JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 12th **PAPER CODE :- CWT-12**
CHAPTER :- PROBABILITY

ANSWER KEY

1.	(A)	**2.**	(B)	**3.**	(A)	**4.**	(B)	**5.**	(A)	**6.**	(A)	**7.**	(A)
8.	(A)	**9.**	(B)	**10.**	(B)	**11.**	(B)	**12.**	(A)	**13.**	(B)	**14.**	(D)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(D)	**19.**	(B)	**20.**	(B)	**21.**	**82**
22.	(C)	**23.**	**375**	**24.**	2	**25.**	(A)	**26.**	**2**	**27.**	157	**28.**	84
29.	**(C)**	**30.**	5										

SOLUTIONS

1. (A)

Sol. $P(H) = \frac{1}{4} = P(S)$; $P(F) = \frac{3}{4}$;

E : even number of draws are needed

P(E) = P (F S or F F F S or.......)

$= \frac{P(F)\cdot P(S)}{1-P(F)P(F)} = \frac{3/16}{1-(9/16)} = \frac{3}{7} \Rightarrow$ (A)

2. (B)

Sol. 7^{α} always ends with 7, 9, 3, 1.

$\therefore$ $7^{\alpha} + 7^{\beta}$ is divisible by 5

$\Rightarrow$ Favourable events = (25 × 25) × 2

and sample space = 100 × 100

$\therefore$ Required probability = $\frac{2(25\times 25)}{100\times 100} = \frac{1}{8}$

3. (A)

Sol. $P\left(\frac{\overline{A}}{B}\right) = \frac{1-P(AB)}{p(B)} = \frac{1-x}{x^2} \le 1$

$\Rightarrow$ $x^2 + x - 1 \ge 0$

$\Rightarrow$ $x \ge \frac{\sqrt{5}-1}{2}$ and $x \le \frac{-1-\sqrt{5}}{2}$

As x is +ve

$\therefore$ $x \ge \frac{\sqrt{5}-1}{2}$

4. (B)

Sol. A : Event that first man speaks truth

B : Event that second man speaks truth

R : Day is rainy

$P(R) = \frac{P(A\cap B).P(R)}{P(A\cap B).P(R) + P(A'\cap B').P(R')}$

$= \frac{\frac{4}{5}.\frac{2}{3}.\frac{3}{4}}{\frac{4}{5}.\frac{2}{3}.\frac{3}{4} + \frac{1}{5}.\frac{1}{3}.\frac{1}{4}} = \frac{24}{25}$

5. (A)

Sol. Let A represents the event 'A hits the target', B represents the event 'B hits the target', C represents the event 'C hits the target' and E be the event that exactly two of A, B and C hit the target.

Then $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$ and $P(C) = \frac{2}{3}$

$\therefore$ $P(C^c/E)$

$= \frac{P(A)P(B)P(C^c)}{P(A)P(B)P(C^c) + P(A)P(B^c)P(C) + P(A^c)P(B)P(C)}$

$= \frac{6}{13}$

6. (A)

Sol. X = sum ; Y = product ; P(X=9/Y=0) = $\frac{P(X=9\cap Y=0)}{P(Y=0)}$

X = 9 $\cap$ Y = 0 = {09, 90}

Y = 0 = {01, 02,.......,09, 10, 20 ,....., 09}

P(X=9/Y=0) = 2/19]

7. (A)

Sol. Let us first count the number of elements in F. Total number of functions from A to B is $3^4 = 81$.

The number of functions which do not contain x(y) [z] in its range is 2^4.

$\therefore$ the number of functions which contain exactly two elements in the range is 3 . 2^4 = 48.

The number of functions which contain exactly one element in its range is 3.

Thus, the number of onto functions from A to B is 81 – 48 + 3 = 36

[using principle of inclusion exclusion]

n (F) = 36.

Let f $\in$ F. We now count the number of ways in which $f^{-1}(x)$ consists of single element.

We can choose preimage of x in 4 ways.

The remaining 3 elements can be mapped onto {y, z} is $2^3 - 2 = 6$ ways.

$\therefore$ f^{-1} (x) will consists of exactly one element in

4 × 6 = 24 ways.

Thus, the probability of the required event is 24/36 = 2/3

8. (A)

Sol. Let E_1 denote the event that the letter came from TATANAGAR and E_2 the event that the letter came from CALCUTTA. Let A denote the event that the two consecutive alphabets visible on the envelope are TA. We have $P(E_1) = 1/2$, $P(E_2) = 1/2$, $P(A / E_1) = 2/8$, $P(A / E_2) = 1/7$. Therefore, by Bayes' theorem we have

$P(E_2 / A) =$

$$\frac{P(E_2)P(A/E_2)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$

$= \frac{4}{11}$

9. (B)

Sol. T_1 and T_2 plays and |||ly T_2 and T_3 and T_3 and T_1

Total games played is 3

P(game ends in tie) i.e. every team wins exactly one game

Case-1: T_1 v/s T_2 $\Rightarrow$ T_1 wins

T_2 v/s T_3 $\Rightarrow$ T_2 wins

T_3 v/s T_1 $\Rightarrow$ T_3 wins

P(ties) = $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$

Case-2: T_1 v/s T_2 $\Rightarrow$ T_2 wins

T_2 v/s T_3 $\Rightarrow$ T_3 wins

T_3 v/s T_1 $\Rightarrow$ T_1 wins

P(ties) = $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$

Hence P(ties) = $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ **Ans.**

10. (B)

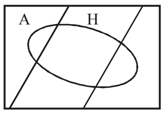
Sol. H : Victim was hit

A : Event that Mr. A was given the live bullet ; P(A) = $\frac{1}{3}$

B : Mr. B had live bullet ; P(B) = $\frac{1}{3}$

C : Mr. C has live bullet ; P(C) = $\frac{1}{3}$

$P(C/H) = \frac{P(C\cap H)}{P(H)} = \frac{P(C)\cdot P(H/C)}{P(H)}$



$P(H) = \frac{1}{3}P(H\cap C) + P(H\cap B) + P(H\cap A)$

$= \frac{1}{3}$ [P(H/C) + P(H/B) + P(H/A)]

$=$ [0.8 + 0.7 + 0.6] = $\frac{0.21}{3}$

$P(C/H) = \frac{0.8}{0.21} = \frac{8}{21}$ **Ans.**

11. (B)

Sol. Let x_i be any element of set P, we have following possibilities

(i) $x_i \in A$, $x_i \in B$; (ii) $x_i \in A$, $x_i \notin B$;

(iii) $x_i \notin A$, $x_i \in B$; (iv) $x_i \notin A$, $x_i \notin B$

Clearly, the element $x_i \in A \cap B$ if it belongs to A and B both. Thus out of these 4 ways only first way is favorable. Now the element that we want to be in the intersection can be chosen in 'n' different ways. Hence required probability is $n.(3/4)^{n-1}$.

12. (A)

Sol. $P(A^C) = 0.3$ $\Rightarrow P(A) = 0.7$

$P(B) = 0.4$ $\Rightarrow P(B^C) = 0.6$

$P(AB^C) = 0.5$ $\Rightarrow P(A) - P(AB) = 0.5$

or, $P(AB) = 0.7 - 0.5 = 0.2$

$P[B/(A\cap B^C)] = \frac{P[B\cap(A\cup B^C)]}{P(A\cup B^C)}$ ...(1)

Now, $B \cap (A\cup B^C) = A\cap B$

$\therefore P[B\cap(A\cup B^C)] = P(A\cap B) = 0.2$ ...(2)

and $P(A\cup B^C) = P(A) + P(B^C) - P(A\cap B^C)$

$= 0.7 + 0.6 - 0.5 = 0.8$ ...(3)

From (1), (2) and (3)

Required probability = $\frac{0.2}{0.8} = \frac{1}{4}$

13. (B)

Sol. Let E = the event that A gets six, P(E) = $\frac{1}{6}$

F = the event that B gets six, P(F) = $\frac{1}{6}$

$\therefore$ P(B wins) = $P(\bar{E}F$ or $\bar{E}\,\bar{F}\,\bar{E}F$ or $\bar{E}\,\bar{F}\,\bar{E}\,\bar{F}\,\bar{E}F$ ....)

(Since B can win the game in 2nd, 4th, 6th ...... throw)

$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\frac{1}{6} + \left(\frac{5}{6}\right)^5\frac{1}{6} + \ldots\ldots\ldots$

$= \frac{5}{36}\left(1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right)$

$= \frac{5}{36}\,\frac{1}{1-\frac{25}{36}} = \frac{5}{11}$

14. (D)

Sol. Let E_1 = the even that six shows when a dice is thrown

E_2 = the number less than or equal to 2 shows where a dice is thrown

$P(E_1) = 1/6$ and $P(E_2) = 2/6 = 1/3$

E = the event that six turns up in the last thrown = the event that E_1 happen in the all previous throws and E_2 happens in the last throw

$P(E) = P(E_1E_2 \text{ or } E_1E_1E_2 \text{ or } E_1E_1E_1E_2.....)$

$= P(E_1).\ P(E_2) + P(E_1).\ P(E_1).\ P(E_2) + P(E_1).\ P(E_1).\ P(E_1).\ P(E_2) + ...$

$$= \frac{1}{3}.\frac{1}{6} + \left(\frac{1}{3}\right)^2.\frac{1}{6} + \left(\frac{1}{3}\right)^3.\frac{1}{6} + ...$$

$$P(E) = \frac{1}{3}.\frac{1}{6} \times \frac{1}{1-1/3} \Rightarrow P(E) = \frac{1}{12}$$

15. (A)

Sol. Total no. of selection of 2 squares out of 64 squares on a chess board = ${}^{64}C_2 = 32 \times 63$

the no. of ways of selecting two consecutive squares from rows or from columns

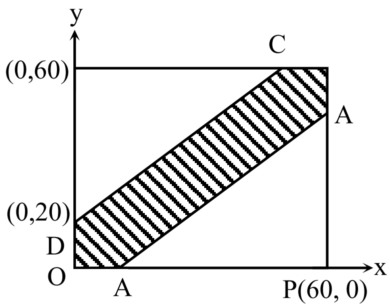
= (7 + 7 + … to 8 terms) + (7 + 7 + …. to 8 terms)

= 56 + 56 = 112

$$\text{Probability} = \frac{112}{32.63} = \frac{1}{18}$$

16. (C)

Sol. A and B arrives at the place of the meeting 'a' minute and 'b' minute after 5 P.M. Their meeting is possible only if $|a - b| \le 20$.



$0 \le a \le 60$ and $0 \le b \le 60$

$-20 \le a - b \le 20$

$-20 \le x - y \le 20 \Rightarrow y \le x + 20$ and $y \ge x - 20$

Required probability

$$= \frac{\text{Area of OABQCDO}}{\text{Area of square OPQR}}$$

$$= \frac{Ar(OPQR) - 2Ar(\Delta APB)}{Ar(OPQR)}$$

$$= \frac{\left[60 \times 60 - \frac{2}{2} \times 40 \times 40\right]}{60 \times 60} = 5/9$$

17. (A)

Sol. Total number of ways in which 8 persons can speak = 8!.

Now 3 positions out of 8 positions can be chosen in ${}^8C_3$i.e. 56 ways and at these positions we can put A_1, A_2 and A_3 in the required order.

Further the remaining persons can speak in 5! ways

$\Rightarrow$ total number of favourable ways = 56(5!)

$\Rightarrow$ required probability = $\frac{56(5!)}{8!}$ = 1/6.

18. (D)

Sol. Probability that the first critic favours the book, $P(E_1) = \frac{5}{5+2} = \frac{5}{7}$

Probability that the second critic favours the book, $P(E_2) = \frac{4}{4+3} = \frac{4}{7}$

Probability that the third critic favours the book, $P(E_3) = \frac{3}{3+4} = \frac{3}{7}$

Majority will be in favour if at least two critics favour the book

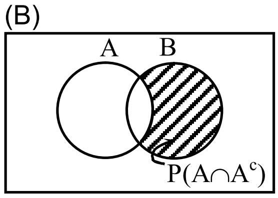
$= P(E_1 \cap E_2 \cap \overline{E}_3) + P(E_1 \cap \overline{E}_2 \cap E_3) + P(\overline{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$

$= P(E_1)\ P(E_2)\ P(\overline{E}_3) + P(E_1)\ P(\overline{E}_2)\ P(E_3) + P(\overline{E}_1 \cap E_2 \cap E_3) + P(E_1)\ P(E_2)\ P(E_3)$

$$= \frac{5}{7} \times \frac{4}{7} \times \left(1 - \frac{3}{7}\right) + \frac{5}{7} \times \left(1 - \frac{4}{7}\right) \times \frac{3}{7} + \left(1 - \frac{5}{7}\right) \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{209}{343}$$

19. (B)

Sol.



$P(A) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$

$$P\left(\frac{A}{B^c}\right) = \frac{P(B \cap A^c)}{P(A^c)}$$

$$= \frac{P(A \cup B) - P(A)}{1 - P(A)}$$

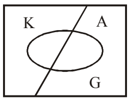
$[\because P(A \cup B) + P(B) - P(A \cap B)]$

$$= \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{4}.\frac{4}{3} = \frac{1}{3}$$

20. (B)

Sol. $P(k) = p$; $P(G) = 1 - p$

$P(A/k) = 1$; $P(A/G) = \frac{1}{m}$



$P(A) = P(A \cap k) + P(A \cap G) = p \cdot 1 + \frac{(1-p)}{m}$

$P(k/A) = \frac{P(k \cap A)}{P(A)} = \frac{mp}{(m-1)p+1}$ **Ans.**

21. **82**

Sol. no. of numbers = 5 ! = 120

E = Event that the number is divisible by 4.

So last two digits to see **12**, 13, 14, 15
21, 23, **24**, 25
31, **32**, 34, 35
41, 42, 43, 45
51, **52**, 53, 54

of the above 4 nos are divisible by 4

Son(E) = no. of numbers divisible by 4

= 3!.4 = 24

Probability = $\frac{n(E)}{n(S)} = \frac{24}{120} = \frac{1}{5}$ = 0.2

$\Rightarrow$ integral part of $(\sqrt{2} + 1)^5$ will be 82.

22. (C)

Sol. A = Rose bush has withered

B_1 = Gardener did not water the rose bush

$P(B_1) = 2/3$

B_2 = Gardener watered the rose bush

$P(B_2) = 1/3$

$P(A/B_1) = \frac{3}{4}$; $P(A/B_2) = \frac{1}{2}$

$P(B_1/A) = \frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1)+P(B_2).P(A/B_2)}$

$= \frac{\frac{2}{3}.\frac{3}{4}}{\frac{2}{3}.\frac{3}{4}+\frac{1}{3}.\frac{1}{2}} = \frac{6}{6+2} = \frac{3}{4} \Rightarrow 75\%$ Ans

23. **375**

Sol. Let

E = the event that A reports it is six

E_1 = six turns up when a dice is thrown

E_2 = six does not turns up when a dice is thrown

Probability that six actually showed up if A reported that of was a six = $P(E_1/E)$

$P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)}$

$= \frac{P(E_1).P(E_1/E)}{P(E_1).P(E_1/E)+P(E_2).P(E_2/E)}$

$P(E/E_1)$ = Probability that the person reports six
when it actually showed
= Probability that he speak thruth
= 3/4

$P(E/E_2)$ = Probability that the person reports six
when it did not show six

= Probability that he lies = $1 - \frac{3}{4}$

$= \frac{1}{4}$

also $P(E) = \frac{1}{6}$ and $P(E_2) = \frac{5}{6}$

$P(E_1/E) = \frac{\frac{1}{6}.\frac{3}{4}}{\frac{1}{6}.\frac{3}{4}+\frac{5}{6}.\frac{1}{4}} = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{5}{24}} = \frac{3}{8}$

$\Rightarrow 1000P = 1000.\frac{3}{8} = 375$

24. 2

Sol. $P(A_1) = P(\omega) + P(BB\omega) + P(BBBB\omega)$

$= \frac{3}{10} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{3}{6}$

$= \frac{3}{10} + \frac{1}{12} + \frac{1}{12\times7} = \frac{332}{840} = \frac{83}{210}$

$\Rightarrow P(A_2) = P(B\omega) + P(BBB\omega) + P(BBBBB\omega)$

$= \frac{5}{10}.\frac{3}{9} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{3}{7} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{1}{6}.\frac{3}{5}$

$= \frac{1}{6} + \frac{1}{28} + \frac{1}{420} = \frac{86}{420} = \frac{43}{210}$

$P(B) = P(R) + P(BR) + P(BBR) + P(BBBR) + P(BBBBR) + P(B^5R)$

$= \frac{2}{10} + \frac{5}{10}.\frac{2}{9} + \frac{5}{10}.\frac{4}{9}.\frac{2}{8} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7} +$

$\frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{2}{6} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{1}{6}.\frac{2}{5}$

$= \frac{1}{5} + \frac{1}{9} + \frac{1}{18} + \frac{1}{42} + \frac{1}{26} + \frac{1}{630}$

$= \frac{126+70+35+15+5+1}{630} = \frac{2\times126}{630} = \frac{2}{5}$

25. (A)

Sol. $N = 10^{99} = 2^{99} \cdot 5^{99}$

$\therefore$ number of divisors of N

$= (100)(100) = 10^4$

now $10^{88} = 2^{88} \cdot 5^{88}$

Hence divisors which are integral multiple of $2^{88} \cdot 5^{88}$ must be of the form of $2^a \cdot 5^b$ where $88 \le a, b \le 99$. Thus there are 12 × 12 ways to choose a and b and hence there are 12 × 12 divisors which are integral multiple if $2^{88} \cdot 5^{88}$.

Hence $p = \frac{144}{10000} = \frac{9}{625}$

$\therefore$ m + n = 634 **Ans.**

26. 2

Sol. Prob. of selection of way box is $= \frac{1}{N+1}$

Let E be the event that the wall clock selected is effective then

$P(E) = P(B_1)\,P(E/B_1) + P(B_2)\,P(E/B_2) + \ldots\ldots + P(B_{N+1})\,P(E/B_{N+1})$

$= \frac{1}{N+1}\left[1 + \frac{N-1}{N} + \frac{N-2}{N} + \ldots\ldots\frac{1}{N} + 0\right]$

$= \frac{1+2+\ldots..+N}{N(N+1)} = \frac{1}{2}$

$P(B_1/E)$

$= \frac{P(B_k).P(E/B_k)}{RE} = \frac{\frac{1}{N+1}\frac{(N-K+1)}{N}}{1/2}$

$= \frac{2N-2K+2}{N^2+N}$

27. 157

Sol. P(atleast 28)=

$P((10,10,10) \text{ or } (10,10,9) \text{ or } (10,10,8) \text{ or } (10,9,9))$

Now P(30) = $\frac{1}{64}$ (Given) i.e. P(10 and 10 and 10) = $\frac{1}{64}$. Hence $(P(10))^3 = \frac{1}{64} \Rightarrow$

P(10) = $\frac{1}{4}$

$\therefore$ P(10 in one shot) = $\frac{1}{4}$ ......(1)

P(8 points in one shot) = $\frac{1}{5}$ and P(less than 8 points) = $\frac{2}{5}$

$\therefore$ P($\geq$ 8 points) = $1 - \frac{2}{5} = \frac{3}{5}$ or P(8 or 9 or 10) = $\frac{3}{5}$

Hence P(8) + P(9) + P(10) = $\frac{3}{5}$

$\frac{1}{5}$ + P(9) + $\frac{1}{4} = \frac{3}{5} \Rightarrow$ P(9) = $\frac{3}{20}$

P(atleast 28 in 3 shots) = P(10, 10, 10) + 3 P(10, 10, 9) + 3 P(10, 10, 8) + 3 P(9, 9, 10)

$= \frac{1}{64} + 3\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{20} + 3\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{5} + 3\cdot\frac{3}{20}\cdot\frac{3}{20}\cdot\frac{1}{4}$

$= \frac{1}{64} + \frac{9}{320} + \frac{3}{80} + \frac{27}{1600}$

$= \frac{25}{1600} + \frac{45}{1600} + \frac{60}{1600} + \frac{27}{1600} = \frac{157}{1600}$

Hence p = 157 **Ans.**]

28. 84

Sol. Required probability

= 1 – P (the roots of $x^2 + 2px + q = 0$ are non-real)

The roots of $x^2 + 2px + q = 0$ will be non-real if $(2p)^2 - 4q < 0$ i.e. if $p^2 < q$.

We enumerate the possible values of p and q.

When q = 1, there is no value of p

When q = 2, 3, 4, possible value of p is 1

When q = 5, 6, 7, 8, 9 possible values of p are 1 and 2

When q = 10, possible values of p are 1, 2 and 3.

Thus, the number of pairs for which $2x^2 + 2px + q = 0$ have non-real roots is

0 + 3 × 1 + 5 × 2 + 1 × 3 = 16

Also, total number of possible pairs is 10 × 10 = 100

Thus, probability of the required event

$= 1 - \frac{16}{100} = 0.84$.

Therefore, 100 k = 100 × 0.84 = 84

29. **(C)**

Sol. Let E_1, denote the event "a coin with two heads" is selected and E_2, denote the event "a fair coin is selected". Let A be the event "the toss results in heads". Then,

$P(E_1) = \frac{1}{n+1}$, $P(E_2) = \frac{n}{n+1}$, $P\left(\frac{A}{E_1}\right) = 1$

and $P\left(\frac{A}{E_2}\right) = \frac{1}{2}$.

$\therefore P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)$

$\Rightarrow \frac{7}{12} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$

$\left[\because P(A) = \frac{7}{12}\right]$

$\Rightarrow$ 12 + 6n = 7n + 7 $\Rightarrow$ n = 5.

30. 5

Sol. $\left[{}^6C_3 \times \left(\frac{10}{20}\right)^3 \times \left(\frac{10}{20}\right)^3\right] \times \frac{10}{20}$

$= 20 \times \left(\frac{1}{2}\right)^7$

$= \frac{5}{(2)^5} = \frac{5}{32}$

$\therefore 32k = 32 \times \frac{5}{32} = 5$