JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-8**
CHAPTER :- STRAIGHT LINE

ANSWER KEY

1.	(C)	**2.**	(A)	**3.**	(A)	**4.**	(A)	**5.**	(D)	**6.**	(A)	**7.**	(B)
8.	(A)	**9.**	(B)	**10.**	(C)	**11.**	(C)	**12.**	(C)	**13.**	(B)	**14.**	(B)
15.	(A)	**16.**	(B)	**17.**	(B)	**18.**	(D)	**19.**	(A)	**20.**	(A)	**21.**	6
22.	6	**23.**	4	**24.**	5	**25.**	3	**26.**	20	**27.**	2	**28.**	4
29.	6	**30.**	3										

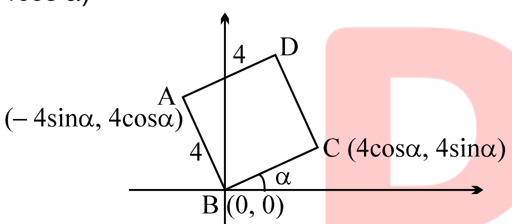
SOLUTIONS

1. (C)

Sol. $\therefore$ slope of line AC = $\dfrac{4\sin\alpha - 4\cos\alpha}{4\cos\alpha + 4\sin\alpha}$

$= \dfrac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}$

$\therefore$ eqn of line $y - 4\sin\alpha = \dfrac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha}(x - 4\cos\alpha)$



$y\cos\alpha + y\sin\alpha - 4\sin\alpha \cdot \cos\alpha - 4\sin^2\alpha$
$= x\sin\alpha - x\cos\alpha - 4\sin\alpha\cos\alpha + 4\cos^2\alpha$
$x(\cos\alpha - \sin\alpha) + y(\cos\alpha + \sin\alpha) = 4$

2. (A)

Sol. Mid point of QS = Mid point of PS

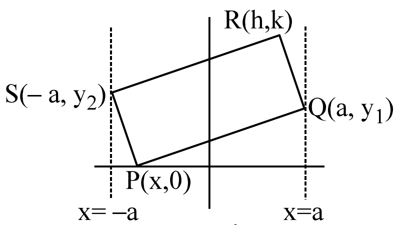
$0 = h + x_1$

$\Rightarrow x_1 = -h$ hence p $(-h, 0)$

equation of $PQ \equiv y = mx + c$

passes through $(-h, 0)$

$c = mh$



$\therefore$ $y = mx + mh$

since Q lies on it

$\Rightarrow$ $y_1 = ma + mh$

now $m_{QR} = -\dfrac{1}{m} = \dfrac{y_1 - K}{a - h}$

$-\dfrac{1}{m} = \dfrac{m(a-h) - K}{a - h}$

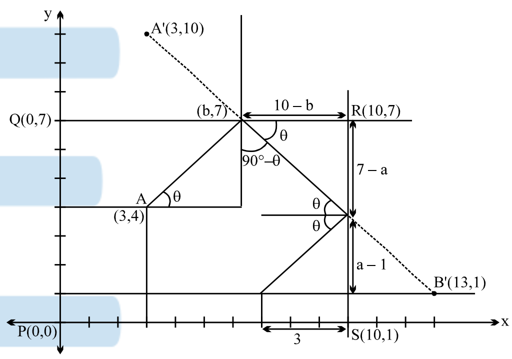
simplifying $h + mk = a + (a + h)m^2 \Rightarrow$ a st. line]

3. (A)

Sol. $\tan\theta = \dfrac{a-1}{3} = \dfrac{7-a}{10-b}$

also $\tan\theta = \dfrac{7-4}{b-3} = \dfrac{3}{b-3}$

hence $\dfrac{3}{b-3} = \dfrac{a-1}{3} = \dfrac{7-a}{10-b}$



from 1st two relations

$9 = ab - b - 3a = 3$

$3a + 6 = ab - b$ ....(1)

from last two

$10a - ab - 10 + b = 21 - 3a$

$13a - ab + b = 31$

or $ab - b = 13a - 31$ ....(2)

hence from (1) and (2)

$3a + 6 = 13a - 31 \Rightarrow 10a = 37 \Rightarrow a = 3.7$ **Ans.**]

4. (A)

Sol. First two family of lines passes through (1, 1) and (3, 3) respectively.

$\Rightarrow$ Point of intersection of lines belonging to third family of lines will lie on the line $y = x$.

$\Rightarrow \alpha x + x - 2 = 0$ and $6x + \alpha x - \alpha = 0 \Rightarrow$

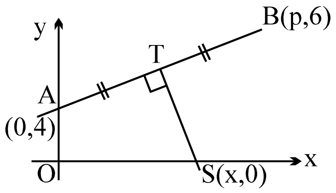
$\dfrac{2}{\alpha+1} = \dfrac{\alpha}{\alpha+6} \Rightarrow 2\alpha + 12 = \alpha^2 + \alpha$

$\Rightarrow (\alpha - 4)(\alpha + 3) = 0 \Rightarrow \alpha = 4$ or -3

5. (D)

Sol. $m_{AB} = \frac{2}{p}$; $m_{ST} = -\frac{p}{2}$; mid point of AB = $\frac{p}{2}, 5$

equation of TS $\quad y - 5 = -\frac{p}{2}\left(x - \frac{p}{2}\right)$



put $\quad y = 0 \quad\quad x = \frac{10}{p} + \frac{p}{2}$

Hence $\;p = \pm 2, \pm 10\;$ for x to be an integer]

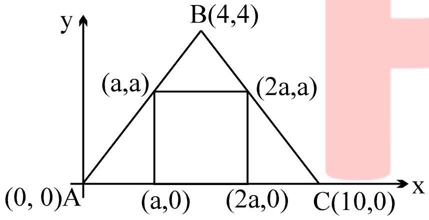
6. (A)

Sol. Equation of BC = $y - 0 = -\frac{4}{6}(x - 10)$

$\therefore$ $2x + 3y = 20$. Note that (a, 2a) lies on it

hence $\quad 4a + 3a = 20$

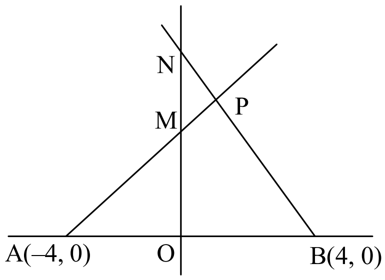
$\Rightarrow \quad a = \frac{20}{7}$



$\therefore \quad$ Area = $\frac{400}{49}$ Ans.]

7. (B)

Sol. Let M = (0, h), N $\equiv$ (0, h + 4)



Equation of AM $\frac{x}{-4} + \frac{y}{h} = 1$ ... (1)

Equation of BNO $\frac{x}{4} + \frac{y}{h+4} = 1$ ... (2)

Eliminating h from (1) and (2), we get

$x^2 + 2xy - 16 = 0$

8. (A)

Sol. let the equation of a line is

$\frac{x}{a} + \frac{y}{b} = 1$; passes through (2, 3); ab = 22 (given Δ = 11)

$\frac{2}{a} + \frac{3}{b} = 1 \quad \Rightarrow \quad 2b + 3a = ab$

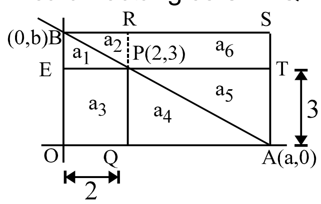
$(2b + 3a)^2 = a^2b^2$

$4b^2 + 9a^2 + 12ab = a^2b^2$

$4b^2 + 9a^2 = (ab)^2 - 12ab = 484 - 264 = 220$ Ans.

Alternatively: Area of D = $a_1 + a_4 + a_4$

Area of rectangles OBRQ + OETA



$= (a_1 + a_2 + a_3) + (a_3 + a_4 + a_5)$

$= 2(a_1 + a_3 + a_4)$

$= 2$ area of Δ AOB $= 2\left(\frac{ab}{2}\right) = 22 \Rightarrow ab = 22$

hence $\quad 2a + 3b = 22 \quad$ or $(2a + 3b)^2 = 484$

$4a^2 + 9b^2 + 12ab = 484$

$4a^2 + 9b^2 = 484 - 12(22)$

$= 484 - 264 = 220$ Ans.]

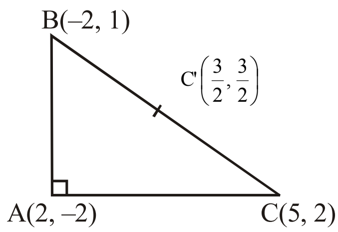
9. (B)

Sol. Let C(a, b)

Co-ordinates of G : $\frac{2 + (-2) + a}{3} = \frac{5}{3}$

$\Rightarrow \quad a = 5$

$\frac{1 - 2 + b}{3} = \frac{1}{3} \Rightarrow b = 2$



Hence C(5, 2)

ΔABC is right $\angle$ Δ

$\therefore$ circumcentre of ΔABC is mid point of BC

$\therefore$ Co-ordinates of circumcentre is $\left(\frac{3}{2}, \frac{3}{2}\right)$

Ans.]

10. (C)

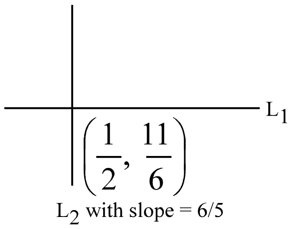
Sol. Equation of L_2, $y-\frac{11}{6} = \frac{-6}{5}\left(x-\frac{1}{2}\right)$

if $y = \frac{1}{30}$, $\frac{1}{30} - \frac{11}{6} = \frac{-6x}{5} + \frac{3}{5}$

$\frac{1}{6} - \frac{55}{6} - 3 = -6x$

$6x = 3+\frac{54}{6} = 12$

$x = 2$ **Ans.**]



11. (C)

Sol. Equation of family of lines through AB & AC

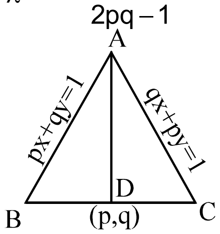
$(px + qy - 1) + \lambda (qx + py - 1) = 0$ .....(1)

Let this be equation of AD

it passes through D(p , q)

$\Rightarrow (p^2 + q^2 - 1) + \lambda (pq + pq - 1) = 0$

$\lambda = \frac{-\left(p^2+q^2-1\right)}{2pq-1}$



Put in equation (1)

$(px+qy-1) - \frac{\left(p^2+q^2-1\right)}{2pq-1}(qx+py-1) = 0$

$\Rightarrow (2pq-1)(px+qx-1) = \left(p^2+q^2-1\right)(qx+py-1)$

12. (C)

Sol. $3x - 4y + 6 + \lambda(x + y + 2) = 0$

slope $= -\frac{3+\lambda}{\lambda-4} = -1 \Rightarrow \quad 3+\lambda = \lambda - 4$

$\Rightarrow \quad \lambda = \infty$

hence slope is $= -\left(\frac{(3/\lambda)+1}{1-(4/\lambda)}\right) = -1$

line is $x + y + 2 = 0$ **Ans.**]

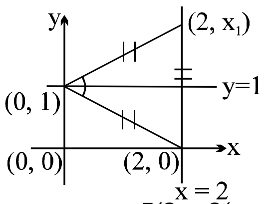
13. (B)

Sol. $2^2 + (x_1 - 1)^2 = x_1^2$

$4 + x_1^2 + 1 - 2x_1 = x_1^2$

$5 = 2x_1$ or $x_1 = 5/2$

equation of the altitude from (2, 5/2) is



$y - 5/2 = 2(x - 2)$

$2y - 5 = 4(x - 2) \Rightarrow 4x - 2y - 3 = 0$

solving it with $y = 1$, $x = 5/4$ Ans.]

14. (B)

Sol. $mx + b = bx + m$

$(m - b)x = m - b$

$\Rightarrow x = 1 (m \neq b$ since for $m = b$, x coordinate will become zero)

$\Rightarrow m - b = 1$ (as x coordinate of the point of intersection of the lines is $m - b$)

$x = 1 \Rightarrow y = b + m = 9$

$b + 1 + b = 9 \quad (m = 1 + b)$

$\therefore b = 4$ and $m = 5$

sum of x-intercepts of lines is

$-\left(\frac{b}{m}+\frac{m}{b}\right) = -\left(\frac{4}{5}+\frac{5}{4}\right) = -\frac{41}{20}$ Ans.]

15. (A)

Sol. We have $(a + c)^2 + 4b^2 - 4ab - 4bc = 0$

$(a + c)^2 + (2b)^2 - 4b(a + c) = 0$

$a^2 + 2a (c - 2b) + c^2 + 4b^2 - 4bc = 0$

$a^2 + 2a (c - 2b) + (c - 2b)^2 = 0$

$(a + c - 2b)^2 = 0 \Rightarrow a = 2b - c$

$\Rightarrow \left(2b-(a+c)\right)^2 = 0 \Rightarrow$ a, b, c are in A.P.

$\Rightarrow ax + by + 2b - a = 0$ (Put $c = 2b - a$)

$\Rightarrow a (x - 1) + b (y + 2) = 0 \Rightarrow (x - 1) + \lambda (y + 2) = 0$

so, fixed point is (1, – 2) **Ans.**]

16. (B)

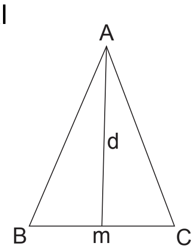
Sol. The vertex A is (2, –1)

Let's drow a $\perp$ AM with length 'd'

Now, as ΔABC is equilateral

BC = mc = x

$\therefore d = \left|\frac{2-1-2}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$



Now in ΔAMC, $\angle C = 90°$

$\therefore \frac{d}{x} = \tan 60°$

$\therefore x = \frac{1}{\sqrt{6}}$

$BC = 2x = \frac{2}{\sqrt{6}}$ is the length

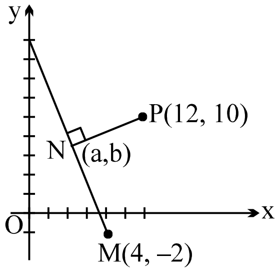
Area of $\Delta = \frac{\sqrt{3}}{4} \times (\text{length})^2 = \frac{\sqrt{3}}{4} \times \frac{4}{6} = \frac{\sqrt{3}}{6}$

17. (B)

Sol. Let shortest distance between the cheese and the line $y = -5x + 18$ is the line that passes through (12, 10) and is perpendicular to it. The equations of this line is $y = \frac{x}{5} + C$

put $x = 12$ and $y = 10$ we get

$C = \frac{38}{5} \Rightarrow$ equation of PN is $y = \frac{x}{5} + \frac{38}{5}$



solving we get a = 2 and b = 8

$\therefore$ a + b = 10 Ans.]

18. (D)

Sol. $x + a(a^2 + 1)y = a$

$$\frac{x}{a} + \frac{y}{\left(\frac{1}{a^2+1}\right)} = 1$$

$\therefore$ Area of triangle formed by the straight line and co-ordinate axes is –

$$= \frac{1}{2} \cdot \frac{a}{a^2+1} = \frac{1}{2} \cdot \frac{1}{a + \frac{1}{a}}$$

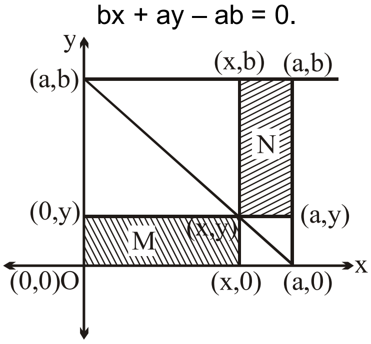
$$\le \frac{1}{2} \cdot \frac{1}{2} \left(\because a + \frac{1}{a} \ge 2\right)$$

$\le \frac{1}{4}$ **Ans.**]

19. (A)

Sol. We have, $\frac{x}{a} + \frac{y}{b} = 1$

As, (x, y) lies on it, so
bx + ay – ab = 0.



Now, Area of M = xy.

Area of N = (a – x) (b – y)

$= ab - ay - bx + xy = xy - \underbrace{(bx + ay - ab)}_{\text{zero}} = xy.$

$\therefore$ Area of M = Area of N = 1. **Ans.**]

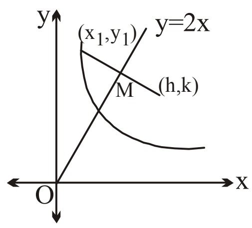
20. (A)

Sol. $\frac{k - y_1}{h - x_1} = -\frac{1}{2}$

$2k - 2y_1 = x_1 - h$

$x_1 + 2y_1 = h + 2k$ ....(1)

again $M\left(\frac{h + x_1}{2}, \frac{k + y_1}{2}\right)$ lies on y = 2x



$\frac{k + y_1}{2} = 2\left(\frac{h + x_1}{2}\right) \Rightarrow 2h + 2x_1 = k + y_1$

$2x_1 - y_1 = -2h + k$ ....(2)

solving (1) and (2) for x_1, y_1, we get

$x_1 = \frac{4k - 3h}{5}$; $y_1 = \frac{4h + 3k}{5}$

$\therefore$ $x_1y_1 = 1$

(4k – 3h)(4h + 3k) = 25

$16kh + 12k^2 - 12h^2 - 9hk = 25$

$12h^2 - 7kh - 12k^2 + 25 = 0$

$\therefore$ C_1 is $12x^2 - 7xy - 12y^2 + 25 = 0$

$\therefore$ b = – 7; c = – 12; d = 25

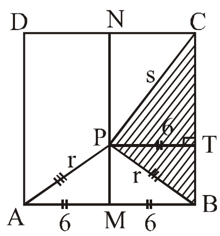
So, (b + c + d) = 6 **Ans.**]

21. 6

Sol. Here, PA = r, PC = s

Also, PB = r

$\therefore$ Area (ΔPCB) $= \frac{1}{2}$ (BC) (PT) $= \frac{1}{2}$ (12) (6) = 36



$\Rightarrow$ $\Delta = 36.$

So, $\sqrt{\Delta} = 6.$]

22. 6

Sol. Equation of l_1, l_2, l_3 and l_4 are

$y = mx + 1$ ....(1) $\rightarrow l_1$

$y = mx + m$ ....(2) $\rightarrow l_2$

$x + my = 0$ ....(3) $\rightarrow l_3$

$my + x = 1$ ....(4) $\rightarrow l_4$

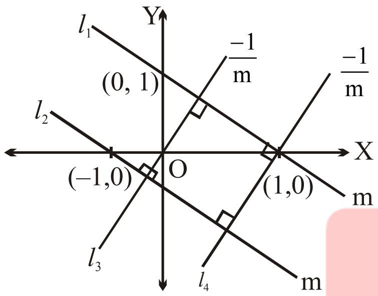
distance between l_1 and l_2 = distance between l_3 and l_4

$$\therefore \left|\frac{m-1}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{1+m^2}} \Rightarrow |m-1| = 1 \Rightarrow$$

$m - 1 = \pm 1$

$m = 2$; $m = 0$ (rejected)

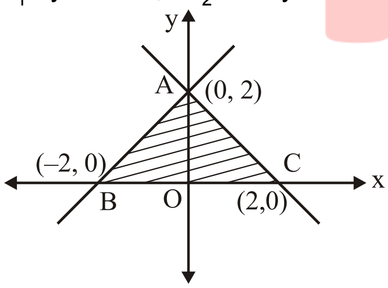
So, side of the square = $\frac{1}{\sqrt{5}}$



$\Rightarrow$ Area = $\frac{1}{5} \Rightarrow (p+q)_{least} = 6$. **Ans.**]

23. 4

Sol. $L_1 : y = x + 2$, $L_2 : x + y = 2$



$\therefore$ Area (ΔABC) = $\frac{1}{2}(4)(2) = 4$ **Ans.**]

24. 5

Sol. Equation of altitudes through A, B, C are

$y = x$, $x = 2y$, $x = 3y$

let $A(\alpha, \alpha)$, $B(2\beta, \beta)$, $C(3\gamma, \gamma)$

$$\therefore \text{Slope of AB} = \frac{\beta-\alpha}{2\beta-\alpha} = -3 \Rightarrow \beta = \frac{4\alpha}{7}$$

$$\text{Slope of BC} = \frac{\gamma-\beta}{3\gamma-2\beta} = -1 \Rightarrow \frac{7\gamma-4\alpha}{21\gamma-8\alpha} = -1 \Rightarrow \gamma = \frac{3}{7}\alpha$$

$$\therefore \text{Centroid} = \left(\frac{\alpha+2\beta+3\gamma}{3}, \frac{\alpha+\beta+\gamma}{3}\right) \equiv (x, y)$$

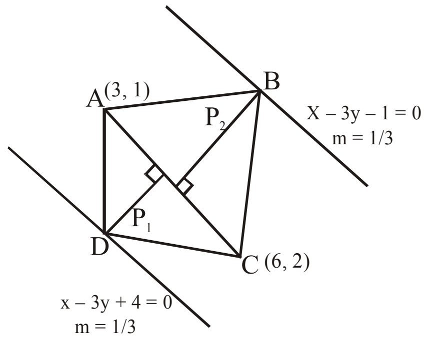
$$\therefore \frac{y}{x} = \frac{\alpha+\beta+\gamma}{\alpha+2\beta+3\gamma} = \frac{\alpha+\frac{4\alpha}{7}+\frac{3\alpha}{7}}{\alpha+\frac{8\alpha}{7}+\frac{9\alpha}{7}}$$

$$= \frac{14\alpha}{24\alpha} = \frac{7}{12} \equiv \frac{m}{n}$$

$\therefore (n - m) = 12 - 7 = 5$. **Ans.**]

25. 3

Sol.



equation of AC $x - 3y = 0$

$|AC| = \sqrt{10}$

Area of quadrilateral ABCD = $\frac{1}{2}AC(p_1 + p_2)$

$$\frac{1}{2}\sqrt{10}\left[\frac{4}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right] = \frac{5}{2}$$

$\therefore p = 5$ & $q = 2$

So, $p - q = 5 - 2 = 3$

26. 20

Sol. Equation of line $\frac{ax}{c-1} + \frac{by}{c-1} + 1 = 0$ has two independent parameter

$5a + 5b + 20c = t$

$5a + 5b = t - 20c$

$$\frac{5a}{c-1} + \frac{5b}{c-1} = \frac{t-20c}{c-1}$$

R.H.S. be independent of c if t = 20

27. 2

Sol. Point $P \equiv (3 + r\cos\alpha, 4 + r\sin\alpha)$ lies on the line passing through (3, 4) and $\tan\alpha = -2$.

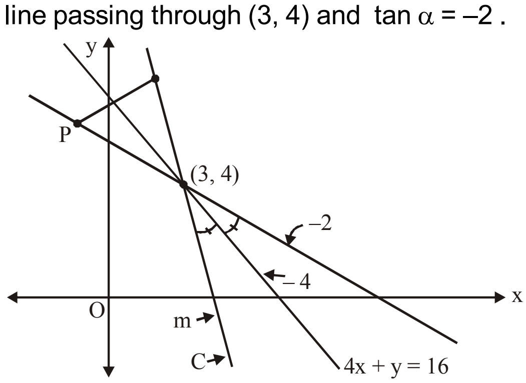


Image of P in the line mirror $4x + y = 16$ is the straight line, let whose slope is m

$$\therefore \quad \left|\frac{m-(-4)}{1+m(-4)}\right| = \left|\frac{-4+2}{1+(-4)(-2)}\right|$$

$$\frac{m+4}{1-4m} = \pm\left(\frac{-2}{9}\right) \Rightarrow m = -38 \text{ or } m = -2$$

$\therefore$ Curve 'C' is

$y - 4 = -38(x - 3)$

$38x + y = 118$

$y = 21x$

$\Rightarrow \quad x = 2$ **Ans.**]

28. 4

Sol. O(0, 0) and (a, a^2) lying on same side of each of the lines

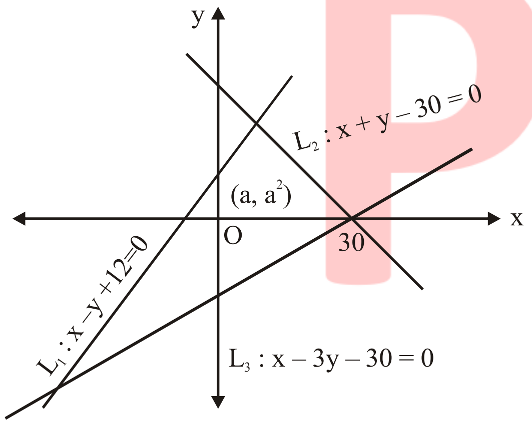
$L_1 : x - y + 12 = 0$

$a - a^2 + 12 \geq 0$

$\Rightarrow \quad a^2 - a - 12 \leq 0$

$\Rightarrow \quad (a - 4)(a + 3) \leq 0$

$a \in [-3, 4]$ ......(i)



$L_2 : x + y - 30 = 0$

$a + a^2 - 30 \leq 0 \Rightarrow a^2 + a - 30 \leq 0$

$\Rightarrow (a + 6)(a - 5) \leq 0$

$\Rightarrow a \in [-6, 5]$ ......(ii)

$L_3 : x - 3y - 30 = 0$

$a - 3a^2 - 30 \leq 0 \Rightarrow 3a^2 - a + 30 \geq 0$

$\Rightarrow a \in R$ ......(iii)

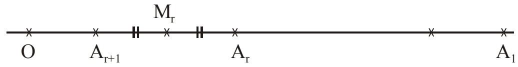
Intersection of (i), (ii) and (iii)

$a \in [-3, 4]$

$\therefore$ Sum of all possible integral values of a is 4.

29. 6

Sol.



$$OM_r = OA_{r+1} + \frac{OA_r - OA_{r+1}}{2} = \frac{OA_r + OA_{r+1}}{2} = \frac{OA_1\left(\frac{1}{2}\right)^{r-1} + (OA_1)\left(\frac{1}{2}\right)^{r}}{2}$$

$$OM_r = OA_r \cdot \frac{3}{2^2}\left(\frac{1}{2}\right)^{r-1} = OA_1 \cdot 3 \cdot \left(\frac{1}{2}\right)^{r+1}$$

$$\sum_{r=1}^{\infty} OM_r = 3 \cdot OA_1 \cdot \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{3}{2}OA_1 = 6 \text{ **Ans.**]}$$

30. 3

Sol. $\because$ 1 and 2 lies between roots,

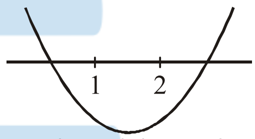
$\therefore \ (k - 5) f(1) < 0$

$\Rightarrow (k - 5)(k - 5 - 2k + k - 4) < 0$

$\Rightarrow (k - 5)(-9) < 0 \Rightarrow k - 5 > 0 \Rightarrow k > 5$

and $(k - 5) f(2) < 0$

$\Rightarrow (k - 5)\left(4(k-5) - 4k + k - 4\right) < 0$



$\Rightarrow (k - 5)(k - 24) < 0 \Rightarrow 5 < k < 24$

$\therefore \ k \in (5, 24)$

prime numbers = 7, 11, 13, 17, 19, 23

$\therefore$ points will be = (7, 17) (11, 19) and (13, 23)

Clearly, P is centroid

$$\therefore \quad P = \left(\frac{31}{3}, \frac{59}{3}\right)$$

$$\therefore \quad \frac{p+q}{10} = \frac{\frac{90}{3}}{10} = 3 \text{ **Ans.**]}$$