JEE MAIN : CHAPTER WISE TEST PAPER-6

SUBJECT :- MATHEMATICS | **DATE..**
CLASS :- 11th | **NAME..**
CHAPTER :- BINOMIAL THEOREM | **SECTION..**

(SECTION-A)

1. Let a and b be the coefficient of x^3 in $(1 + x + 2x^2 + 3x^3)^3$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^3$, respectively then
(A) $a \neq b$ (B) $a > b$
(C) $a < b$ (D) $a + b = 44$

2. Let the term independent of x in the expansion of $\left(x^2 - \frac{1}{x}\right)^9$ has the value p and q be the sum of the coefficients of its middle terms, then (p – q) equals
(A) 0 (B) $2 \cdot {}^9C_4$ (C) ${}^9C_5$ (D) ${}^9C_3$

3. $(2n+1)(2n+3)(2n+5)\,.......\,(4n-1)$ is equal to
(A) $\frac{(4n)!}{2^n.(2n)!(2n)!}$ (B) $\frac{(4n)!\ n!}{2^n.(2n)!(2n)!}$
(C) $\frac{(4n)!\ n!}{(2n)!(2n)!}$ (D) $\frac{(4n)!\ n!}{2^n\ !(2n)!}$

4. The sum of the series
$$\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} + \frac{5}{3!+4!+5!} + ...+ \frac{2008}{(2006)!+(2007)!+(2008)!}$$ is equal to
(A) $\frac{(2008)!+2}{2\cdot(2008)!}$ (B) $\frac{(2008)!+1}{2\cdot(2008)!}$
(C) $\frac{(2008)!-2}{2\cdot(2008)!}$ (D) $\frac{(2008)!-3}{2\cdot(2008)!}$

5. The expression $(x + y + z)^{100} + (x - y - z)^{100}$ is simplified by expanding it and combining like terms. Number of terms in the simplified expression is
(A) 2601 (B) 2652
(C) 2401 (D) 2500

6. Let n be the smallest positive integer larger than 150 so that the number ${}^nC_{151}$ is divisible by ${}^nC_{150}$ but is not equal to it. The sum of the digits of n, is
(A) 5 (B) 8 (C) 9 (D) 11

7. The co–efficient of x^4 in the expansion of $(1 - x + 2x^2)^{12}$ is :
(A) ${}^{12}C_3$
(B) ${}^{13}C_3$
(C) ${}^{14}C_4$
(D) ${}^{12}C_3 + 3\ {}^{13}C_3 + {}^{14}C_4$

8. Co-efficient of α^t in the expansion of, $(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + ...(\alpha + q)^{m-1}$
where $\alpha \neq -q$ and $p \neq q$ is :
(A) $\frac{{}^mC_t\left(p^t - q^t\right)}{p-q}$
(B) $\frac{{}^mC_t\left(p^{m-t} - q^{m-t}\right)}{p-q}$
(C) $\frac{{}^mC_t\left(p^t + q^t\right)}{p-q}$
(D) $\frac{{}^mC_t\left(p^{m-t} + q^{m-t}\right)}{p-q}$

9. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to
(A) ${}^{47}C_5$ (B) ${}^{52}C_5$ (C) ${}^{52}C_4$ (D) ${}^{49}C_4$

10. In the expansion of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{15}$, the 11th term is a :
(A) positive integer
(B) positive irrational number
(C) negative integer
(D) negative irrational number .

11. Coefficient of x^n in the polynomial $(x + {}^nC_0)(x + 3 \cdot {}^nC_1)(x + 5 \cdot {}^nC_2)\,...............(x + (2n + 1) \cdot {}^nC_n)$ when n = 9 is
(A) 2560 (B) 5120
(C) 4096 (D) 5160

12. If $\sum_{k=10}^{2006} {}^kC_{10}$ simplifies to ${}^nC_p$ where p is prime then (n + p) has the value equal to
(A) 2017 (B) 2018
(B) 2019 (D) 2020

13. The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^nC_k : {}^nC_{k+1} : {}^nC_{k+2}$. The sum of all possible values of (n + k) is
(A) 18 (B) 21 (C) 28 (D) 32

14. Define $a_k = (k^2 + 1)k!$ and $b_k = a_1 + a_2 + a_3 + \ldots\ldots + a_k$. Let $\frac{a_{100}}{b_{100}} = \frac{m}{n}$ where m and n are relatively prime natural numbers. The value of $(n - m)$ is equal to

(A) 99 (B) 100 (C) 101 (D) 102

15. Number of different terms in the sum $(1 + x)^{2009} + (1 + x^2)^{2008} + (1 + x^3)^{2007}$, is

(A) 3683 (B) 4007
(C) 4017 (D) 4352

16. In the binomial expansion of $\left(\sqrt{x} + \frac{1}{2\cdot\sqrt[4]{x}}\right)^n$, the first three coefficients form an arithmetic progression, then sum of coefficients of all the terms is

(A) $\left(\frac{3}{2}\right)^5$ (B) $\left(\frac{3}{2}\right)^6$ (C) $\left(\frac{3}{2}\right)^7$ (D) $\left(\frac{3}{2}\right)^8$

17. The coefficient of x^7 in the expansion of $(1 + 2x - x^2)^4$ is

(A) – 8 (B) 12 (C) 6 (D) – 6

18. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

(A) $(n-1)a_n$
(B) $n\,a_n$
(C) $n\,a_n / 2$
(D) can not be determined

19 When $(3x + 5)^{100}$ is expanded, the largest power of 2 dividing the coefficient of x^{39} is

(A) 6 (B) 7 (C) 2^6 (D) 2^7

20. If the value of the sum

$$29\binom{30}{0}+28\binom{30}{1}+27\binom{30}{2}+\ldots\ldots+1\binom{30}{28}+0\binom{30}{29}-\binom{30}{30}$$

where $\binom{n}{r} = {}^nC_r$, is equal to $k\,2^{32}$, then the value of k is equal to

(A) 7 (B) 14 (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

(SECTION-B)

21. Let α and β be non-zero complex numbers. If the coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{\beta x}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(\alpha x - \frac{1}{\beta x^2}\right)^{11}$ then find the value of $(\alpha\beta)$.

22. If the coefficient of first three terms in the expansion $\left(\sqrt{x} + \frac{1}{2\cdot\sqrt[4]{x}}\right)^n$ where $n \in N$ form an arithmetic progression, then find the number of terms in the expansion having integral powers of x.

23. Let P be the 7^{th} term from the beginning and Q be the 7^{th} term from the end in the expansion of $\left(\sqrt[3]{3} + \frac{1}{\sqrt[3]{4}}\right)^n$ where $n \in N$. If $12P = Q$, then find the value of n.

24. Let $f(n) = \sum_{r=0}^{n}\sum_{k=r}^{n}\binom{k}{r}$. Find the total number of divisors of $f(9)$.

25. Find the coefficient of t^{203} in the expression $(t + 1)(t^2 + 2)(t^3 + 3)\ldots\ldots\ldots(t^{20} + 20)$.

26. Let $1+\sum_{r=1}^{10}\left(3^r\cdot{}^{10}C_r + r\cdot{}^{10}C_r\right) = 2^{10}(\alpha\cdot 4^5 + \beta)$ where $\alpha, \beta \in N$ and $f(x) = x^2 - 2x - k^2 + 1$. If α, β lies between the roots of $f(x) = 0$, then find the smallest positive integral value of k.

27. If sum of coefficients in the expansion of $(1 + ix)^{2009}$ is $(u + iv)$, $(u, v \in R$ and $i^2 = -1)$, then find the value of $(\log_2 u + \log_2 v)$.

28. Find the sum of all possible value(s) of 'x' for which the sixth term of

$$\left[\left(2^{\log\left(10-3^x\right)}\right)^{\frac{1}{2}} + \left(2^{(x-2)\log 3}\right)^{\frac{1}{5}}\right]^m$$

is equal to 21 and binomial coefficients of second, third and fourth terms are the first, third & fifth terms of an arithmetic progression. [Take every where base of log as 10]

29. If the co-efficients of 3 consecutive terms in the expansion of $(1 + x)^n$, $n \in N$ are in the ratio 1 : 7 : 35, then find n.

30. If the coefficients of $(2r + 4)^{th}$, $(r - 2)^{th}$ terms in the expansion of $(1 + x)^{18}$ are equal, find the value of r.