JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-6**
CHAPTER :- BINOMIAL THEOREM

ANSWER KEY

1.	(D)	**2.**	(D)	**3.**	(B)	**4.**	(C)	**5.**	(A)	**6.**	(D)	**7.**	(D)
8.	(B)	**9.**	(C)	**10.**	(B)	**11.**	(B)	**12.**	(B)	**13.**	(A)	**14.**	(A)
15.	(C)	**16.**	(D)	**17.**	(A)	**18.**	(C)	**19**	(A)	**20.**	(D)	**21.**	1
22.	3	**23.**	9	**24.**	8	**25.**	43	**26.**	5	**27.**	2008	**28.**	2
29.	23	**30.**	6										

SOLUTIONS

1. (D)

Sol. $(1 + z)^3$ where $z = x(1 + 2x + 3x^2)$

$1 + {}^3C_1 z + {}^3C_2 z^2 + {}^3C_3 z^3$

coefficient of x^3 in $(1 + z)^3$

$${}^3C_1(3) + {}^3C_2(4) + {}^3C_3(1) = 22$$

$\Rightarrow$ $a = 22$

now again $(1 + y)^3$

where $y = x(1 + 2x + 3x^2 + 4x^3)$

$$(1 + y)^3 = 1 + {}^3C_1 y + {}^3C_2 y^2 + {}^3C_3 y^3$$

$\therefore$ coefficient of x^3 is

$${}^3C_1(3) + {}^3C_2(4) + {}^3C_3(1)$$
$$= 9 + 12 + 1 = 22$$

$\Rightarrow$ $b = 22$

Hence $a = b$ $\Rightarrow$ $a + b = 44$ Ans.

2. (D)

Sol. T_{r+1} in $\left(x^2 - \frac{1}{x}\right)^9$ is ${}^9C_r\, x^{2(9-r)}\left(-\frac{1}{x}\right)^r$

$= {}^9C_r \cdot x^{18-3r} \cdot (-1)^r$

for term independent of x, $18 - 3r = 0 \Rightarrow r = 6$

$\therefore$ 7th term is independent of x and equals ${}^9C_6$

$= {}^9C_3 = 84$

Also there are 10 terms, hence 5th term and 6th are the two middle term

$$T_5 = {}^9C_4 \cdot x^6$$
$$T_6 = -{}^9C_5 \cdot x^3$$

$\therefore$ q = coefficient of 5th + coefficient of 6th term

$$= {}^9C_4 - {}^9C_5 = 0$$

hence $p = 84$; $q = 0$

$\therefore$ $p - q = {}^9C_3$ Ans.

3. (B)

Sol. $E = (2n + 1)(2n + 3)(2n + 5) \ldots\ldots (4n - 1)$

Multiply numerator and denominator by $(2n + 2)(2n + 4) \ldots\ldots (4n)$ and also by $(2n)!$ and $n!$.

$$E = \frac{(2n)!\,(2n+1)(2n+2)(2n+3)\ldots\ldots(4n-1).4n}{(2n)!\,(2n+2)(2n+4)\ldots\ldots(2n+2n)}$$

$$= \frac{(4n)! \quad \times \quad (n)!}{(2n)!\,2^n\left[(n+1)(n+2)\ldots\ldots(2n)\right]n!} = \frac{(n!).(4n)!}{2^n.\left((2n)!\right)^2} \Rightarrow \text{B}]$$

4. (C)

Sol. $T_n = \frac{n}{(n-2)! + (n-1)! + n!} = \frac{n}{(n-2)!\cdot[1 + n - 1 + n(n-1)]}$

$$= \frac{n}{(n-2)!\cdot n^2} = \frac{1}{(n-2)!\cdot n} = \frac{n-1}{(n-1)!\cdot n} = \frac{1}{(n-1)!}\left[1 - \frac{1}{n}\right] = \frac{1}{(n-1)!} - \frac{1}{n!}$$

hence sum $= \sum_{n=3}^{2008}\left(\frac{1}{(n-1)!} - \frac{1}{n!}\right)$ sum $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{4!} - \frac{1}{5!} \ldots\ldots\ldots\ldots + \frac{1}{(2007)!} - \frac{1}{(2008)!}$

$$= \frac{1}{2!} - \frac{1}{(2008)!} = \frac{(2008)! - 2}{2\cdot(2008)!}\ ]$$

5. (A)

Sol. Let $y + z = t$

$\therefore$ $E = (x + t)^{100} + (x - t)^{100}$

$= 2[{}^{100}C_0 + {}^{100}C_2\, t^2 + {}^{100}C_4\, t^4 + \ldots\ldots\ldots\ldots + {}^{100}C_{100}\, t^{100}]$

number of terms $= \underbrace{1 + 3 + 5 + 7 + \ldots\ldots + 101}_{51 \text{ terms}} = (51)^2 = 2601$ Ans.

6. (D)

Sol. Let ${}^nC_{151} = q \cdot {}^nC_{150}$ where $q \neq 1$

$$\frac{n!}{(151)!(n-151)!} = \frac{q\cdot n!}{(150)!(n-150)!}$$

$\frac{1}{151} = \frac{q}{n-150}$ $\Rightarrow n - 150 = q \cdot 151$ or $n = 151q + 150$

when $q = 2$

smallest n will be

$n = 302 + 150$

$n = 452 \Rightarrow$ sum of the digit = 11 Ans.]

7. (D)

Sol. $(1 - x + 2x^2)^{12} = \left((1 - x) + 2x^2\right)^{12}$

$= (1 - x)^{12} + {}^{12}C_1(1 - x)^{11}\, 2x^2 + {}^{12}C_2\,(1 - x)^{10}\, 4x^4 + \ldots\ldots$

required co-efficient of x^4 =
co-eff. of x^4 in $(1-x)^{12}$ + 24 co-eff. of x^2 in $(1-x)^{11}$ + 4 . ${}^{12}C_2$
$= {}^{12}C_4 + 24 \,.\, {}^{11}C_2 + 4 \,.\, {}^{12}C_2 = {}^{12}C_4 + \frac{2.12.11.10.3}{1.2.3} + 4 \,.\, {}^{12}C_2$
$= {}^{12}C_4 + 6 \,.\, {}^{12}C_3 + 4 \,.\, {}^{12}C_2 = {}^{12}C_4 + 2 \,.\, {}^{12}C_3 + 4\,({}^{12}C_3 + {}^{12}C_2)$
$= {}^{12}C_4 + 2 \,.\, {}^{12}C_3 + 4 \,.\, {}^{13}C_3 = {}^{12}C_4 + {}^{12}C_3 + {}^{12}C_3 + 4 \,.\, {}^{13}C_3$
$= {}^{13}C_4 + {}^{13}C_3 + {}^{12}C_3 + 3 \,.\, {}^{13}C_3 = {}^{14}C_4 + 3 \,.\, {}^{13}C_3 + {}^{12}C_3 \Rightarrow$ D]

8. (B)

Sol. $E = (\alpha + p)^{m-1}$

$\left[1+\frac{\alpha+q}{\alpha+p}+\left(\frac{\alpha+q}{\alpha+p}\right)^2+......+\left(\frac{\alpha+q}{\alpha+p}\right)^{m-1}\right] = (\alpha + p)^{m-1}\left[\frac{\left(\frac{\alpha+q}{\alpha+p}\right)^m-1}{\left(\frac{\alpha+q}{\alpha+p}\right)-1}\right]$

$= \frac{(\alpha+p)^m[(\alpha+p)^m-(\alpha+q)^m]}{(\alpha+p)^m(p-q)}$

hence coefficient of α^t in $\frac{(\alpha+p)^m-(\alpha+q)^m}{p-q}$

$= \frac{(p+\alpha)^m-(q+\alpha)^m}{p-q} = \frac{{}^mC_t\left(p^{m-t}-q^{m-t}\right)}{p-q}$]

9. (C)

Sol. $\underbrace{\underbrace{{}^{47}C_4 + {}^{47}C_3}_{{}^{48}C_4} + {}^{48}C_3}_{{}^{49}C_4} + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$. **Ans.**]

10. (B)

Sol. Note that $\frac{17}{4} + 3\sqrt{2} = \left(\frac{3}{2}+\sqrt{2}\right)^2$.

Hence we have $\left(3-\frac{3}{2}-\sqrt{2}\right)^{15} = \left(\frac{3}{2}-\sqrt{2}\right)^{15}$]

11. (B)

Sol. Coefficient of x^n in the given polynomial is
${}^nC_0 + 3 \cdot {}^nC_1 + 5 \cdot {}^nC_2 + ... + (2n+1) \cdot {}^nC_n$
$= ({}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n) + 2({}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + ... + n \cdot {}^nC_n)$
$= 2^n + 2S$
now $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$
differentiating w.r.t. x and put x = 1
$S = {}^nC_1 + 2 \cdot C_2 + 3 \cdot C_3 + ... + n \cdot C_n = n \cdot 2^{n-1}$
hence coefficient of $x^n = 2^n + 2 \cdot n \cdot 2^{n-1} = 2^n(n+1)$
$\therefore$ coefficient OF $X^9 = 2^9 \cdot 10 = 5120$ **Ans.**]

12. (B)

Sol. $S = {}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + {}^{13}C_{10} + ... + {}^{2006}C_{10}$

$= \underbrace{{}^{11}C_0 + {}^{11}C_1} + {}^{12}C_2 + {}^{13}C_3 + ... + {}^{2006}C_{1996}$

$= \underbrace{{}^{12}C_1 + {}^{12}C_2} + ...$

${}^{13}C_2$ and so on

$\therefore \quad S = {}^{2007}C_{1996} = {}^{2007}C_{11} = {}^nC_p$

$\therefore \quad n + p = 2018$ Ans.]

13. (A)

Sol. $\frac{{}^nC_k}{{}^nC_{k+1}} = \frac{1}{2} \Rightarrow \frac{n!}{k!(n-k)!} \cdot \frac{(k+1)!(n-k-1)!}{n!} = \frac{1}{2}$ or $\frac{k+1}{n-k} = \frac{1}{2}$

$2k + 2 = n - k \quad n - 3k = 2 \quad ....(1)$

|||ly $\frac{{}^nC_{k+1}}{{}^nC_{k+2}} = \frac{2}{3}$

$\frac{n!}{(k+1)!(n-k-1)!} \cdot \frac{(k+2)!(n-k-2)!}{n!} = \frac{2}{3}$

$\frac{k+2}{n-k-1} = \frac{2}{3}$

$3k + 6 = 2n - 2k - 2$

$2n - 5k = 8 \quad ....(2)$

From (1) and (2) n = 14 and k = 4

$\therefore \quad n + k = 18$ **Ans.**]

14. (A)

Sol. $a_k = (k^2+1)\,k! = \left(k(k+1)-(k-1)\right)k! = k(k+1)! - (k-1)k!$

so $\quad k(k+1)! - (k-1)k!$

$a_1 = 1 \cdot 2! - 0$

$a_2 = 2 \cdot 3! - 1 \cdot 2!$

$a_3 = 3 \cdot 4! - 2 \cdot 3!$

$\vdots$

$a_k = k(k+1)! - (k-1)k!$

$a_1 + a_2 + ... + a_k = k(k+1)!$

hence $\quad b_k = k(k+1)!$

$\therefore \quad \frac{a_k}{b_k} = \frac{(k^2+1)k!}{k(k+1)!} = \frac{(k^2+1)}{k(k+1)} = \frac{k^2+1}{k^2+k}$;

$\frac{a_{100}}{b_{100}} = \frac{10001}{10100} = \frac{m}{n}$; $\therefore$ (n – m) = 99 **Ans.**]

15. (C)

Sol. Number of terms in $(1 + x)^{2009} = 2010$ **....(1)**

+ additional terms in $(1 + x^2)^{2008} = x^{2010} + x^{2012} + \ldots\ldots + x^{4016} = 1004$ **....(2)**
+ additional terms in $(1 + x^3)^{2007} = x^{2010} + x^{2013} + \ldots\ldots + x^{4014} + \ldots. + x^{6021} = 1338$ **....(3)**
– (common to 2 and 3) = $x^{2010} + x^{2016} + \ldots\ldots + x^{4014}) = 335$
Hence total = 2010 + 1004 + 1338 – 335
= 4352 – 335 = 4017 **Ans.**

16. (D)

Sol. ${}^nC_0 + {}^nC_2 \dfrac{1}{2^2} = 2\,{}^nC_1 \dfrac{1}{2}$

$\Rightarrow 1 + \dfrac{n(n-1)}{8} = n = \dfrac{n^2 - n + 8}{8}$

$\Rightarrow n^2 - 9n + 8 = 0 \Rightarrow n = 1, 8$ n = 1 (rejected)

$\therefore$ sum of coefficient = $\left(1 + \dfrac{1}{2}\right)^8 = \left(\dfrac{3}{2}\right)^8$]

17. (A)

Sol. $[1 + (2x - x^2)]^4$
$= {}^4C_0 + {}^4C_1 (2x - x^2) + {}^4C_2 (2x - x^2)^2 + {}^4C_3(2x - x^2)^3 + {}^4C_4(2x - x^2)^4$
Hence x^7 would appear only in $(2x - x^2)^4$
$\therefore t_{r+1}$ in $(2x - x^2)^4$ is ${}^4C_r(2x)^{4-r}(-1)^r x^{2r}$
put r = 3
$-\ {}^4C_3 \cdot 2 \cdot x \cdot x^6 = -8x^7$
$\therefore$ Coefficient of x^7 is – 8 **Ans.**]

18. (C)

Sol. $a_n = \dfrac{1}{{}^nC_0} + \dfrac{1}{{}^nC_1} + \dfrac{1}{{}^nC_2} + \ldots\ldots$

$S = \dfrac{0}{{}^nC_0} + \dfrac{1}{{}^nC_1} + \dfrac{2}{{}^nC_2} + \dfrac{3}{{}^nC_3} + \ldots\ldots + \dfrac{n}{{}^nC_n}$

$S = \dfrac{n}{{}^nC_0} + \dfrac{n-1}{{}^nC_n} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$2S = n\left[\dfrac{1}{{}^nC_0} + \dfrac{1}{{}^nC_1} + \dfrac{1}{{}^nC_2} + \ldots\ldots\right] = \dfrac{na_n}{2}$ **Ans.**]

19 (A)

Sol. $T_{r+1} = {}^{100}C_r (3x)^{100-r} \cdot 5^r$
$= {}^{100}C_r \cdot 3^{100-r} \cdot 5^r \cdot x^{100-r}$
$\therefore 100 - r = 39 \Rightarrow r = 61$
$\therefore$ Coefficient of $x^{39} = {}^{100}C_{39} \cdot 3^{39} \cdot 5^{61}$.

Now ${}^{100}C_{39} = \dfrac{(100)!}{39!\,(61)!}$

$= \dfrac{50+25+12+6+3+1}{(19+9+4+2+1)+(30+15+7+3+1)}$

$= \dfrac{97}{(35)+(56)} = \dfrac{97}{91} = 6.$

Hence largest power of 2 that divides the coefficient of x^{39} is 6 **Ans.**]

20. (D)

$29\binom{30}{0} + 28\binom{30}{1} + 27\binom{30}{2} + \ldots\ldots$

Sol. S = $+1\binom{30}{28} + 0\binom{30}{29} - \binom{30}{30}$
$-\binom{30}{0} + 0\binom{30}{1} + 1\binom{30}{2} + \ldots\ldots$

Also S = $+27\binom{30}{28} + 28\binom{30}{29} + 29\binom{30}{30}$

$\Rightarrow 2S = 28({}^{30}C_0 + {}^{30}C_1 + {}^{30}C_2 + \ldots\ldots\ldots + {}^{30}C_{30})$
$= 28(2^{30})$

Hence, $S = 14 \times 2^{30} = \dfrac{7}{2} \times 2^{32}$. $\Rightarrow k = \dfrac{7}{2}$.]

21. 1

Sol. The coefficient of x^7 in $\left(\alpha x^2 + \dfrac{1}{\beta x}\right)^{11} = {}^{11}C_5(\alpha)^6 (\beta)^{-5}$

Also, the coefficient of x^{-7} in $\left(\alpha x - \dfrac{1}{\beta x^2}\right)^{11} =$ ${}^{11}C_6(\alpha)^5(\beta)^{-6}$
Now according to question we get

${}^{11}C_5 \cdot \dfrac{(\alpha)^6}{(\beta)^5} = \dfrac{{}^{11}C_6 \cdot (\alpha)^5}{(\beta)6} \Rightarrow \alpha\beta = 1$ **Ans.**]

22. 3

Sol. $T_{r+1} = {}^nC_r \left(\sqrt{x}\right)^{n-r} \cdot \left(\dfrac{1}{2\cdot\sqrt[4]{x}}\right)^r$

Also, ${}^nC_0$, $\dfrac{{}^nC_1}{2}$, $\dfrac{{}^nC_2}{4}$ (in that order) are in A.P.

$\Rightarrow 2\left(\dfrac{{}^nC_1}{2}\right) = {}^nC_0 + \dfrac{{}^nC_2}{4} \qquad \Rightarrow n = 8$

$\therefore T_{r+1} = \dfrac{{}^8C_r}{2^r} \cdot (x)^{\frac{16-3r}{4}}$; r = 0, 1, 2,3 ,4, 5, 6, 7, 8

But $\left(\dfrac{16-3r}{4}\right)$ is an integer for r = 0, 4, 8.]

23. 9

Sol. P = $^nC_6\left(3^{\frac{1}{3}}\right)^{n-6}\cdot\left(4^{\frac{-1}{3}}\right)^{6}$

Q = $^nC_{n-6}\left(3^{\frac{1}{3}}\right)^{6}\cdot\left(4^{\frac{-1}{3}}\right)^{n-6}$

$\therefore \frac{Q}{P} = 12 \Rightarrow (12)^{\frac{n-6}{3}} = (12)^1 \Rightarrow \frac{n-6}{3} = 1 \Rightarrow$

n = 9 **Ans.**]

24. 8

Sol. $\sum_{k=r}^{n} {}^kC_r = {}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + ... + {}^nC_r$

$= 1 + {}^{r+1}C_1 + {}^{r+2}C_2 + {}^{r+3}C_3 + ... + {}^nC_{n-r}$

$= {}^{r+1}C_0 + {}^{r+1}C_1 + {}^{r+2}C_2 + ... + {}^nC_{n-r}$

$= {}^{r+2}C_1$ ... ${}^{r+3}C_2$ and so on finally ${}^{n+1}C_{n-r}$

now, $^{n+1}C_{n-r} = {}^{n+1}C_{r+1}$

$\therefore f(n) = \sum_{r=0}^{n} {}^{n+1}C_{r+1} = {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 +$ ..... $+ {}^{n+1}C_{n+1}$

$= {}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + ... + {}^{n+1}C_{n+1} - 1$

$f(n) = (2^{n+1}) - 1$

$f(9) = 2^{10} - 1 = 1023 = 3 \cdot 11 \cdot 31$

hence number of divisors are (1 + 1)(1 + 1)(1 + 1) = 8 Ans.]

25. 43

Sol. $(t + 1)(t^2 + 2)(t^3 + 3) .....(t^{20} + 20) = t^{1+2+3+....+20}\left(1+\frac{1}{t}\right)\left(1+\frac{2}{t^2}\right).....\left(1+\frac{20}{t^{20}}\right)$

$= t^{210}\left(1+\frac{1}{t}\right)\left(1+\frac{2}{t^2}\right).....\left(1+\frac{20}{t^{20}}\right)$

$\Rightarrow$ Coefficient of t^{203} in $(t + 1)(t^2 + 2) .... (t^{20} + 20)$ = coefficient of t^{-7} in $\left(1+\frac{1}{t}\right)\left(1+\frac{2}{t^2}\right).....\left(1+\frac{20}{t^{20}}\right)$

As 7 = 1 + 2 + 4 = 2 + 5 = 3 + 4 = 1 + 6

Hence required coefficient = 1 · 2 · 4 + 2 · 5 + 3 · 4 + 1 · 6 + 7 = 8 + 10 + 12 + 6 + 7 = 43 **Ans.**]

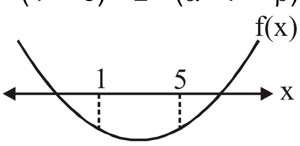
26. 5

Sol. We have $1+\sum_{r=1}^{10}\left(3^r\cdot{}^{10}C_r + r\cdot{}^{10}C_r\right)$

$= 1+\sum_{r=1}^{10}3^r\cdot{}^{10}C_r + 10\sum_{r=1}^{10}{}^9C_{r-1} = 1 + 4^{10} - 1 + 10\cdot 2^9$

$= 4^{10} + 5.2^{10} = 2^{10}(4^5 + 5) = 2^{10}(\alpha \cdot 4^5 + \beta)$, so

α = 1 and β = 5



Now f (1) < 0 and f (5) < 0

So f (1) < 0 $\Rightarrow$ $-k^2 < 0 \Rightarrow$ $k \neq 0$

and f (5) < 0 $\Rightarrow$ $16 - k^2 < 0$ $\Rightarrow$

$k^2 - 16 > 0$ $\Rightarrow$ $k \in (-\infty, 4) \cup (4, \infty)$

Hence smallest positive integral value of k = 5 **Ans.**]

27. 2008

Sol. Sum of all the coefficients is $(1 + i)^{2009} = ((1 + i)^2)^{1004}(1 + i) = 2^{1004}(1 + i) = u + iv$

$\Rightarrow u = 2^{1004}$ and $v = 2^{1004}$

Hence $(\log_2 u + \log_2 v) = 1004 + 1004 = 2008$ **Ans.**]

28. 2

Sol. Let a = $2^{\log(10-3^x)}$ and b = $2^{(x-2)\log 3}$

hence $T_{r+1} = {}^mC_r\, a^{\frac{m-r}{2}}\, b^{\frac{r}{5}}$

$\therefore$ $T_6 = {}^mC_5\, a^{\frac{m-r}{2}}\, b^{\frac{r}{5}}$ ....(1)

now ${}^mC_1, {}^mC_2, {}^mC_3$ are in A.P. $\Rightarrow$ m = 7 or – 2 (rejected)

substituting m = 7 in equation (1)

${}^7C_5 \cdot a \cdot b = 21$

$\therefore$ ab = 1

$2^{\log(10-3^x)}.2^{(x-2)\log 3} = 1$

$\log(10 - 3^x) + (x - 2)\log 3 = 0$

$\log(10 - 3^x)(3^{x-2}) = 0$

$\frac{(10-3^x)3^x}{9} = 1 \Rightarrow$ x = 0 or 2

Hence Sum = 2 **Ans.**]

29. 23

Sol. Given, ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1 : 7 : 35$

$\Rightarrow \frac{r+1}{n-r} = \frac{1}{7} \Rightarrow 8r = n - 7$ ....(1)

Again, $\frac{r+2}{n-(r+1)} = \frac{1}{5}$ $\Rightarrow 6r = n - 11$ ....(2)

$\therefore$ On solving, we get n = 23 and r = 2. **Ans.**]

30. 6

Sol. $T_{2r+4} = {}^{18}C_{2r+3}\, x^{2r+3}$

$T_{r-2} = {}^{18}C_{r-3}\, x^{r-3}$

${}^{18}C_{2r-3} = {}^{18}C_{r-3}$

2r + 3 = r – 3 2r + 3 + r – 3 = 18

r = – 6 (reject) 3r = 18

r = 6]