\dfrac{1}{a} + \dfrac{1}{b}\right) = 2 + 4 = 6$. **Ans.**]

19. (A)

Sol. Let $\alpha = 2\theta$

length of perpendicular from 'O' on PQ i.e. $lx + my - 1 = 0$

$p = \left|\dfrac{1}{\sqrt{l^2+m^2}}\right|$



now $\cos\theta = \dfrac{p}{a} = \dfrac{1}{a\sqrt{l^2+m^2}}$

$\sec\theta = a\sqrt{l^2+m^2} \Rightarrow \theta$

$= \sec^{-1}(a\sqrt{l^2+m^2})$

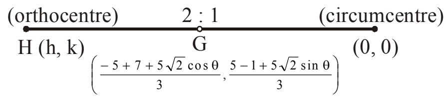
$\therefore \dfrac{\alpha}{2} = \sec^{-1}(a\sqrt{l^2+m^2})$ $\therefore$ α = $2\sec^{-1}(a\sqrt{l^2+m^2})$ $\Rightarrow$ (A)]

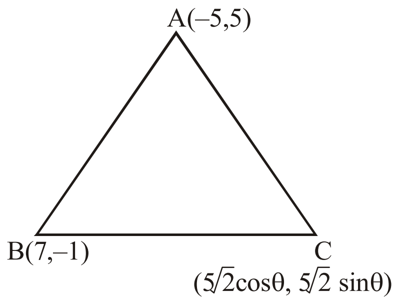
20. (C)

Sol. If vertex C lies on the circle whose director circle has equation $x^2 + y^2 = 100$, then vertex C must lie on circle $x^2 + y^2 = 50$, whose centre is (0, 0) and radius = $5\sqrt{2}$. Also OA = OB = $5\sqrt{2}$, where O is the origin. (All the three vertices i.e. A, B and C lie on $x^2 + y^2 = 50$)

Clearly O (0, 0) is circumcentre of triangle ABC.

Let C be $(5\sqrt{2}\cos\theta,\ 5\sqrt{2}\sin\theta)$





Now, $\dfrac{2(0)+1(h)}{3} = \dfrac{2+5\sqrt{2}\cos\theta}{3}$

$\Rightarrow h = 2 + 5\sqrt{2}\cos\theta$ ...(1)

and $\dfrac{2(0)+1(k)}{3} = \dfrac{2+5\sqrt{2}\sin\theta}{3} \Rightarrow k$

$= 4 + 5\sqrt{2}\sin\theta$ .....(2)

$\therefore$ On eliminating θ between equation (1) and (2), we get locus of orthocentre (h, k) of ΔABC, is $(x-2)^2 + (y-4)^2 = 50$ i.e., $x^2 + y^2 - 4x - 8y - 30 = 0$ **Ans.**]

21. 32

Sol. $S - S_1 = 0$

$(y-k)^2 - y^2 = 0$

$k^2 - 2ky = 0$

$k(k - 2y) = 0$

$k = 2y$ or $1 = \dfrac{2y}{k}$

The combined equation of the straight lines joining the origen to the points of intersection

$y = \dfrac{k}{2}$ & $x^2 + y^2 = 16\left(\dfrac{2y}{k}\right)^2$

$k^2x^2 + (k^2 - 64)y^2 = 0$

This equation represents a pair of $\perp$ lines

$k^2 + k^2 - 64 = 0$

$2k^2 = 64$

$k^2 = 32$

22. 4

Sol. Equation of S_3 is $(x-0)(x-1)+(y-0)^2+\lambda y = 0$

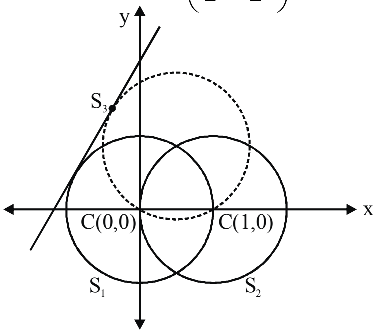
i.e. $x^2+y^2-x+\lambda y = 0$

i.e. $\sqrt{\frac{1}{4}+\frac{\lambda^2}{4}-0} = 1 \Rightarrow \lambda = \pm\sqrt{3}$

$\therefore$ Circle S_3 lie above x-axis $\Rightarrow \lambda = -\sqrt{3}$

i.e. $S_2 \equiv x^2+y^2-x-\sqrt{3}\,y = 0$

its centre $S_3 \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



Slope of line joining c_1 & $c_2 = \sqrt{3}$

$\therefore$ Slope of common tangent is $\sqrt{3}$

i.e. $\sqrt{3}\,x - y + k = 0$ is required common tangent and touches circle S_1

$\Rightarrow \left|\frac{k}{\sqrt{3+1}}\right| = 1 \Rightarrow k = \pm 2$

$\therefore$ For the given figure k = 2

$\therefore$ Required common tangent is $\sqrt{3}\,x - y + 2 = 0$

$\therefore$ $a = \sqrt{3}$, $b = -1$,

$\therefore$ $(a^2 - b) = 3 + 1 = 4$

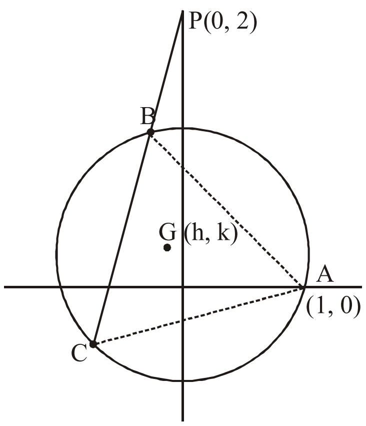
23. 0

Sol. The equation BC can be written as $y = mx + 2$

solve $x^2 + y^2 = 1$ with $y = mx + 2$

$(m^2+1)x^2 + 4mx + 3 = 0.$

$x_1 + x_2 = \frac{-4m}{m^2+1}.$



$||^{ly}$ $\frac{(y-2)^2}{m^2} + y^2 = 1$

$(m^2+1)y^2 - 4y + (4-m^2) = 0$

$y_1 + y_2 = \frac{4}{m^2+1}$

$3h = x_1 + x_2 + 1$ and $3k = y_1 + y_2$

$3h = \frac{-4m}{m^2+1} + 1$ and $3k = \frac{4}{m^2+1}$

$\frac{3h-1}{3k} = -m$

$3k(m^2+1) = 4 \Rightarrow 3k\left(\left(\frac{3h-1}{3k}\right)^2+1\right) = 4$

$\frac{(3h-1)^2}{3k} + 3k = 4$

$x^2 + y^2 - \frac{2}{3}x - \frac{4}{3}y + 1 = 0.$

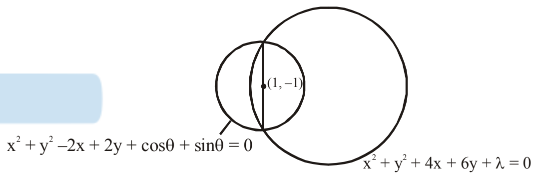
$a = \frac{-2}{3}$; $b = \frac{-4}{3}$, $c = \frac{1}{9}$

(a + b + 18c) = 0. **Ans.**]

24. 4

Sol. RA = $6x + 4y + c - \cos\theta - \sin\theta = 0$

It passes through (1, –1)



$6 - 4 + c = \cos\theta + \sin\theta$

$c = \cos\theta + \sin\theta - 2$

$c_{max} = \sqrt{2} - 2$

$c_{min} = -\sqrt{2} - 2$

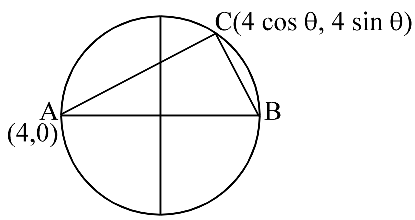
sum = $\lambda_1 = -4$

$|\lambda_1| = 4$]

25. 62

Sol. $A = \frac{1}{2}\cdot 8\cdot 4\sin\theta = |\,16\sin\theta\,|$

now $|\sin\theta|$ can be $= \frac{1}{16}, \frac{2}{16}, \ldots\ldots\ldots \frac{15}{16}$



i.e. 15 points in each quadrant

$\Rightarrow$ 60 + 2 more with $\sin\theta = 1$

$\Rightarrow$ total = 62 **Ans.**]

26. 6

Sol. $x - 2 - m_1 (y + 2) = 0$

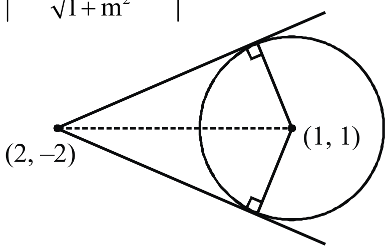
$x - 2 - m_2 (y + 2) = 0$

Both lines are passing through (2, –2) and tangents to the circle $(x - 1)^2 + (y - 1)^2 = 2$

$\therefore$ Applying p = r

$x - my - 2 - 2m = 0$

$$\left|\frac{1-m-2-2m}{\sqrt{1+m^2}}\right| = \sqrt{2}$$



$\Rightarrow$ $(3m + 1)^2 = 2(1 + m^2)$

$\Rightarrow$ $7m^2 + 6m - 1 = 0$

$\Rightarrow$ $m = \frac{1}{7}, -1$

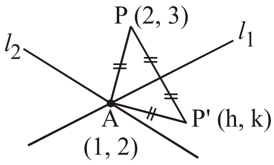
$\therefore$ $\frac{1}{\frac{1}{7}} + \frac{1}{(-1)} = 7 - 1 = 6$

27. 4

Sol. AP = AP'

$\Rightarrow (h - 1)^2 + (k - 1)^2 = 1 + 1 = 2$

$\therefore$ radius of circle = $\sqrt{2}$



$\therefore$ maximum distance = $\sqrt{5} + \sqrt{2}$

and minimum distance = $\sqrt{5} - \sqrt{2}$

$\therefore$ Sum, $S = 2\sqrt{5}$ $\Rightarrow$ [S] = 4 **Ans.**

28. 4

Sol. tangent at (2, 2) :

$2x + 2y - \frac{5}{2}(x + 2) - \frac{3}{2}(y + 2) + 8 = 0$

$\frac{-x}{2} + \frac{y}{2} = 0 \Rightarrow y = x$

cc' : $\frac{x - \frac{5}{2}}{\cos 45°} = \frac{y - \frac{3}{2}}{\sin 45°} = \sqrt{2}\,r \Rightarrow c' =$

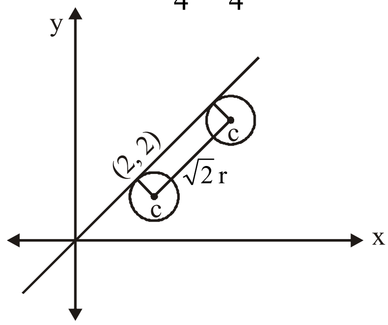
$\left(\frac{5}{2} + r \cdot \frac{3}{2} + r\right)$

$\frac{5}{2} + r = 4 \Rightarrow r = \frac{3}{2}$

$\Rightarrow$ c' (4, 3)

$\therefore$ circle : $x^2 + y^2 - 8x - 6y + c = 0$

$r^2 = 25 - c = \frac{25}{4} + \frac{9}{4} - 8$



$c = 25 - \frac{1}{2} = \frac{49}{2}$

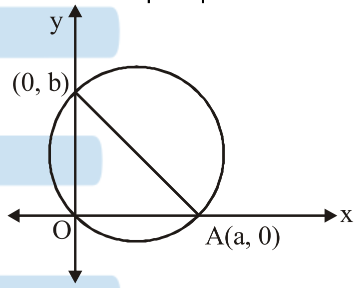
$\therefore$ $\frac{r + a + c}{8} = \frac{\frac{3}{2} + \frac{49}{2} + 6}{8} = \frac{26 + 6}{8} = 4$ **Ans.**]

29. 4

Sol. $x(x - a) + y(y - b) = 0$

$x^2 - ax - by + y^2 = 0$

$\left(3\sqrt{2}\right)^2 = \frac{a^2}{4} + \frac{b^2}{4}$



$a^2 + b^2 = 72$

3h = a, 3k = b

Locus of centroid $x^2 + y^2 = 8$

$r = 2\sqrt{2}$

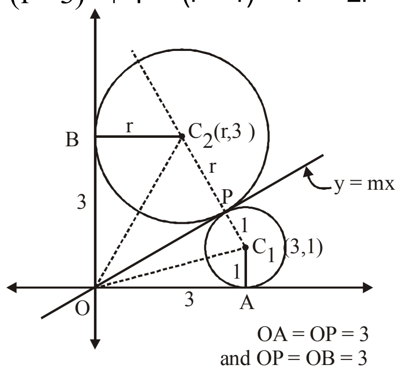
Radius of director circle = 4 **Ans.**]

30. 5

Sol. $C_1C_2 = r + 1$

$\sqrt{(r-3)^2 + 4} = r + 1$

$(r-3)^2 + 4 = (r + 1)^2 = r^2 + 2r + 1$



OA = OP = 3
and OP = OB = 3

$\Rightarrow$ 8r = 12 $\Rightarrow$ $r = \frac{3}{2} \equiv \frac{p}{q}$

$\therefore$ p + q = 5 **Ans.**]