

JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12th

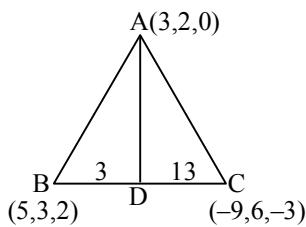
CHAPTER :- VECTOR & 3D

PAPER CODE :- CWT-11

ANSWER KEY											
1.	(B)	2.	(B)	3.	(B)	4.	(B)	5.	(A)	6.	(D)
8.	(D)	9.	(D)	10.	(A)	11.	(C)	12.	(C)	13.	(D)
15.	(D)	16.	(B)	17.	(B)	18.	(B)	19.	(B)	20.	(C)
22.	17	23.	2	24.	1	25.	4	26.	58	27.	1
29.	5	30.	3							28.	3

SOLUTIONS

1. (B)



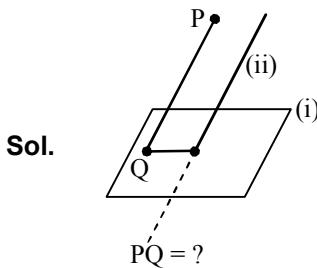
$$AB = \sqrt{4+1+4} = 3$$

$$AC = \sqrt{144+16+9} = 13$$

$$AD = \sqrt{\left(\frac{-27+65}{3+13}\right)^2 + \left(\frac{18+39}{3+13}\right)^2 + \left(\frac{-9+26}{3+13}\right)^2}$$

$$= \sqrt{\left(\frac{19}{8}\right)^2 + \left(\frac{57}{16}\right)^2 + \left(\frac{17}{16}\right)^2}$$

2. (B)



$$2x + 5y - 6z = 16 \quad \dots \text{(i)}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{-2} \quad \dots \text{(ii)}$$

$$P(3, -4, 5)$$

DR's of line (ii) = DR's of line PQ

$$\text{Eq. of line } PQ \Rightarrow \frac{x-3}{2} = \frac{y+4}{1} = \frac{z-5}{-2} = \lambda$$

∴ Coordinate of the point Q $(3 + 2\lambda, -4 + \lambda, 5 - 2\lambda)$

Put the coordinates of Q in eqn. (i) $\Rightarrow \lambda = ?$
distance between P & Q

$$2(3 + 2\lambda) + 5(-4 + \lambda) - 6(5 - 2\lambda) = 16$$

$$6 + 4\lambda - 20 + 5\lambda - 30 + 12\lambda = 16$$

$$\lambda = \frac{20}{7}$$

$$Q = \left(3 + \frac{40}{7}, -4 + \frac{20}{7}, 5 - \frac{40}{7}\right)$$

$$Q = \left(\frac{61}{7}, \frac{-8}{7}, \frac{-5}{7}\right)$$

$$PQ = \sqrt{\left(\frac{61}{7} - 3\right)^2 + \left(\frac{-8}{7} + 4\right)^2 + \left(\frac{-5}{7} + 5\right)^2}$$

$$PQ = \frac{\sqrt{(40)^2 + (20)^2 + (40)^2}}{7}$$

$$PQ = \frac{60}{7}$$

3. (B)

$$\vec{x} + \vec{y} + \vec{z} = 0$$

$$\Rightarrow \vec{y} + \vec{z} = -\vec{x} \Rightarrow |\vec{y} + \vec{z}|^2 = |\vec{x}|^2$$

$$\Rightarrow |\vec{y}|^2 + |\vec{z}|^2 + 2\vec{y} \cdot \vec{z} = |\vec{x}|^2$$

$$\Rightarrow 4 + 4 + 2(2)(2)\cos\theta = 4$$

$$\Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = 120^\circ$$

$$\Rightarrow \operatorname{cosec}^2\theta + \cot^2\theta = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

4. (B)

$$\left(\sqrt{x^2 + y^2}\right)^2 + \left(\sqrt{y^2 + z^2}\right)^2 + \left(\sqrt{z^2 + x^2}\right)^2 =$$

$$36$$

$$\Rightarrow 2(x^2 + y^2 + z^2) = 36$$

$$\Rightarrow x^2 + y^2 + z^2 = 18$$

$$\Rightarrow \sqrt{x^2 + y^2 + z^2} = \sqrt{18} \Rightarrow OP = 3\sqrt{2}$$

5. (A)

Sol. We have $\sin\alpha = \frac{3}{5}$, $\cos\beta = \frac{2+2}{\sqrt{5}\sqrt{5}} = \frac{4}{5}$

$$\text{So, } (\cos^2\alpha + \sin^2\beta) = \frac{16}{25} + \frac{9}{25} = 1. \text{ s}$$

Ans.

6. (D)

Sol. Direction ratio's of normal of the plane are $<2, 3, 4>$.

$$(A) \frac{x-0}{-6} = \frac{y-1}{9} = \frac{z-0}{-3} = t ; \text{ Now, } -6$$

$$(2) + 9(3) - 3(4) \neq 0.$$

$$(B) \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-1}{3} = t ; \text{ Now, } 1(2) + 1(3) + 3(4) \neq 0.$$

$$(C) \frac{x-1}{2} = \frac{y-0}{3} = \frac{z-2}{-1} = t ; \text{ Now, } 2(2) + 3(3) - 1(4) \neq 0.$$

$$(D) \frac{x-1}{2} = \frac{y-4}{-4} = \frac{z-0}{2} = t \text{ Now, } 2(2) - 4(3) + 2(4) = 0.$$

7. (A)

Sol. Direction ratios of joining $(-1, 2, 4)$ and $(1, 0, 5)$ are

$$1 + 1, 0 - 2, 5 - 4 = (2, -2, 1)$$

\therefore D.C. of this line are

$$\ell = \frac{2}{\sqrt{4+4+1}} = 2/3$$

$$m = \frac{-2}{\sqrt{4+4+1}} = -2/3$$

$$n = \frac{1}{\sqrt{4+4+1}} = 1/3$$

\therefore required projection

$$= \left| (4-3)\frac{2}{3} + (6-4)(-2/3) + (3-5)\frac{1}{3} \right|$$

$$= \left| \frac{2}{3} - \frac{4}{3} - \frac{2}{3} \right| = 4/3$$

$\left. \begin{array}{l} \text{As projection of line joining } (x_1, y_1, z_1) \text{ and} \\ (x_2, y_2, z_2) \text{ to the line whose d.c.'s are } \ell, m, n \text{ is} \\ |(x_2 - x_1)\ell + (y_2 - y_1)m + (z_2 - z_1)n| \end{array} \right\}$

8. (D)

Sol. Here L_1 is parallel to $2\hat{i} + \hat{j} + 4\hat{k}$ and L_2 is parallel to $4\hat{i} - 3\hat{j} + \hat{k} \Rightarrow L_1 \neq L_2$.

We have (Lines in parametric form) as

$$L_1 : x = 1 + 2s, y = -1 + s, z = 2 + 4s$$

$$\text{and } L_1 : x = -2 + 4t, y = -3t, z = -1 + t$$

\therefore The lines intersect if there are s and t for which

$$2s - 4t = -3 \quad \dots\dots(1)$$

$$s + 3t = 1 \quad \dots\dots(2)$$

$$4s - t = -3 \quad \dots\dots(3)$$

Any solution of this system must correspond to a point of intersection of L_1 and L_2 and if no solution exists, then L_1 and L_2 are skew.

\therefore From (1) and (2), we get

$$s = \frac{-1}{2}, t = \frac{1}{2}s, \text{ which does not satisfy (3)}$$

$\Rightarrow L_1$ and L_2 are skew line.

9. (D)

Sol. \because DR's of the normal of the plane $\Rightarrow 2, -3, 6$

& DR's of x-axis is $1, 0, 0$

& If ' θ ' is the angle between the plane

$$\text{& x-axis, then } \sin \theta = \frac{2(1) + (-3)(0) + (6)(0)}{\sqrt{2^2 + (-3)^2 + 6^2} \sqrt{1}} =$$

$$\frac{2}{7}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{7}\right)$$

$$\therefore a = \frac{2}{7}$$

10. (A)

Sol. Given line is

$$\Rightarrow \vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$$

.....(i)

$$\text{& } \vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5 \dots\dots(ii)$$

$$\text{By (i)} \Rightarrow \begin{cases} \vec{a} = (2\hat{i} - 2\hat{j} + 3\hat{k}) \\ \vec{b} = (\hat{i} - \hat{j} + 4\hat{k}) \end{cases}$$

$$\text{By (ii)} \Rightarrow \vec{n} = (\hat{i} + 5\hat{j} + \hat{k})$$

$$\therefore \vec{b} \cdot \vec{n} = 0$$

Therefore, the line is parallel to the plane. Thus, the distance between the line & the plane is equal to the length of the

perpendicular from a point $\vec{a} = (2\hat{i} - 2\hat{j} + 3\hat{k})$ on the line to the given plane.

Hence, the required distance

$$= \frac{|(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5|}{\sqrt{1+5^2+1}}$$

$$= \frac{10}{3\sqrt{3}}$$

11. (C)

Sol. Line of shortest distance will be along

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore x = (\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \frac{(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \cos^{-1}(\cos \sqrt{6} x) = \cos^{-1}(\cos 1) = 1$$

12. (C)

Sol. Equation of line passing through given point P and normal to given plane is

$$\vec{r} = (\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

⇒ Image point Q

$$= ((1+2\lambda)\hat{i} + (3-\lambda)\hat{j} + (4+\lambda)\hat{k})$$

Now mid point of

$$PQ = [(\lambda+1)\hat{i} + (3-\lambda/2)\hat{j} + (4+\lambda/2)\hat{k}]$$

satisfies given plane that gives $\lambda = -2$

So position vector of image is $-3\hat{i} + 5\hat{j} + 2\hat{k}$

13. (D)

Sol. Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = t$

from given equation, we have

$$(\vec{a} \cdot \vec{a})(\vec{m} - \vec{b}) - \{\vec{a} \cdot (\vec{m} - \vec{b})\}\vec{a} + (\vec{b} \cdot \vec{b})(\vec{m} - \vec{c}) - \{\vec{b} \cdot (\vec{m} - \vec{c})\}\vec{b} + (\vec{c} \cdot \vec{c})(\vec{m} - \vec{a}) - \{\vec{c} \cdot (\vec{m} - \vec{a})\}\vec{c} = 0$$

$$\Rightarrow |\vec{a}|^2(\vec{m} - \vec{b}) + |\vec{b}|^2(\vec{m} - \vec{c}) + |\vec{c}|^2(\vec{m} - \vec{a}) - [(\vec{a} \cdot \vec{m})\vec{a} + (\vec{b} \cdot \vec{m})\vec{b} + (\vec{c} \cdot \vec{m})\vec{c}] + (\vec{a} \cdot \vec{b})\vec{a} + (\vec{b} \cdot \vec{c})\vec{b} + (\vec{c} \cdot \vec{a})\vec{c} = 0$$

also $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

$$\Rightarrow t(3\vec{m} - \vec{a} - \vec{b} - \vec{c}) - t^2\vec{m} = 0$$

$$\Rightarrow 2\vec{m} = \vec{a} + \vec{b} + \vec{c} \Rightarrow \vec{m} = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$$

14. (C)

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\alpha + 1)\vec{d} = (\beta + 1)\vec{a}$$

$$\Rightarrow \vec{d} = \frac{\beta + 1}{\alpha + 1}\vec{a}$$

$$\text{So } \vec{a} + \vec{b} + \vec{c} = \alpha\vec{d} = \alpha \left(\frac{\beta + 1}{\alpha + 1} \right) \vec{a}$$

$$\Rightarrow \vec{a} \left\{ 1 - \frac{\beta + 1}{\alpha + 1} \alpha \right\} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow [\vec{a} \vec{b} \vec{c}] \neq 0 \Rightarrow \alpha = -1$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$$

15. (D)

$$\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} = \vec{d}$$

$$\therefore \{ \vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} \} \times \vec{c} = \vec{d} \times \vec{c}$$

$$\text{or } (\vec{x} \times \vec{a}) \times \vec{c} + (\vec{x} \cdot \vec{b})(\vec{c} \times \vec{c}) = \vec{d} \times \vec{c}$$

$$= (\vec{x} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{x} = (\vec{d} \times \vec{c})$$

$$\vec{a} \times \{(\vec{x} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{x}\} = \vec{a} \times (\vec{d} \times \vec{c})$$

$$= -(\vec{a} \cdot \vec{c})(\vec{a} \times \vec{x}) = \vec{a} \times (\vec{d} \times \vec{c})$$

$$\therefore \vec{a} \times \vec{a} = 0$$

$$\vec{x} \times \vec{a} = \frac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a} \cdot \vec{c}}$$

$$\vec{a} \times (\vec{x} \times \vec{a}) = \vec{a} \times \frac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a} \cdot \vec{c}}$$

$$(\vec{a} \cdot \vec{a})\vec{x} - (\vec{a} \cdot \vec{x})\vec{a} = \vec{a} \times \frac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a} \cdot \vec{c}}$$

$$\vec{a}^2 \vec{x} = (\vec{a} \cdot \vec{x})\vec{a} + \vec{a} \frac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a} \cdot \vec{c}}$$

$$\vec{x} = \frac{(\vec{a} \cdot \vec{x})\vec{a}}{\vec{a}^2} + \vec{a} \times \frac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a} \cdot \vec{c})\vec{a}^2}$$

$$\vec{x} = \lambda \vec{a} + \vec{a} \times \frac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a} \cdot \vec{c})\vec{a}^2} \Rightarrow \lambda = \frac{\vec{a} \cdot \vec{x}}{\vec{a}^2}$$

16. (B)

$$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$$

$$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$$

$$\Rightarrow \vec{d} = \vec{c} + \lambda \vec{b}$$

$$\Rightarrow \vec{d} = (7 + \lambda)\hat{i} + (-3 + \lambda)\hat{j} + (4 + \lambda)\hat{k}$$

$$\Rightarrow \vec{a} \cdot \vec{d} = 0$$

$$\Rightarrow 7 + \lambda + 2(4 + \lambda) = 0 \Rightarrow \lambda = -5$$

$$\text{So } \vec{d} = 2\hat{i} - 8\hat{j} - \hat{k}$$

17. (B)

$$\vec{b} \text{ perpendicular } \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0$$

$$\Rightarrow \tan^2 \alpha - \tan \alpha - 6 = 0$$

$$\Rightarrow \tan \alpha = 3, -2$$

also \vec{a} make an obtuse angle with z-axis
therefore $\vec{a} \cdot \hat{k} < 0 \Rightarrow \sin 2\alpha < 0$

$$\text{if } \tan \alpha = 3 \text{ then } \sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} = \frac{4}{3} > 0$$

Now $\tan 2\alpha > 0, \sin 2\alpha < 0 \Rightarrow \alpha \in \text{third quadrant and } \tan \alpha = -2$

$$\Rightarrow \tan(\pi - \alpha) = 2 \Rightarrow \alpha = (2n + 1)\pi - \tan^{-1} 2, n \in \mathbb{N}$$

18. (B)

$$\vec{c} = \lambda(\vec{a} \times \vec{b})$$

$$1 = (\vec{a} \times \vec{b}) \cdot \vec{c} = \frac{\vec{c} \cdot \vec{c}}{\lambda} = \frac{|\vec{c}|^2}{\lambda} = \frac{1}{3\lambda} \Rightarrow \lambda = \frac{1}{3}$$

$$\vec{c}^2 = \lambda^2 (\vec{a} \times \vec{b})^2$$

$$\frac{1}{3} = \frac{1}{9} = (a^2 b^2 \sin^2 \theta) = \frac{1}{9} \times 2 \times 3 \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{4}$$

19. (B)

Sol. Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$$\text{then } \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{\vec{a} \cdot \vec{d}}{ad}$$

$$\text{i.e. } \frac{(\lambda\vec{b} + \mu\vec{c}) \cdot \vec{b}}{b} = \frac{(\lambda\vec{b} + \mu\vec{c}) \cdot \vec{d}}{d}$$

$$\text{i.e. } \frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})] \cdot (2\hat{i} + \hat{j})}{\sqrt{5}}$$

$$= \frac{[\lambda(2\hat{i} + \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})] \cdot (\hat{j} + 2\hat{k})}{\sqrt{5}}$$

$$\text{i.e. } \lambda(4+1) + \mu(2-1) = \lambda(1) + \mu(-1+2)$$

$$\text{i.e. } 4\lambda = 0 \text{ i.e. } \lambda = 0$$

$$\therefore \vec{a} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

20. (C)

Sol. We have $\begin{vmatrix} 2 & 3 & 4 \\ 1 & \alpha & 2 \\ 1 & 2 & \alpha \end{vmatrix} = 15$

$$\Rightarrow |2(\alpha^2 - 4) + 3(2 - \alpha) + 4(2 - \alpha)| = 15$$

$$\Rightarrow |2\alpha^2 - 7\alpha + 6| = 15$$

$$\Rightarrow \text{Either } (2\alpha^2 - 7\alpha + 6) = 15 \text{ or } (2\alpha^2 - 7\alpha + 6) = -15$$

$$\text{So, either } (2\alpha^2 - 7\alpha - 9) = 0 = (\alpha + 1)(2\alpha - 9)$$

$$\therefore \alpha = -1, \frac{9}{2} \text{ or } 2\alpha^2 - 7\alpha + 21 = 0$$

has non-real roots as discriminant is negative.

$$\text{Hence } \alpha = \frac{9}{2}. \text{ Ans.}$$

21. 7

Sol. $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$

P(1, 2, 3)

A _____ B
 $M(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$
 $\therefore M(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

As lines PM and AB are perpendicular, therefore

$$3(3\lambda + 6 - 1) + 2(2\lambda + 7 - 2) - 2(-2\lambda + 7 - 3) = 0$$

$$17\lambda = -17$$

$$\lambda = -1$$

$$\therefore M(3, 5, 9)$$

$$\therefore PM = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$$

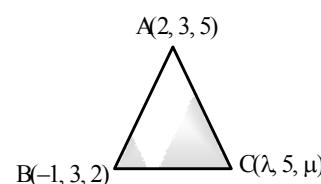
$$PM = \sqrt{4+9+36}$$

$$PM = 7$$

22. 17

Sol. Mid point of BC is $\left(\frac{\lambda-1}{2}, 4, \frac{2+\mu}{2}\right)$

DR's of median through A are -



$$\frac{\lambda-1}{2} - 2, 4 - 3, \frac{2+\mu}{2} - 5$$

$$\text{i.e. } \frac{\lambda-5}{2}, 1, \frac{\mu+8}{2}$$

The medium is equally inclined to axes,
 \therefore D.R's must be

$$\text{equal } \frac{\lambda-5}{2} = 1 = \frac{\mu+8}{2} \Rightarrow \lambda = 7 \text{ and } \mu = 10$$

23. 2

Sol. Here $d_1 = d \cos(90^\circ - \alpha)$, $d_2 = d \cos(90^\circ - \beta)$ and $d_3 = d \cos(90^\circ - \gamma)$

$$d_1 = d \sin \alpha, d_2 = d \sin \beta, d_3 = d \sin \gamma$$

$$\Rightarrow d_1^2 + d_2^2 + d_3^2 = kd^2$$

$$\Rightarrow d^2(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma) = kd^2$$

$$\therefore k = 2.$$

24. 1

Sol. Let the vertices A, B, C, D quadrilateral be

$(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ & (x_4, y_4, z_4) and the equation of plane PQRS be

$$u = ax + by + cz + d = 0$$

let $u_r = a_r x + b_r y + c_r z + d$ where $r = 1, 2, 3, 4$

$$\text{Then } \frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA}$$

$$= \left(-\frac{u_1}{u_2}\right) \left(-\frac{u_2}{u_3}\right) \left(-\frac{u_3}{u_4}\right) \left(-\frac{u_4}{u_1}\right) = 1.$$

25. 4

Sol. Point $(3, 2, 1)$ & $(2, -3, -1)$

lies on $11x + my + nz = 28$

$$\text{i.e., } 33 + 2m + n = 28$$

$$\Rightarrow 2m + n = -5 \quad \dots (1)$$

$$\text{and } 22 - 3m - n = 28$$

$$\Rightarrow -3m - n = 6 \quad \dots (2)$$

From (1) and (2)

$$m = -1 \text{ and } n = -3.$$

$$|m + n| = |-1 - 3| = 4$$

26. 58

Sol. Since, both the planes are parallel

$$P_1 : 4x - 6y + 12z + 10 = 0$$

$$P_2 : 4x - 6y + 12z + d = 0$$

$$b = -6, c = 12$$

$$\text{Now, } \left| \frac{d-10}{2\sqrt{4+9+36}} \right| = 3$$

$$|d-10| = 42 \Rightarrow d = 52 \text{ or } -32$$

$$\therefore P_2 \text{ is } 4x - 6y + 12z + 52 = 0$$

$$\text{or } 4x - 6y + 12z - 32 = 0$$

\therefore Point $(-3, 0, -1)$ is lying between planes P_1 and P_2

\therefore On substituting the point in the equation of the planes both expressions must be of opposite sign.

$$\text{From } P_1 : 4 \times (-3) - 6 \times 0 + 12(-1) + 10 = \text{ve}$$

$$\text{From } P_2 : 4 \times (-3) - 6 \times 0 + 12(-1) + 52 = +\text{ve}$$

$\therefore d$ must be 52

$$\text{Hence, } (b + c + d) = -6 + 12 + 52 = 58$$

Ans.

27. 1

Sol. The vectors are coplanar if we can find two scalars λ, μ such that

$$x\vec{i} + \vec{j} + \vec{k} = \lambda(\vec{i} + y\vec{j} + \vec{k}) + \mu(\vec{i} + \vec{j} + z\vec{k})$$

$$\Rightarrow x = \lambda + \mu, 1 = \lambda y + \mu, 1 = \lambda + \mu z$$

$$\therefore 1-x = 1-\lambda-\mu, y = \frac{1-\mu}{\lambda}, z = \frac{1-\lambda}{\mu}$$

$$\therefore 1-y = \frac{\lambda-1+\mu}{\lambda} \text{ and } 1-z = \frac{\mu-1+\lambda}{\mu}$$

$$\therefore \frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z}$$

$$= \frac{1}{1-\lambda-\mu} + \frac{\lambda}{\lambda+\mu-1} + \frac{\mu}{\lambda+\mu-1}$$

$$= \frac{1-\lambda-\mu}{1-\lambda-\mu} = 1.$$

28. 3

Sol. If two vectors are perpendicular then $\cos \theta = 0$

$$\therefore a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$$

$$2(4) + P(-2) + 2(-1) = 0$$

$$8 - 2P - 2 = 0$$

$$-2P = -6$$

$$P = 3$$

29. 5

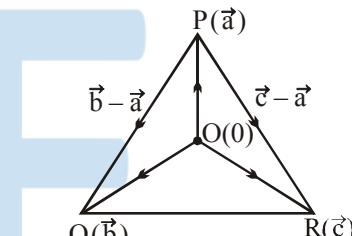
Sol. The line contained by the planes is along the vector $(2\hat{i} + \hat{j}) \times (\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 2\hat{j} - 3\hat{k}$

Since it is parallel to the plane $kx + y + z + 2 = 0$, so

$$k(1) + 1(-2) + 1(-3) = 0 \Rightarrow k = 5. \text{ Ans.}$$

30. 3

Sol. Given that $\vec{a} + k_1\vec{b} + k_2\vec{c} = \vec{0}$



$$\text{Now, } \frac{\text{Area } (\Delta PQR)}{\text{Area } (\Delta OQR)}$$

$$= \frac{\frac{1}{2} |(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})|}{\frac{1}{2} |(\vec{b} \times \vec{c})|} = 4$$

$$\Rightarrow \frac{|((1+k_1)\vec{b} + k_2\vec{c}) \times (k_1\vec{b} + (1+k_2)\vec{c})|}{|\vec{b} \times \vec{c}|} = 4$$

$$\Rightarrow (1+k_1)(1+k_2) - k_1 k_2 = 4$$

$$\text{Hence, } k_1 + k_2 = 3. \text{ Ans.}$$