JEE MAIN ANSWER KEY & SOLUTIONS

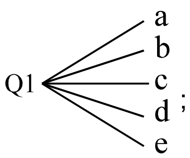
SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-7**
CHAPTER :- PERMUTATION & COMBINATION

ANSWER KEY

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(A)	**5.**	(C)	**6.**	(D)	**7.**	(D)
8.	(C)	**9.**	(C)	**10.**	(B)	**11.**	(A)	**12.**	(C)	**13.**	(B)	**14.**	(C)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(D)	**19.**	(C)	**20.**	(C)	**21.**	10
22.	42	**23.**	770	**24.**	208	**25.**	1275	**26.**	7000	**27.**	710	**28.**	151200
29.	629	**30.**	300										

SOLUTIONS

1. (D)

Sol. 

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Question No. 1 can be printed in 5! ways
Sim., Question No.2 can be printed in 5! ways
and so on
$\therefore$ Total ways $(5!)^{20}$

2. (B)

Sol. 4 couples $<$ 4 H, 4 W (number of ways = Total – all (HW) pair together)
4 husbands can be seated in 3! ways and 4 women (any) can be seated in 4! ways
Total ways when women and husbands are alternate = 4! × 3!
Now, $\boxed{H_1W_1}$, $\boxed{H_2W_2}$, $\boxed{H_3W_3}$, $\boxed{H_4W_4}$ are always together = 3! · 2!
$\therefore$ Required no. of ways = 4! · 3! – 3! · 2! = 132

3. (C)

Sol. 10 $<$ 3m, 7w 3 women can be selected in ${}^7C_3$ ways and can be paired with 3 men in 3! ways. Remaining 4 women can be grouped into two couples in $\frac{4!}{2!\cdot 2!\cdot 2!}$ = 3
$\therefore$ total = ${}^7C_3$ · 3! · 3 = 630 **Ans.**]

4. (A)

Sol. n(A ∪ B) = n(A) + n(B) – n(A ∩ B)
where A = A's together and B = B's together
now n(A) = $\frac{5!}{3!}$ = 20 $\boxed{A\ A\ A\ A}$ B B B C
n(B) = $\frac{6!}{4!}$ = 30 $\boxed{B\ B\ B}$ A A A A C
n(A ∩ B) = 3! = 6 $\boxed{A\ A\ A\ A}$ $\boxed{B\ B\ B}$ C
n(A ∪ B) = 50 – 6 = 44 Ans.]

5. (C)

Sol. Each a_i can be replaced by $2b_i - 1$, where b_i is a positive integer.

Now $32 = \sum_{i=1}^{4}(2b_i - 1) = \left(2\sum_{i=1}^{4} b_i\right) - 4 \quad \Rightarrow$

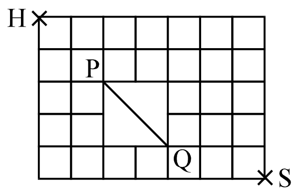
$18 = \sum_{i=1}^{4} b_i$ ... (i)

Now assign 1 to each b_i, so $14 = \sum_{i=1}^{4} b_i'$

$\therefore$ By using beggar method, number of ways = $\frac{17!}{14!\,3!} = \frac{17\times 16\times 15}{6}$ = 17 × 40 = 680 **Ans.**]

6. (D)

Sol. Number of way from H to P = $\frac{4!}{2!2!}$ = 6



Number of ways from P to Q = 1 (obvious)

Number of ways from Q to S = $\frac{4!}{1!3!}$ = 4

$\therefore$ Total ways = 6 × 4 = 24 Ans.]

7. (D)

Sol. Let X = ${}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + {}^{2n+1}C_{n+3} + \ldots\ldots + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$
using ${}^nC_r = {}^nC_{n-r}$ X = ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n$
adding 2X = ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots\ldots + {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}$ = 2^{2n+1} $\Rightarrow$ X = $2^{2n} = 4^n$]

8. (C)

Sol. If number of sides is n, then
total number of diagonals of a convex polygon
$= {}^nC_2 - n = 44$ (given)
$\Rightarrow n(n-1) - 2n = 88 \quad \Rightarrow n^2 - 3n - 88 = 0$
$\Rightarrow \quad (n-11)(n+8) = 0$
$\Rightarrow \quad n = 11 \quad \Rightarrow$ undecagon **Ans.**]

9. (C)

Sol. 'MEENANSHU'. Number of letters = 9 (EE = 2, NN = 2)
$\therefore$ Number of ways = (Total words formed) – n

$$(A \cup B) = \frac{9!}{2!\cdot 2!} - \left[\frac{2\cdot 8!}{2!} - 7!\right]$$

$= 18 \cdot 7! - 7 \cdot 7! = (18-7)7! = 11 \cdot 7!$
k = 11 **Ans.**]

10. (B)

Sol. In 132!, we have
number of 2's = 130
number of 5's = 32

$$\therefore \frac{2^{130}\cdot 5^{32}}{2^{104}\cdot 5^{19}} \Rightarrow \text{Number of 5's} = 13 \text{ Ans.]}$$

11. (A)

Sol. A person starting from london will have 13 options also the next person who starts at 2nd Station will have 12 options
So, total no. of options $\sum 13 = 78$ & no. of people = 75.
$\therefore$ no. of different set of tickets
$= {}^{78}C_{75}$ or ${}^{78}C_{78-75} = {}^{78}C_3$.

12. (C)

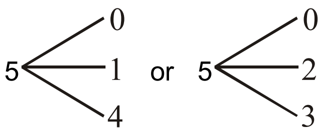
Sol. If m parallel lines in a plane are interasected by n parallel line, the no of parallelogram that are formed.
$= {}^mC_2 \times {}^nC_2$.
No. of parallelograms = 1260.
m = 8 , n = ?
$\therefore {}^8C_2 \times {}^nC_2 = 1260$

$$\Rightarrow 28 \times \frac{n(n-1)}{2} = 1260 \quad \left\{\because {}^rC_r = \frac{n!}{r!\times(n-r)!}\right\}$$

$\Rightarrow n^2 - n = 90$
$\Rightarrow n^2 - n = 90 = 0$
$\Rightarrow n^2 - 10n + 9n - 90 = 0$
$\Rightarrow (n-10)(n+9) = 0$
n = 10 & – 9
$\because n \neq -ve$
$\boxed{\therefore n = 10}$

13. (B)

Sol.



$$= \frac{5!}{0!\cdot 1!\cdot 4!}\times 3! + \frac{5!}{0!\cdot 2!\cdot 3!}\times 3!$$

= 30 + 60 = 90 ways]

14. (C)

Sol. One pair of 2 and 5 makes 10 (i.e. 2 × 5 = 10) which gives one zero at the end.
Now 5^6, 10^{11}, 15^{16}, 20^{21}, 25^{26}, 30^{31} contain 6, 11, 16, 21, 52, 31 fives.
Therefore total number of fives
= 6 + 11 + 16 + 21 + 2 × 26 + 31 = 137
Also, number of 2's are greater than number of 5's. Thus, number of zero at the end 137.]

15. (A)

Sol. P_1 must win atleast n + 1 games.
Total no. of ways is

$$\sum_{r=1}^{n} {}^{2n}C_{n+r} = {}^{2n}C_{n+1} + {}^{2n}C_{n+2} + \ldots\ldots\ldots\ldots + {}^{2n}C_{2n}.$$

$$= \frac{1}{2}(2^{2n} - {}^{2n}C_n).$$

16. (C)

Sol. Treat each of V_1 V_2 ....V_n varieties as n beggars.
Total ways when m mangoes can be taken without any restriction $= {}^{m+n-1}C_m$
Now ${}^{m+n-1}C_m$ = All of same variety + (m – 1) of same variety and 1 diff. + (m–2) of same variety and 2 diff. + ... + all 'm' of diff. variety.
At least two mangoes of the same variety (say x) = Total – All m of different variety
$= {}^{m+n-1}C_m - {}^nC_m \Rightarrow$ (C)]

17. (A)

Sol. Required number of ways = (number of ways in which 16 players can be divided in 8 couples) – (number of ways when S_1 and S_2 are in the same group)

$$= \frac{16!}{2^8\cdot 8!} - \frac{(14)!}{2^7\cdot 7!} = \frac{16\cdot 15\cdot 14!}{2\cdot 2^7\cdot 8\cdot 7!} - \frac{(14)!}{2^7\cdot 7!}$$

$$= \frac{(14)!}{2^7\cdot 7!}\left[\frac{16\cdot 15}{16} - 1\right] = \frac{(14)\cdot(14)!}{14\cdot(6!)\cdot 2^6} = \frac{(14)!}{2^6\cdot 6!} \textbf{ Ans.]}$$

18. (D)

Sol. Line v/s line : $^3C_2 \cdot 1 = 3$
Circle v/s circle : $^4C_2 \cdot 2 = 12$
Parabola v/s Parabola : $^5C_2 \cdot 4 = 40$
Line v/s Circle : $^3C_1 \cdot {^4C_1} \cdot 2 = 24$
Line v/s Parabola : $^3C_1 \cdot {^5C_1} \cdot 2 = 30$
Circle v/s Parabola: $^4C_1 \cdot {^5C_1} \cdot 4 = 80$
Total = 189 **Ans.**]

19. (C)

Sol.
a b a b a b   1st row in 6! ways
× × × × × ×

× × × × × ×
a b a b a b

2nd row in 6! ways
a's and b's in a row in 2! ways

x x x x x x
1 2 3 4 5 6

6 students in column 1, 3, 5 have the same paper and those in 2, 4, 6 have the same paper. 6 such people of two sets can sit in 6 ! 6 ! ways and can interchange in 2 ways]
∴ total no. of ways = $(6!)^2 .2$

20. (C)

Sol. 1st card can be taken in 104 ways , 2nd can be taken in 102 ways , 3rd can be taken in 100 ways and so on .
Hence number of selections ,

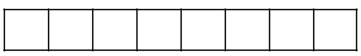
$$\frac{104 . 102 . 100 .... , 54}{26 !}$$

$$2^{26} . \frac{(52 . 51 . 50 .... 27) . 26 !}{26 ! \; 26 !}$$

$$= \frac{2^{26} . 52 !}{(26 !)^2} = {^{52}C_{26}} . 2^{26} \;]$$

21. 10

Sol.
$A's = 4$
$N = 1$
$B = 1$
$L = 1$
$V = 1$



required number = $\frac{8 !}{4 !}$ – number of ways when ends in A

$= \frac{8 !}{4 !} - \frac{7 !}{3 !} = 2 \cdot \frac{7 !}{3 !} - \frac{7 !}{3 !} = \frac{7 !}{3 !} \Rightarrow$ m + n = 10. **Ans.**]

22. 42

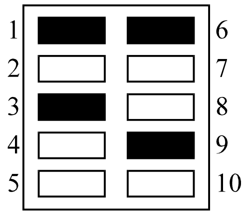
Sol. Put 1 ball in each of 3 boxes. Now remaining balls (k – 3)

use beggar $\underbrace{0\ 0.......\ 0}_{(k-3)}\ \underbrace{\varnothing\ \varnothing}_{2}$ $^{k-1}C_2 = 820$

$$\frac{(k-1)(k-2)}{2} = 820 \quad \Rightarrow \quad k = 42 \ \textbf{Ans.}]$$

23. 770

Sol.



With only five buttons pressing number of combination = $^{10}C_5$
total in redesign = $2^{10} - (^{10}C_0 + {^{10}C_{10}}) = 2^{10} - 2$
additional combinations = $(2^{10} - 2) - {^{10}C_5}$
= 1022 – 252 = 770 Ans.]

24. 208

Sol. T' S = 3
A' S = 1
E' S = 1
R' S = 1
S = 1

$4 \times \frac{4!}{3!} = 16$

$^4C_2 \times \frac{4!}{2!} = 72$

$^5C_4 \times 4! = 120$

208

25. 1275

Sol. Let he gives x, y and z shares to his sons.
now, x + y + z = 101 ....(1)
number of non negative integral solution of the equation (1) = $^{103}C_2$ (Total ways)
When x assumes the value 51 or more than such cases are to be rejected.
now, x + y + z = 50 ....(2)
(where x = 51 or more)
number of solution of (2) = $^{52}C_2$
Total number of such cases 3 . $^{52}C_2$
(when y takes 51 when z takes 51 values.)
∴ required number of ways = $^{103}C_2$ – 3 . $^{52}C_2$ = 1275 **Ans.**]

26. 7000

Sol. for distribution of 3 blue balls
number of ways = ${}^6C_3$ [O O O ∅ ∅ ∅]
(beggar)
||ly 4 white and 2 red balls can be distributed in ${}^7C_3$ and ${}^5C_3$ ways.
Hence using fundamental principle of counting
Total ways = ${}^5C_3 \cdot {}^6C_3 \cdot {}^7C_3$
= 10 × 20 × 35
= 7000 Ans.]

27. 710

Sol. No. of words when it contains

(a) 3 alike, 1 different $\longrightarrow 1 \times {}^8C_1 \times \frac{4!}{3!}$ = 32

(b) 2 alike, 2 alike $\longrightarrow 1 \times \frac{4!}{2!\,2!}$ = 6

(c) 2 alike, 1 different, 1 different $\longrightarrow 2 \times {}^8C_2 \times \frac{4!}{2!}$ = 672

Total number of required words = 710. **Ans.**]

28. 151200

Sol. Total number of ways = ${}^9C_7 \cdot 7!$
now there are 6 pairs of consecutive places e.g. ab, bc,...... when we can place 56 or 65.
This can be done in 6 · 2 = 12 ways and remaining places can be filled in ${}^7C_5 \cdot 5!$.
Here number of ways in which all the seven places can be filled with consecutive 65 or 56
= $12 \cdot {}^7C_5 \cdot 5!$
Hence the required number of ways
= ${}^9C_7 \cdot 7! - ({}^7C_5 \cdot 5! \cdot 12)$
= $36 \cdot 7! - 2 \cdot 6! \cdot 2! = 6 \cdot 6!\,[42 - 7] = 6 \cdot 35 \cdot 6! = 151200$]

29. 629

Sol. A A A, M, N, R, Y

Number of words starting with A = $\frac{6!}{2!}$ = 360

Number of words starting with M = $\frac{6!}{3!}$ = 120

Number of words starting with N = $\frac{6!}{3!}$ = 120

Number of words having 1st 3 letter as [R A A] = 4 ! = 24

Number of words having first 4 letter as [R A M A][A] or [R A M A][N] = 2 ! + 2 ! = 4

All above maintained words will be above "Ramayan" in dictionary, hence number of such words
= 360 + 120 + 120 + 24 + 4 = 628
Hence rank of " RAMAYAN" = 629 **Ans.**

30. 300

Sol. Given : Girls – 5 & boys – 7
team members : girls –2 & boys – 3
Total no. of ways = ${}^5C_2 \cdot {}^7C_3$
Consider when A & B are always included = ${}^5C_1\ {}^5C_2$ as only 1 boy & 2 girls are to we selected.
required no. of ways = total No.of ways – when A & B are always included

$= {}^5C_2 \cdot {}^7C_3 - {}^5C_1\ {}^5C_2$

$= {}^5C_2\left[{}^7C_3 - {}^5C_1\right]$

$= = \frac{5!}{2!3!}\left[\frac{7!}{3!4!} - \frac{5!}{4!}\right]$

$= 10\left[\frac{7 \times 6 \times 5}{3 \times 2} - 5\right]$

$10[35 - 5] = 30 \times 10 = 300.$