JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 11th **PAPER CODE :- CWT-3**
CHAPTER :- TRIGONOMETRY

ANSWER KEY

1.	(B)	**2.**	(B)	**3.**	(C)	**4.**	(A)	**5.**	(B)	**6.**	(D)	**7.**	(C)
8.	(C)	**9.**	(A)	**10.**	(A)	**11.**	(C)	**12.**	(C)	**13.**	(B)	**14.**	(D)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(B)	**19.**	(B)	**20.**	(B)	**21.**	1
22.	0	**23.**	1	**24.**	0	**25.**	27	**26.**	0	**27.**	3	**28.**	11
29.	5	**30.**	4										

SOLUTIONS

1. (B)

Sol. $a\sin^2 x + b\cos^2 x = c \Rightarrow (b-a)\cos^2 x = c-a$

$\Rightarrow (b-a) = (c-a)(1+\tan^2 x)$

$b\sin^2 y + a\cos^2 y = d \Rightarrow (a-b)\cos^2 y = d-b$

$\Rightarrow (a-b) = (d-b)(1+\tan^2 y)$

$\therefore \tan^2 x = \frac{b-c}{c-a}, \tan^2 y = \frac{a-d}{d-b}$

$\therefore \frac{\tan^2 x}{\tan^2 y} = \frac{(b-c)(d-b)}{(c-a)(a-d)}$ .....(i)

But $a\tan x = b\tan y$,

i.e., $\frac{\tan x}{\tan y} = \frac{b}{a}$ .....(ii)

From (i) and (ii), $\frac{b^2}{a^2} = \frac{(b-c)(d-b)}{(c-a)(a-d)}$

$\Rightarrow \frac{a^2}{b^2} = \frac{(c-a)(a-d)}{(b-c)(d-b)}$.

2. (B)

Sol. The expression reduces to $\cot^n \frac{A-B}{2} + \cot^n \frac{B-A}{2}$

If n is even, answer is (B) and if n is odd answer is (C).

3. (C)

Sol. We have $\sin\alpha = 1/\sqrt{5} \Rightarrow \cos\alpha = 2/\sqrt{5}$

and $\sin\beta = 3/5 \Rightarrow \cos\beta = 4/5$

$\sin(\beta-\alpha) = \sin\beta\cos\alpha - \sin\alpha\cos\beta$

$= \frac{3}{5}\cdot\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\cdot\frac{4}{5} = \frac{2}{5\sqrt{5}} = 0.1789$

Now $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = 0.7071 = \sin\frac{3\pi}{4}$

Since $0 < 0.1789 < 0.7071$

$\therefore \sin 0 < \sin(\beta-\alpha) < \sin\frac{\pi}{4} \Rightarrow 0 < (\beta-\alpha) < \frac{\pi}{4}$

Also, $\sin\pi < \sin(\beta-\alpha) < \sin\frac{3\pi}{4}$

$\therefore (\beta-\alpha) \in [0, \pi/4]$ and $[3\pi/4, \pi]$.

4. (A)

Sol. The given equation may be written as

$\frac{2}{\cos 2\alpha} = \frac{\sin\beta}{\cos\beta} + \frac{\cos\beta}{\sin\beta} = \frac{\sin^2\beta + \cos^2\beta}{\cos\beta\sin\beta}$

$= \frac{1}{\cos\beta.\sin\beta}$

$\Rightarrow \cos 2\alpha = \sin 2\beta$

$\Rightarrow \cos 2\alpha = \cos\left(\frac{\pi}{2} - 2\beta\right) \Rightarrow 2\alpha = \frac{\pi}{2} - 2\beta$

$\Rightarrow 2\alpha + 2\beta = \frac{\pi}{2} \Rightarrow \alpha + \beta = \frac{\pi}{4}$.

5. (B)

Sol. We have

$\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)} = k$

$\Rightarrow x = k\cos\theta$, $y = k\cos\left(\theta - \frac{2\pi}{3}\right)$,

$z = k\cos\left(\theta + \frac{2\pi}{3}\right)$

$\Rightarrow x + y + z = k\left[\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)\right]$

$= k[(0) = 0$

$\Rightarrow x + y + z = 0$.

6. (D)

Sol. $\sin 6\theta = 2\sin 3\theta\cos 3\theta$

$= 2[3\sin\theta - 4\sin^3\theta][4\cos^3\theta - 3\cos\theta]$

$=24\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta) - 18\sin\theta\cos\theta - 32\sin^2\theta\cos^2\theta$

$= 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3\sin 2\theta$

On comparing, $x = \sin 2\theta$.

7. (C)

Sol. $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

$=\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$

$=\frac{1}{4}(3)+\frac{1}{4}(3)=\frac{3}{2}$.

8. (C)

Sol. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}+\cos\frac{7\pi}{8}+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)$

$\left(1+\cos\frac{5\pi}{8}+\cos\frac{3\pi}{8}+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}-\cos\frac{\pi}{8}+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)$

$\left(1+\cos\frac{5\pi}{8}-\cos\frac{5\pi}{8}+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$

$=\frac{1}{4}\left(2+2\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(2+2\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$

$=\frac{1}{4}\left(2+\cos\frac{3\pi}{4}+\cos\pi\right)\left(2+\cos\frac{\pi}{4}+\cos\pi\right)$

$=\frac{1}{4}\left(1+\cos\frac{3\pi}{4}\right)\left(1+\cos\frac{\pi}{4}\right)=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1+\cos\frac{\pi}{4}\right)$

$=\frac{1}{4}\left(1-\cos^2\frac{\pi}{4}\right)=\frac{1}{4}\left(1-\frac{1}{2}\right)=\frac{1}{8}$.

Aliter :

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)=\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$=\frac{1}{4}\left(2\sin\frac{\pi}{8}.\sin\frac{3\pi}{8}\right)^2$

$=\frac{1}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right)^2=\frac{1}{8}$.

9. (A)

Sol. $3\tan A-4=0\Rightarrow\tan A=\frac{4}{3}\Rightarrow\sin A=-\frac{4}{5},\cos A=-\frac{3}{5}$

$\therefore\ 5\sin 2A+3\sin A+4\cos A$

$=10\sin A\cos A+3\sin A+4\cos A$

$=10\left(\frac{12}{25}\right)-\frac{12}{5}-\frac{12}{5}=0$.

10. (A)

Sol. We have

$\cot A=\frac{\cos A}{\sin A}=\frac{2\cos^2 A}{2\sin A\cos A}=\frac{1+\cos 2A}{\sin 2A}$

Putting $A=7\frac{1^o}{2}\Rightarrow\cot 7\frac{1^o}{2}=\frac{1+\cos 15^o}{\sin 15^o}$

On simplification,

we get $\cot 7\frac{1^o}{2}=\sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$.

11. (C)

Sol. $2A=(A+B)+(A-B)$

$\Rightarrow\ \tan 2A=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}=\frac{p+q}{1-pq}$.

12. (C)

Sol. $\cos 2(\alpha+\beta)=2\cos^2(\alpha+\beta)-1, 2\sin^2\beta=1-\cos 2\beta$

L.H.S.

$=-\cos 2\beta+2\cos(\alpha+\beta)[2\sin\alpha\sin\beta+\cos(\alpha+\beta)]$

$=-\cos 2\beta+2\cos(\alpha+\beta)\cos(\alpha-\beta)$

$=-\cos 2\beta+(\cos 2\alpha+\cos 2\beta)=\cos 2\alpha$.

13. (B)

Sol. Let $f(x)=\sin\theta+\cos\theta=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$

But

$-1\le\sin\left(\theta+\frac{\pi}{2}\right)\le 1\Rightarrow-\sqrt{2}\le\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)\le\sqrt{2}$

.

Hence the maximum value of $(\sin\theta+\cos\theta)$

i.e., of $\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)=\sqrt{2}$.

$\therefore\ \sin\left(\theta+\frac{\pi}{4}\right)=1\Rightarrow\sin\left(\theta+\frac{\pi}{4}\right)=\sin\frac{\pi}{2}$

$\Rightarrow\ \theta+\frac{\pi}{4}=\frac{\pi}{2}\Rightarrow\theta=\frac{\pi}{4}=45^o$.

14. (D)

Sol. Since $\left(x-\frac{1}{x}\right)^2\ge 0,\ \forall\ x\in R,$

we have $x^2+\frac{1}{x^2}\ge 2$ and

Hence, $f(x)=\cos^2 x+\frac{1}{\cos^2 x}\ge 2$.

15. (A)

Sol. Let $y = \frac{\tan x}{\tan 3x} = \frac{\tan x}{\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}$

$$y = \frac{1 - 3\tan^2 x}{3 - \tan^2 x} = \frac{\frac{1}{3} - \tan^2 x}{1 - \frac{1}{3}.\tan^2 x}$$

Hence, y should never lie between $\frac{1}{3}$ and 3 whenever defined.

16. (C)

Sol. We have $\cos 2\theta + 2\cos\theta = 2\cos^2\theta - 1 + 2\cos\theta$

$$= 2\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{3}{2}$$

Now $2\left(\cos\theta + \frac{1}{2}\right)^2 \geq 0$ for all θ

$\therefore 2\left(\cos\theta + \frac{1}{2}\right)^2 - \frac{3}{2} \geq \frac{-3}{2}$ for all θ.

$\Rightarrow \cos 2\theta + 2\cos\theta \geq \frac{-3}{2}$ for all θ

Also max. value of this expression is 3.

17. (A)

Sol. $A + B + C = \pi$

$\therefore \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\frac{C}{2} \Rightarrow \frac{\frac{1}{3} + \frac{2}{3}}{1 - \frac{1}{3}.\frac{2}{3}} = \frac{9}{7} = \cot\frac{C}{2}$

$\therefore \tan\frac{C}{2} = \frac{7}{9}$.

18. (B)

Sol. $\cos[\pi - (B + C)] = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2\cos B \cos C$

$\Rightarrow \tan B \tan C = 2$.

19. (B)

Sol. $B = A + C \Rightarrow \tan B = \tan(A + C)$

$\Rightarrow \tan B = \frac{\tan A + \tan C}{1 - \tan A \tan C}$

$\Rightarrow \tan A \tan B \tan C = \tan B - \tan A - \tan C$.

20. (B)

Sol. Let $u = \cos\theta\left\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\right\}$

$\Rightarrow (u - \sin\theta\cos\theta)^2 = \cos^2\theta(\sin^2\theta + \sin^2\alpha)$

$\Rightarrow u^2\tan^2\theta - 2u\tan\theta + u^2 - \sin^2\alpha = 0$

Since $\tan\theta$ is real, therefore

$\Rightarrow 4u^2 - 4u^2(u^2 - \sin^2\alpha) \geq 0$

$\Rightarrow u^2 - (1 + \sin^2\alpha) \leq 0$

$\Rightarrow |u| \leq \sqrt{1 + \sin^2\alpha}$.

21. 1

Sol. We have $\sin A, \cos A$ and $\tan A$ are in *G.P.*

$\cos^2 A = \sin A \tan A = \frac{\sin^2 A}{\cos A} \Rightarrow \cos^3 A - \sin^2 A = 0$

Hence $\cos^3 A + \cos^2 A = \sin^2 A + \cos^2 A = 1$

22. 0

Sol. $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore \sin\theta\cos\theta = 0$.

23. 1

Sol. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta = 1.$

Trick : Put $\theta = 0^o$, we get the value of expression equal to 1. Again put $\theta = 45^o$, the value remains 1, it means that the expression is independent of θ and is equal to 1.

24. 0

Sol. $(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

and $\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

Both gives,

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

25. 27

Sol. $A - B = \frac{\pi}{4} \Rightarrow \tan(A - B) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\Rightarrow \tan A - \tan B - \tan A \tan B = 1$

$\Rightarrow \tan A - \tan B - \tan A \tan B + 1 = 2$

$\Rightarrow (1 + \tan A)(1 - \tan B) = 2 \Rightarrow y = 2$

Hence, $(y + 1)^{y+1} = (2 + 1)^{2+1} = (3)^3 = 27$.

26. 0

Sol. $\frac{\cos 12^o - \sin 12^o}{\cos 12^o + \sin 12^o} + \frac{\sin 147^o}{\cos 147^o}$

$= \frac{1 - \tan 12^o}{1 + \tan 12^o} + \tan 147^o$

$= \tan 33^o - \tan 33^o = 0$.

27. 3

Sol. $\tan 20^o \tan 40^o \tan 60^o \tan 80^o$

$= \frac{\sin 20^o \sin 40^o \sin 80^o \tan 60^o}{\cos 20^o \cos 40^o \cos 80^o}$

Here $N^r = (\sin 20^o \sin 40^o \sin 80^o)$

$= \frac{\sin 20^o}{2}(2 \sin 40^o \sin 80^o)$

$= \frac{\sin 20^o}{2}(\cos 40^o - \cos 120^o)$

$= \frac{1}{2}\sin 20^o\left(1 - 2\sin^2 20^o + \frac{1}{2}\right)$

$= \frac{1}{2}\sin 20^o\left(\frac{3}{2} - 2\sin^2 20^o\right) = \frac{\sin 60^o}{4} = \frac{\sqrt{3}}{8}$

Now, we take $D^r = \cos 20^o \cos 40^o \cos 80^o$

$= \frac{\sin 2^3 20^o}{2^3 \sin 20^o} = \frac{\sin 160^o}{8 \sin 20^o} = \frac{\sin 20^o}{8 \sin 20^o} = \frac{1}{8}$

$\therefore$ Hence $\tan 20^o \tan 40^o \tan 80^o = \frac{\sqrt{3}/8}{1/8}$

Therefore

$\tan 20^o \tan 40^o \tan 60^o \tan 80^o = \sqrt{3}.\sqrt{3} = 3$.

28. 11

Sol. $32\sin\frac{A}{2}\sin\frac{5A}{2} = 16(\cos 2A - \cos 3A)$

$= 16(2\cos^2 A - 1 - 4\cos^3 A + 3\cos A)$

$= 16\left(2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right) = 11$.

29. 5

Sol. Let $3 = r\cos\alpha, 4 = r\sin\alpha$, so $r = 5$

$f(\theta) = r.(\cos\alpha\cos\theta + \sin\alpha\sin\theta) = 5.\cos(\theta - \alpha)$

$\therefore$ The maximum value of $f(\theta) = 5.1 = 5$.

{Since the maximum value of $\cos(\theta - \alpha) = 1$}.

Aliter : As we know that, the maximum value of $a\sin\theta + b\cos\theta$ is $+\sqrt{a^2 + b^2}$ and the minimum value is $-\sqrt{a^2 + b^2}$. Therefore, the maximum value is $(3\cos\theta + 4\sin\theta) = +\sqrt{3^2 + (-4)^2} = 5$ and the minimum value is – 5.

30. 4

Sol. $f(x) = 4\sin^2 x + 3\cos^2 x \sin^2 x + 3$ and $0 \le |\sin x| \le 1$

$\therefore$ Maximum value of $\sin^2 x + 3$ is 4.