## JEE MAIN ANSWER KEY & SOLUTIONS

**SUBJECT :- MATHEMATICS**

**CLASS :- 11th** **PAPER CODE :- CWT-5**

**CHAPTER :- SEQUENCE & SERIES**

## ANSWER KEY

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (A) | **2.** | (D) | **3.** | (C) | **4.** | (D) | **5.** | (D) | **6.** | (A) | **7.** | (C) |
| **8.** | (C) | **9.** | (B) | **10.** | (A) | **11.** | (A) | **12.** | (B) | **13.** | (C) | **14.** | (C) |
| **15.** | (B) | **16.** | (D) | **17.** | (B) | **18.** | (A) | **19.** | (A) | **20.** | (C) | **21.** | 0 |
| **22.** | 4 | **23.** | 64 | **24.** | 18 | **25.** | 8 | **26.** | 18 | **27.** | 3 | **28.** | 900 |
| **29.** | 64 | **30.** | 1 | | | | | | | | | | |

## SOLUTIONS

**1.** (A)

**Sol.** Given series $3.8+6.11+9.14+12.17+.....$

First factors are 3, 6, 9, 12 whose $n^{th}$ term is $3n$ and second factors are 8, 11, 14, 17

$t_n=[8+(n-1)3]=(3n+5)$

Hence $n^{th}$ term of given series $=3n(3n+5)$.

**2.** (D)

**Sol.** $\log_3 2, \log_3(2^x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in A.P.

$\Rightarrow 2\log_3(2^x-5)=\log_3\left[(2)\left(2^x-\frac{7}{2}\right)\right]$

$\Rightarrow (2^x-5)^2=2^{x+1}-7$

$\Rightarrow 2^{2x}-12\,.\,2^x-32=0$

$\Rightarrow x=2,3$

But $x=2$ does not hold, hence $x=3$.

**3.** (C)

**Sol.** $a_{m+k}=A+(m+k-1)D$

and $a_{m-k}=A+(m-k-1)D$

Adding both equations, we get

$\frac{a_{m+k}+a_{m-k}}{2}=A+(m-1)D=a_m$.

**4.** (D)

**Sol.** Suppose that required numbers $a$ and $b$.

Therefore according to the conditions

$a=\frac{1}{b}$

and $\frac{a+b}{2}=\frac{13}{12}\Rightarrow a+b=\frac{13}{6}$

$\Rightarrow a+\frac{1}{a}=\frac{13}{6}\Rightarrow 6a^2-13a+6=0$

$\Rightarrow \left(a-\frac{3}{2}\right)\left(a-\frac{2}{3}\right)=0\Rightarrow a=\frac{3}{2}$ and $b=\frac{2}{3}$

or $a=\frac{2}{3}$ and $b=\frac{3}{2}$.

**5.** (D)

**Sol.** $l=ar^{n-1}\Rightarrow \frac{l}{a}=r^{n-1}\Rightarrow \frac{\log l-\log a}{\log r}+1=n$.

**6.** (A)

**Sol.** Let $\frac{A}{R}, A, AR$ be the roots of the equation

$ax^3+bx^2+cx+d=0$

then $A^3=$ Product of the roots $=-\frac{d}{a}$

$\Rightarrow A=-\left(\frac{d}{a}\right)^{1/3}$

Since $A$ is a root of the equation.

$\therefore aA^3+bA^2+cA+d=0$

$\Rightarrow a\left(-\frac{d}{a}\right)+b\left(-\frac{d}{a}\right)^{2/3}+c\left(-\frac{d}{a}\right)^{1/3}+d=0$

$\Rightarrow b\left(\frac{d}{a}\right)^{2/3}=c\left(\frac{d}{a}\right)^{1/3}\Rightarrow b^3\frac{d^2}{a^2}=c^3\frac{d}{a}$

$\Rightarrow b^3d=c^3a$.

**7.** (C)

**Sol.** $1+(1+x)+(1+x+x^2)+...+$

$(1+x+x^2+x^3+...+x^{n-1})+...$

Required sum

$=\frac{1}{(1-x)}\{(1-x)+(1-x^2)+(1-x^3)$

$+(1-x^4)+.........\text{uptp } n \text{ terms}\}$

$=\frac{1}{(1-x)}[n-\{x+x^2+x^3+.........\text{upto } n \text{ terms}\}]$

$=\frac{1}{(1-x)}\left[n-\frac{x(1-x^n)}{1-x}\right]=\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$.

**8.** (C)

**Sol.** We have $\frac{a}{1-r}=x$ .....(i)

and $\frac{a^2}{1-r^2}=\frac{a}{1-r}.\frac{a}{1+r}=y$ .....(ii)

$\Rightarrow y=x.\frac{a}{1+r}=x.\frac{x(1-r)}{1+r}\Rightarrow \frac{y}{x^2}=\frac{1-r}{1+r}$

$\Rightarrow \frac{x^2}{y}=\frac{1+r}{1-r}\Rightarrow \frac{x^2}{y}(1-r)=1+r$

$\Rightarrow r\left[1+\frac{x^2}{y}\right]=-1+\frac{x^2}{y}\Rightarrow r=\frac{x^2-y}{x^2+y}$.

**9.** (B)

**Sol.** Let $r$ be the common ratio of the G.P. Then

$S = \frac{a}{1-r} \Rightarrow r = 1 - \frac{a}{S}$

Now $S_n$ = Sum of $n$ terms

$= a\left(\frac{1-r^n}{1-r}\right) = \frac{a}{1-r}(1-r^n) = S\left[1 - \left(1 - \frac{a}{S}\right)^n\right]$.

**10.** (A)

**Sol.** Given that $a, A_1, A_2, b$ are in A.P.

Therefore $A_1 = \frac{a+A_2}{2}, A_2 = \frac{A_1+b}{2}$

$\Rightarrow A_1 + A_2 = \frac{1}{2}(a+b+A_1+A_2)$

$\Rightarrow \frac{1}{2}(A_1+A_2) = \frac{1}{2}(a+b)$ or $A_1 + A_2 = a+b$ .....(i)

and $a, G_1, G_2, b$ are in G.P.

Therefore $G_1^2 = aG_2, G_2^2 = bG_1$ .....(ii)

$\Rightarrow G_1^2 G_2^2 = abG_1G_2 \Rightarrow G_1G_2 = ab$

Hence $\frac{A_1+A_2}{G_1G_2} = \frac{a+b}{ab}$

**11.** (A)

**Sol.** Let $a$ be the first term and $d$ be the common difference of the given A.P. Then as given the $(m+1)^{th}$, $(n+1)^{th}$ and $(r+1)^{th}$ terms are in G.P.

$\Rightarrow a+md, a+nd, a+rd$ are in G.P.

$\Rightarrow (a+nd)^2 = (a+md)(a+rd)$

$\Rightarrow a(2n-m-r) = d(mr-n^2)$

or $\frac{d}{a} = \frac{2n-(m+r)}{mr-n^2}$ .....(i)

Next, $m, n, r$ in H.P. $\Rightarrow n = \frac{2mr}{m+r}$ .....(ii)

From (i) and (ii)

$\frac{d}{a} = \frac{2n-(m+r)}{mr-n^2} = \frac{2}{n}\left(\frac{2n-(m+r)}{(m+r)-2n}\right) = -\frac{2}{n}$.

**12.** (B)

**Sol.** Let $a$ and $b$ be the same first and last terms of the three progressions, each having $(2n+1)$ terms. Then the middle term of the A.P. is $\frac{a+b}{2}$. The middle term of the G.P. is $\sqrt{ab}$. The middle term of the H.P. is $\frac{2ab}{a+b}$. Obviously, these terms are in G.P.

**13.** (C)

**Sol.** Let the two numbers be $p$ and $q$.

$\therefore G_1 = p^{2/3}\ q^{1/3}, G_2 = p^{1/3}\ q^{2/3}$

$\therefore \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{p^{4/3}\ q^{2/3}}{p^{1/3}\ q^{2/3}} + \frac{p^{2/3}\ q^{4/3}}{p^{2/3}\ q^{1/3}}$

$= p + q = 2 \times \left(\frac{p+q}{2}\right) = 2A$.

**14.** (C)

**Sol.** If $\frac{1}{16}, a, b$ are in G.P., then

$a^2 = \frac{b}{16}$ or $16a^2 = b$ .....(i)

and if $a, b, \frac{1}{6}$ are in H.P.,

then $b = \frac{2.a.\frac{1}{6}}{a+\frac{1}{6}} = \frac{2a}{6a+1}$ .....(ii)

From (i) and (ii), $16a^2 = \frac{2a}{6a+1}$

or $2a\left(8a - \frac{1}{6a+1}\right) = 0$ or $8a(6a+1) - 1 = 0$

or $48a^2 + 8a - 1 = 0$, $(\because a \neq 0)$

or $(4a+1)(12a-1) = 0 \ \therefore \ a = -\frac{1}{4}, \frac{1}{12}$

When $a = -\frac{1}{4}$ then from (i), $b = 16\left(-\frac{1}{4}\right)^2 = 1$

When $a = \frac{1}{12}$ then from (i), $b = 16\left(\frac{1}{12}\right)^2 = \frac{1}{9}$

Therefore, $a = -\frac{1}{4}, b = 1$ or $a = \frac{1}{12}, b = \frac{1}{9}$.

**15.** (B)

**Sol.** $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$ n is odd.

Let n = 2 m + 1.

Sum = $1^2 + 2.2^2 + 3^2 + 2.4^2 + \ldots + 2(2m)^2 + (2m+1)^2$.

$= \Sigma(2m+1)^2 + 4[1^2 + 2^2 + 3^2 + \ldots + m^2]$

($\because$ odd place terms + even place terms)

$= \frac{(2m+1)(2m+2)(4m+2+1)}{6} + \frac{4m(m+1)(2m+1)}{6}$

$= \frac{(2m+1)(m+1)}{6}[2(4m+3)+4m]$

$= \frac{(2m+1)(2m+2)(6m+3)}{6}$

$= \frac{(2m+1)^2(2m+2)}{2} = \frac{n^2(n+1)}{2}$

**16.** (D)

**Sol.** 1 + (1 + 2) + (1 + 2 + 3) + .........

S = 1 + (1 + 2) + (1 + 2 + 3) + .........

S = 1 + (1 + 2) + (1 + 2 + 3) + ......... +(1+ 2 + 3 + 4 + – – n).

the last term = (1 + 2 + 3 + ...........+ n).

$= \sum_{1}^{n} n. = \frac{n(n+1).}{2}$ $\quad \therefore S = \sum_{1}^{n} \frac{n(n+1).}{2}$

$\Rightarrow S = \sum_{1}^{n} \frac{n^2}{2} + \sum_{1}^{n} \frac{n}{2}.$

$= S = \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$

$= S = \frac{1}{4}(n)(n+1)\left(\frac{2n+1}{3}+1\right)$

$\Rightarrow S = \frac{1}{4}(n)(n+1)\left(\frac{2n+4}{3}\right)$

$S = \frac{1}{6}(n)(n+1)(n+2)$

**17.** (B)

**Sol.** Given, length of a side of $S_n$.

= length of a diagonal of $S_{n+1}$

⇒length of a side of $S_n$

$= \sqrt{2}$ (length of a side of $S_{n+1}$)

$\Rightarrow \frac{\text{Length of a side of } S_{n+1}}{\text{Length of a side of } S_n} = \frac{1}{\sqrt{2}} \quad \forall\, n \geq 1$

So, the side of $S_1, S_2, ..............S_n$ form a G.P, with common ratio. $\frac{1}{\sqrt{2}}$ & first term 10.

$\therefore$ Side of $S_n = 10\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \frac{10}{2^{\frac{n-1}{2}}}$

since, area of $S_n < 1$ (given)

$\Rightarrow \frac{100}{2^{n-1}} < 1$

$\Rightarrow\ n - 1 \geq 7,\ \Rightarrow n \geq 8$

**18.** (A)

**Sol.** $f(x+y) = f(x)f(y)$

Substitute y = 1

f (x + 1) = 3f(x) $\quad (\because f(1) = 3)$

$\Rightarrow \frac{f(x+1)}{f(x)} = 3$

$\therefore$ the sequence f(n) for natural n is a G.P....

$\sum_{x=1}^{n} f(x) = f(1) + f(2) + ......f(n)$

$= f(1)(1 + 3 + 3^2 + ...... + 3^{n-1})$

$= = \frac{f(1)(3^{n-1})}{3-1} = \frac{3(3^n - 1)}{2} = 120$

$3^n = 81 \ \Rightarrow n = 4$

**19.** (A)

**Sol.** Let three numbers in G.P. are $\frac{a}{r}, a, ar$.

**Condition I :**

$\frac{a}{r} + a + ar = 14 \Rightarrow a\left(\frac{1}{r} + 1 + r\right) = 14$ ....(i)

**Condition II :** $\frac{a}{r} + 1, a + 1$ and $ar - 1$ will be in A.P., then

$2(a+1) = \frac{a}{r} + 1 + ar - 1 = \frac{a}{r}(1 + r^2)$ ......(ii)

From (i) and (ii), we get $a = 4$ and $r = 2$.

So, required numbers are 2, 4, 8.

Hence greatest number is 8.

**20.** (C)

**Sol.** Obviously $T_n = \frac{1^3 + 2^3 + 3^3 + ......... + n^3}{1 + 3 + 5 + .........\text{upto } n \text{ terms}}$

$= \frac{\Sigma n^3}{\frac{n}{2}[2 + (n-1)2]} = \frac{1}{4}\frac{n^2(n+1)^2}{n^2} = \frac{1}{4}(n^2 + 2n + 1)$

**21.** 0

**Sol.** $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a+b}{2}$

$\Rightarrow\ a^{n+1} - ab^n + b^{n+1} - ba^n = 0$

$\Rightarrow (a-b)(a^n - b^n) = 0$

If $a^n - b^n = 0$. Then $\left(\frac{a}{b}\right)^n = 1 = \left(\frac{a}{b}\right)^0$.

Hence $n = 0$.

**22.** 4

**Sol.** We have $ar = 2$ and $S_\infty = 8 = \frac{a}{1-r}$

$\Rightarrow\ 8 = \frac{2}{r(1-r)}\left(\because a = \frac{2}{r}\right)$

$\Rightarrow\ 4r(1-r) = 1 \Rightarrow 4r - 4r^2 - 1 = 0$

$\Rightarrow\ 4r^2 - 4r + 1 = 0 \Rightarrow \left(r - \frac{1}{2}\right)(4r-2) = 0 \Rightarrow r = \frac{1}{2}$

So first term $a = 4$.

**23.** 64

**Sol.** $(32)(32)^{1/6}(32)^{1/36}......\infty = (32)^{1+\frac{1}{6}+\frac{1}{36}+.....\infty}$

$= (32)^{\frac{1}{1-(1/6)}} = (32)^{\frac{1}{5/6}} = (32)^{6/5} = 2^6 = 64$.

**24.** 18

**Sol.** Let two numbers $a$ and $b$.

We have $G = \sqrt{AH} \Rightarrow$ G.M. $= \sqrt{27 \times 12} = 18$.

**25.** 8

**Sol.** Let the numbers be

$\frac{a}{r}, a, ar, 2ar-a$ .....(i)

Where first three numbers are in G.P. and last three are in A.P.

Given that the common difference of A.P. is 6, so

$ar - a = 6$ .....(ii)

Also given $\frac{a}{r} = 2ar - a \Rightarrow \frac{a}{r} = 2(ar-a)+a$

$\Rightarrow \frac{a}{r} = 2(6)+a,$ from (ii)

$\Rightarrow \left(\frac{a}{r}\right) - a = 12 \Rightarrow a(1-r) = 12r \Rightarrow r = -\frac{1}{2}$

From (i) we get, $a\left[\left(-\frac{1}{2}\right)-1\right] = 6$ or $a = -4$

Required numbers from (i) are $8, -4, 2, 8$ .

**26.** 18

**Sol.** $(A.M.)\ (H.M.) = ab = (G.M)^2$

$\Rightarrow 9 \cdot 36 = (G.M)^2 \Rightarrow G.M. = 18$ .

**27.** 3

**Sol.** Let the numbers be $a-d, a, a+d$ .

Then $(a-d)^2, a^2, (a+d)^2$ are in G.P.

$\therefore a^4 = (a-d)^2(a+d)^2 \Rightarrow d^4 - 2a^2d^2 = 0$

$\Rightarrow d = 0, \pm\sqrt{2}a$

Hence $d$ has three values.

**28.** 900

**Sol.** Let say $a_1 = a$, $a_2 = a + d$. & $a_n = a + (n - 1)d$

So, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

a + a + 4d + a + 9d + a + 14d + a + 19d + a + 23d = 225

6a + 69d = 225

3(2a + 23d) = 225

$2a + 23d = \frac{225}{3}$ ...........(1)

Now sum of 24 terms $\Rightarrow$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{24}{2}[2a + (24 - 1)d]$

$S_{24} = 12\ (2a + 23d)$

from eq$^n$ (i)

$S_{24} = 12\left[\frac{225}{3}\right] = 4 \times 225$

$S_{24} = 900$

**29.** 64

**Sol.** Since $(n^m + 1)\ (1 + n + n^2........+n^{127})$

$\therefore \frac{1+n+n^2......+n^{127}}{n^m+1}$ is an integer

$\Rightarrow \frac{1-n^{128}}{1-n} \times \frac{1}{n^{m+1}}$ is an integer

$\Rightarrow \frac{(1-n^{64})(1+n^{64})}{(1-n)(n^{m+1})}$ is an integer when largest

m = 64

**30.** 1

**Sol.** Let G.P be a, ar,...... $ar^n$,......,$ar^{2n}$

So sum of all terms = $S_{all} = \frac{a(1-r^{2n})}{1-r}$

& sum of odd terms = $S_{odd}$ = $\frac{a(1-r^{2n})^n}{1-r^2} = \frac{a(1-r^{2n})}{1-r^2}$

$S_{all} = 5.5_{odd}$

$\Rightarrow (1-r^2) = 5(1-r)$

$\Rightarrow r^2 - 5r + 4 = 0$

$\Rightarrow$ r = 1 or 4 (but r ≠ 1)