JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-4**
CHAPTER :- QUADRATIC EQUATION

ANSWER KEY

1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(D)	**7.**	(D)
8.	(B)	**9.**	(A)	**10.**	(D)	**11.**	(A)	**12.**	(A)	**13.**	(B)	**14.**	(D)
15.	(A)	**16.**	(D)	**17.**	(A)	**18.**	(A)	**19.**	(D)	**20.**	(A)	**21.**	6
22.	0	**23.**	1	**24.**	5	**25.**	9	**26.**	9	**27.**	8	**28.**	5
29.	3	**30.**	2										

SOLUTIONS

1. (B)

Sol. It must be an identity.
So, $a = 0 = b = c$ **Ans.**]

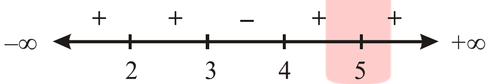
2. (A)

Sol. Here $P(4) = (64k + 48 - 3) = 64k + 45$
$Q(4) = 128 - 20 + k = k + 108$
As $P(4) = Q(4)$
$\Rightarrow 64k + 45 = k + 108$
$\Rightarrow 63k = 63$
$\therefore k = 1$]

3. (C)

Sol. We have $\dfrac{(x-2)^2(3-x)^3(5-x)^4}{(4-x)^5} < 0$

$\Rightarrow \dfrac{(x-2)^2(x-3)^3(x-5)^4}{(x-4)^5} < 0$



ts
$\therefore 3a = 3 \Rightarrow a = 1$ which satisfy the given equation.
$\therefore 1 - 3 + b + 5 = 0 \Rightarrow b = -3$ **Ans.**

4. (C)

Sol. $2\left(\sqrt{50}+\sqrt{14}\right)x^2 + 10x - 2\left(\sqrt{50}-\sqrt{14}\right) = 0$

$\Rightarrow x = \dfrac{-10 \pm \sqrt{100+16\cdot 36}}{4\left(\sqrt{50}+\sqrt{14}\right)} = \dfrac{-5 \pm \sqrt{25+144}}{2\left(\sqrt{50}+\sqrt{14}\right)}$

$= \dfrac{-5 \pm 13}{2\left(\sqrt{50}+\sqrt{14}\right)}$

Positive root

$x = \dfrac{4\left(\sqrt{50}-\sqrt{14}\right)}{36} = \dfrac{\sqrt{50}-\sqrt{14}}{9}$

$= \dfrac{5\sqrt{2}-\sqrt{14}}{9}$. **Ans.**

5. (B)

Sol. $(x - 4) + \dfrac{7}{x+4} > 0 \Rightarrow \dfrac{(x^2-16)+7}{x+4} > 0 \Rightarrow$

$\dfrac{x^2-9}{x+4} > 0$

$\therefore \dfrac{(x-3)(x+3)}{(x+4)} > 0$

$\therefore x \in (-4, -3) \cup x > 3$

–4 –3 3

permissible length = 13

–5 n(S) = 20 15

$\therefore$ Percentage $= \dfrac{13}{20} \times 100 = 65\%$. **Ans.**]

6. (D)

Sol. If $x^2 + 4x + 3 = (x + 3)(x + 1) \geq 0$, $x \in R - (-3, -1)$ ....(1)
The given equation becomes $x^2 + 6x + 8 = 0 \Rightarrow x = -2, -4$ ....(2)
(1), (2) $\Rightarrow x = -4$
If $x^2 + 4x + 3 < 0$, $x \in (-3, -1)$ ....(3)
The equation becomes $-(x^2 + 4x + 3) + 2x + 5 = 0$
or $x^2 + 2x - 2 = 0 \Rightarrow x = -1 \pm \sqrt{3}$ ....(4)
(3), (4) $\Rightarrow x = -1 - \sqrt{3}$
Sum of the roots $= -4 + (-1 - \sqrt{3}) = -5 - \sqrt{3}$.
Ans.]

7. (D)

Sol. $x^2 + 4 = (x^4 + 4x^2 + 4) - 4x^2 = (x^2 + 2)^2 - (2x)^2$
$= (x^2 + 2x + 2)(x^2 - 2x + 2)$.]

8. (B)

Sol. $(6x^2 + 4x - 3)(4x^2 - 4x + 1) = 0$

$\underbrace{(6x^2+4x-3)}_{D>0}\ \underbrace{(2x-1)^2}_{x=\frac{1}{2}} = 0$

$\Rightarrow$ 3 distinct roots

9. (A)

Sol. $\frac{2x-6}{2x-1} > 1 = \frac{2x-6-2x+1}{2x-1} > 0$

$\therefore \frac{-5}{2x-1} > 0 \Rightarrow 2x - 1 < 0 \Rightarrow x < \frac{1}{2}$.]

10. (D)

Sol. Let the roots are α and α^2

$\therefore x^2 - \frac{10x}{9} + c = (x - \alpha)(x - \alpha^2)$

$\alpha + \alpha^2 = \frac{10}{9}$ and $\alpha \cdot \alpha^2 = c$

$9\alpha^2 + 9\alpha - 10 = 0 \Rightarrow 9\alpha^2 + 15\alpha - 6\alpha - 10 = 0$

$\Rightarrow 3\alpha(3\alpha + 5) - 2(3\alpha + 5) = 0$

$\therefore \alpha = \frac{2}{3}$ or $\alpha = \frac{-5}{3}$

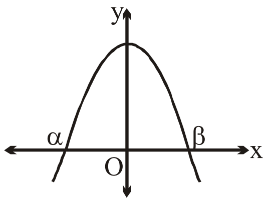
as $c > 0$ and $\alpha^3 = c$, hence $\alpha = \frac{-5}{3}$ is rejected

$\therefore \alpha = \frac{2}{3}$; $c = \alpha^3 = \frac{8}{27}$

$\Rightarrow$ (m + n) = 35. **Ans.**]

11. (A)

Sol. **Note:** sin 6 + sin 10 + cos 10 < 0



So, f(0) > 0 $\Rightarrow$ (k – 1) (k – 2) > 0

$\Rightarrow k \in (-\infty, 1) \cup (2, \infty)$. **Ans.**

12. (A)

Sol. Since 4 is one of the root of equation $x^2 + px + 12 = 0 \Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$.

and the equation $x^2 + px + q = 0$ has equal roots, so D = 0

$\Rightarrow 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$. **Ans.**

13. (B)

Sol. $(x^3 - 1) - k(x - 1) = 0$

$(x - 1)[x^2 + x + 1 - k] = 0$

$\therefore x = 1$ or $x^2 + x + (1 - k) = 0$

For exactly two distinct real solutions,

either $D = 0 \Rightarrow 1 - 4(1 - k) = 0 \Rightarrow 1 - 4 + 4k = 0$

$\therefore k = \frac{3}{4}$

or $x^2 + x + (1 - k) = 0$ < 1 (one root), –2 (other root)

$\Rightarrow 3 - k = 0 \Rightarrow k = 3$.

Hence sum of possible values of $k = 3 + \frac{3}{4} = \frac{15}{4}$. **Ans.**

14. (D)

Sol. $x^2 = 5x + 6 \Rightarrow x^2 - 5x - 6 = 0$ (x – 6) (x + 1) = 0

$\Rightarrow$ x = 6 or x = – 1. **Ans.**

15. (A)

Sol. We have,

$x^2 + px + 3 = 0$ ....(1)

$x^2 + qx + 5 = 0$ ....(2)

and $x^2 + (p + q)x + 24 = 0$ ....(2)

$\therefore$ (1) + (2) – (3) $\Rightarrow x^2 = 16 \Rightarrow x = -4, 4$

So, the common negative root is – 4

$\therefore$ Put x = – 4 in (1), we get $p = \frac{19}{4}$ and put x = – 4 in (2), we get $q = \frac{21}{4}$.

Hence, $(p + q) = \frac{19}{4} + \frac{21}{4} = 10$. **Ans.**]

16. (D)

Sol. Let x_1, x_2, x_3 are the roots of $x^3 - 2x^2 + 9x + 10 = 0$

$\therefore \underbrace{x_1 + x_2}_{\substack{\text{add. inverse} \\ \Rightarrow 0}} + x_3 = 2 \Rightarrow x_3 = 2$

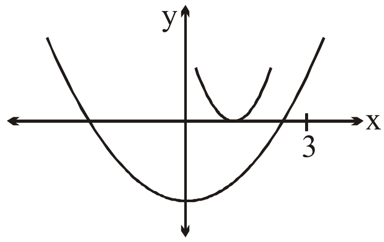
$\therefore 8 - 8 + 2a + 10 = 0 \Rightarrow a = -5 \Rightarrow$ (D).

17. (A)

Sol. $D \geq 0$ ....(1)

f(3) > 0 ....(2)

$\frac{-b}{2a} < 3$ ....(3)



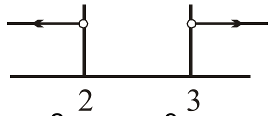
(1) $\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$12 - 4a \geq 0 \Rightarrow a \leq 3$ ....(4)

$9 - 6a + a^2 + a - 3 > 0$

$a^2 - 5a + 6 > 0$

$(a - 3)(a - 2) > 0$



a < 2 or a > 3

$\frac{2a}{2} < 3 \Rightarrow a < 3$

Hence, a < 2 $\Rightarrow$ **(1). Ans.**

18. (A)

Sol. $\because \frac{b}{a} = 4\lambda \Rightarrow \frac{b}{2a} = 2\lambda$ = even integer $\{\because \lambda$ is integer$\}$

$\therefore$ minimum value of f(x) occurs at x = 2

$\{\because f(1) > f(2) < f(4) < f(5)\}$

$\therefore \frac{-b}{2a} = 2$

$\Rightarrow$ sum of the roots, $\frac{-b}{a} = 4$. **Ans.]**

19. (D)

Sol. Given,

$x^2 + 3(1 - 3p)x + 2 = 0$ (roots α, 2α)

$\therefore$ $3\alpha = 3(3p - 1) \Rightarrow \alpha = (3p - 1)$

Also, $2\alpha^2 = 2 \Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

So, for $\alpha = -1$, p = 0 and for $\alpha = 1$, $p = \frac{2}{3}$

$\Rightarrow$ sum of all possible values of $p = 0 + \frac{2}{3} = \frac{2}{3}$. **Ans.**

20. (A)

Sol. Let $y = \frac{3x^2+9x+17}{3x^2+9x+7} \Rightarrow 3x^2(y-1) + 9x(y-1) + (7y-17) = 0$

As x is real, so $D \geq 0$

$\Rightarrow 81(y-1)^2 - 12(y-1)(7y-17) \geq 0 \Rightarrow (y-1)(y-41) \leq 0 \Rightarrow 1 \leq y \leq 41$

But $y \neq 1$, so $1 < y \leq 41 \Rightarrow y_{max} = 41$. **Ans.**

<u>**Aliter**</u>: $y = \frac{3x^2+9x+17}{3x^2+9x+7} = \frac{(3x^2+9x+7)+10}{3x^2+9x+7} = 1 + \frac{10}{3x^2+9x+7}$

$y_{max} = 1 + \frac{10}{(3x^2+9x+7)_{min.}}$

$y_{max} = 1 + \frac{10}{\frac{3}{12}\left(-d/4a\right)}$

$y_{max} = 41$.

Hence B **Ans.**

21. 6

Sol. Given, $x^2 - 2ax + 2 = 0$ (roots x_1, x_2) ; $x^2 - 2bx + 3 = 0$ (roots x_2, x_3) and $x^2 - 2cx + 16 = 0$ (roots x_3, x_1)

Clearly, $x_1x_2 = 2$, $x_2x_3 = 3$ and $x_3x_1 = 6$

$\Rightarrow (x_1x_2)(x_2x_3)(x_3x_1) = 36$ or $x_1x_2x_3 = 6$

(As, x_1, x_2, x_3 are all positive.)

So, $x_1 = \frac{x_1x_2x_3}{x_2x_3} = \frac{6}{3} = 2$

Similarly, $x_2 = \frac{x_1x_2x_3}{x_1x_3} = \frac{6}{6} = 1$

and $x_3 = \frac{x_1x_2x_3}{x_1x_2} = \frac{6}{2} = 3$

Also, we have $2a = x_1 + x_2 = 3$; $2b = x_2 + x_3 = 4$ and $2c = x_3 + x_1 = 5$

Hence, 2 (a + b + c) = 12.

$\Rightarrow$ (a + b + c) = 6. **Ans.**

22. 0

Sol. Let $x = \alpha$ be common zero then

$\alpha^2 + a\alpha + b = 0$ ......(1)

$\alpha^2 + \alpha + \alpha b = 0$ ......(2)

$a\alpha^2 + \alpha + b = 0$ ......(3)

(1) – (2) $\Rightarrow (a-1)(\alpha - b) = 0$

$a \neq 1 \Rightarrow \alpha = b$

(1) or (3) gives $b^2 + ab + b = 0$ ......(4)

and (3) gives $ab^2 + 2b = 0$

$b \neq 0 \Rightarrow ab = -2$

if $ab = -2$, $b^2 + b - 2 = 0$

b = 1 or –2

a = –2 or 1

a = –2 b = 1 and a = 1, b = –2 (does not satisfy the mentioned condition polynomial will be same)

a + 2b = 0.

23. 1

Sol. Since P(x) takes only three negative integral values

$\Rightarrow -4 < \frac{-D}{4a} \leq -3$

$-4 < \frac{(4k-16)}{4} \leq -3$

$-4 < k - 4 \leq -3$

$0 < k \leq 1 \Rightarrow k \in (0, 1]$ **Ans.**

24. 5

Sol. P(1) = 2, P(2) = 5, P(3) = 10

$g(x) = P(x) - (x^2 + 1) \Rightarrow P(x) = (x-1)(x-2)(x-3) + (x^2+1)$

$\therefore |P(0)| = |-6 + 1| = 5$ **Ans.**

25. 9

Sol. First expression = $10 - \frac{3(1-\cos 6x)}{2} + 2\sin 6x$

$= \frac{17}{2} + \left(\frac{3\cos 6x + 4\sin 6x}{2}\right)$

$\therefore$ largest value = $\frac{17}{2} + \frac{5}{2} = 11$

Second expression = 2 cos 2x – 4 cos x + 5

= 2 (2 $\cos^2$x – 1) – 4 cos x + 5

= 4 $\cos^2$ x – 4 cos x + 3

= $(2\cos x - 1)^2 + 2$

least value = 2

$\therefore$ Difference = 9 **Ans.**

26. 9

Sol. $\because$ Roots of $x^2 - 3x + 4 = 0$ are imaginary

$\therefore$ Both the roots should be common

$\therefore \frac{4}{1} = \frac{-2(b-5a)}{-3} = \frac{b}{4} \Rightarrow b = 16$

and b – 5a = 6 $\Rightarrow$ 5a = 10 $\Rightarrow$ a = 2

Arithmetic mean of all the mean = $\frac{\text{sum of all the means}}{\text{number of means}}$

$= \frac{13\cdot(\text{single A.M. between } a \text{ and } b)}{13} = \frac{2+16}{2} =$ 9 **Ans.**

27. 8

Sol. $-\frac{(4-4\cdot(-1)(a-1)}{4\cdot(-1)} < -\left(\frac{4a^2 - 4\cdot 1\cdot(10-2a)}{4\cdot 1}\right)$

$\Rightarrow a < -(a^2 + 2a - 10)$

$\Rightarrow a^2 + 3a - 10 < 0 \Rightarrow -5 < a < 2$

$\because f(a) = a^2 + ka - 20 < 0 \Rightarrow \forall a \in (-5, 2)$

$\therefore f(-5) \le 0 \Rightarrow 25 - 5k - 20 \le 0 \Rightarrow 5k \ge 5 \Rightarrow k \ge 1$

and $f(2) \le 0 \Rightarrow 4 + 2k - 20 < 0 \Rightarrow k \le 8$

$\therefore k \in [1, 8]$

no of integral values of k = 8. **Ans.**

28. 5

Sol. $E = (1 - 2x^2)^2 + 4x^2y^2$

Put $y^2 = 4 - x^2$

$E = 1 + 4x^4 - 4x^2 + 4x^2(4 - x^2) = 1 + 12x^2$

Now, $E_{max} = 49$ at x = 2 and $E_{min} = 1$ at x = 0

$\therefore$ m = 1 and M = 49 .

Hence, $\left(\frac{M}{7} - 2m\right) = 5$. **Ans.**

Aliter: $x^2 + y^2 = 4$

Let x = 2 sin θ , y = 2 cos θ

$E = (1 - 2x^2)^2 + 4x^2y^2$

E = 25 – 24 cos 2θ

$\therefore$ m = 1 and M = 49.

29. 3

Sol. $f(x) = 1 + \log_2(\tan^2 x) = \log_2(2\tan^2 x)$

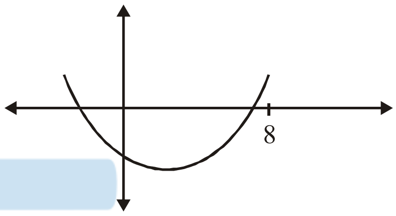
$g(x) = 3 + \log_2(\cot^2 x) = \log_2(8\cot^2 x)$

$2^{f(x)} + 2^{g(x)} = 2\tan^2 x + 8\cot^2 x \ge 2\sqrt{2}\cdot\sqrt{8}$

≥ 8

Now, $h(x) = 3x^2 - (k^2 - 2k)x - 168$

For $h\left(2^{f(x)} + 2^{g(x)}\right) > 0 \ \forall x \in \left(0, \frac{\pi}{2}\right)$



$\Rightarrow h(8) > 0$

$\Rightarrow 192 - 8(k^2 - 2k) - 168 > 0$

$\Rightarrow 24 - 8(k^2 - 2k) > 0$

$\Rightarrow 3 - k^2 + 2k > 0$

$\Rightarrow k^2 - 2k - 3 < 0$

$\Rightarrow (k-3)(k+1) < 0 \Rightarrow k \in (-1, 3)$

30. 2

Sol. $k^2x^2 - 4k^2 - 2(k-1)x^2 + 4(k-1)x = 0$

$k^2(x^2 - 4) - 2(k-1)x(x-2) = 0$

$(x-2)\left(k^2(x+2) - 2x(k-1)\right) = 0$

$x = 2, \quad x(k^2 - 2k + 2) + 2k^2 = 0$

$x = \frac{-2k^2}{k^2 - 2k + 2}$

Now, $\frac{-2k^2}{k^2 - 2k + 2} < -2 \Rightarrow k^2 > k^2 - 2k + 2$

$\Rightarrow k > 1$

$\therefore$ Least integral value of k is 2. **Ans.**