JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12th **PAPER CODE :- CWT-10**

CHAPTER :- DIFFERENTIAL EQUATION

ANSWER KEY

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(B)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(A)	**9.**	(D)	**10.**	(B)	**11.**	(B)	**12.**	(A)	**13.**	(C)	**14.**	(C)
15.	(B)	**16.**	(C)	**17.**	(C)	**18.**	(B)	**19.**	(A)	**20.**	(D)	**21.**	2
22.	48	**23.**	40	**24.**	79	**25.**	3	**26.**	5	**27.**	7	**28.**	1
29.	2	**30.**	5										

SOLUTIONS

1. (B)

Sol. $\frac{1}{1+y^2}.\frac{dy}{dx} + 2x(\tan^{-1}y) = x^3$

Put $\tan^{-1}y = z$

$\therefore \frac{1}{1+y^2}.\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dz}{dx} + (2x)z = x^3$

$\Rightarrow z.e^{x^2} = \frac{1}{2}\int 2e^{x^2}.x^3\,dx + C$

$\Rightarrow 2e^{x^2}(\tan^{-1}y) = x^2e^{x^2} - e^{x^2} + 2C$

$\Rightarrow 2\tan^{-1}y = x^2 - 1 + 2Ce^{-x^2}$

2. (C)

Sol. The intersection of $y - x + 1 = 0$ and $y + x + 5 = 0$ is $(-2, -3)$. Put $x = X - 2$, $y = Y - 3$. The given equation reduces to $\frac{dY}{dX} = \frac{Y-X}{Y+X}$. This is a homogeneous equation, so puttig $Y = \upsilon X$, we get

$X\frac{d\upsilon}{dX} = -\frac{\upsilon^2+1}{\upsilon+1}$

$\Rightarrow \left(-\frac{\upsilon}{\upsilon^2+1} - \frac{1}{\upsilon^2+1}\right)d\upsilon = \frac{dX}{X}$

$\Rightarrow -\frac{1}{2}\log(\upsilon^2+1) - \tan^{-1}\upsilon = \log|X| + C$

$\Rightarrow \log(Y^2+X^2) + 2\tan^{-1}\frac{Y}{X} = C$

$\Rightarrow \log((y+3)^2 + (x+2)^2) + 2\tan^{-1}\frac{y+3}{x+2} = C$

3. (A)

Sol. $2x\cos y\,dx + y^2\cos x\,dx + 2y\sin x\,dy - x^2\sin y\,dy = 0$

$\Rightarrow (2x\cos y\,dx - x^2\sin y\,dy) + (y^2\cos x\,dx + 2y\sin x\,dy) = 0$

$\Rightarrow d(x^2\cos y + y^2\sin x) = 0$

$\Rightarrow x^2\cos y + y^2\sin x = C$

4. (B)

Sol. $x = r\cos\theta$, $y = \sin\theta$,

$\frac{dr}{|-r^2|} = d\theta \to \sin^{-1}r + \theta + \alpha$

$\Rightarrow r = \sin(\theta + \phi)$

$r^2 = y\cos\alpha + x\sin\alpha$

$x^2 + y^2 - x\sin\alpha - y\cos\alpha = 0$

$\therefore$ radius = 1/2

5. (A)

Sol. Rearrange the diff. equation

$xdx + \frac{ydx - xdy}{y^4} = 0$

$x^3dx + \frac{x^2}{y^2}.\frac{ydx - xdy}{y^2} = 0$

$x^3dx + \left(\frac{x}{y}\right)^2\frac{d}{dx}\left(\frac{x}{y}\right) = 0$

integratings $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 + c$

6. (C)

Sol. $\because \frac{dy}{dx} + y = f(x)$

$\therefore$ I.F. $= e^x$

solution will be $y \cdot e^x = \int f(x)e^x\,dx$

(i) If $0 \le x \le 2$, $y \cdot e^x = \int e^{-x}\cdot e^x\,dx = x + c$

$\therefore$ $y(0) = 1$

$\therefore$ $1 = c$

$\therefore$ $y = \frac{x+1}{e^x}$

(ii) If $x > 2$

$y \cdot e^x = \int e^{-2}\cdot e^x\,dx = \int e^{x-2}\,dx$

$\Rightarrow$ $y e^x = e^{x-2} + c_1$

$\Rightarrow$ $y = e^{-2} + c_1e^{-x}$

function is continuous at $x = 2$

$\therefore \frac{3}{e^2} = \frac{1}{e^2} + \frac{c_1}{e^2} \Rightarrow c_1 = 2$

$\therefore y(x) = \begin{cases} \frac{x+1}{e^x}, & 0 \le x \le 2 \\ \frac{1}{e^2} + \frac{2}{e^x}, & x > 2 \end{cases}$

$\therefore y(3) = \frac{1}{e^2} + \frac{2}{e^3} = \left(\frac{e+2}{e^3}\right)$

7. (C)

Sol. $y = u^m$

$dy/dx = mu^{m-1}\frac{du}{dx}$

The given differential equation becomes

$2x^4.u^m.\ mu^{m-1}\frac{du}{dx} + u^{4m} = 4x^6$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4x^{2m-1}}$

For homogeneous equation degree should be same in numerator & denominator so,

$6 = 4m = 4 + 2m - 1 \Rightarrow m = 3/2$

8. (A)

Sol. We have $\frac{y_3}{y_2} = 8 \Rightarrow \ell n\ y_2 = 8x + C_1$

Putting x = 0, we have $C_1 = \log y_2(0) = \log 1 = 0$

$\therefore \log y_2 = 8x$ or $y_2 = e^{8x}$

i.e. $y_1 = \frac{e^{8x}}{8} + C_2$

Again, putting x = 0 , we have $C_2 = -\frac{1}{8}$

So, $y_1 = \frac{1}{8}(e^{8x} - 1) \Rightarrow y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x\right) + C_3$

Putting x = 0, we have $C_3 = \frac{1}{8} - \frac{1}{64} = \frac{7}{64}$

Thus $y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$

9. (D)

Sol. $y = e^{4x} + 2e^{-x}$

$\frac{dy}{dx} = 4e^{4x} - 2e^{-x} \Rightarrow \frac{d^2y}{dx^2} = 16e^{4x} + 2e^{-x}$

$\frac{d^3y}{dx^3} = 64e^{4x} - 2e^{-x}$

$\therefore \frac{\frac{d^3y}{dx^3} - 13\frac{dy}{dx}}{y}$

$= \frac{(64e^{4x} - 2e^{-x}) - 13(4e^{4x} - 2e^{-x})}{e^{4x} + 2e^{-x}}$

$= \frac{12e^{4x} + 24e^{-x}}{e^{4x} + 2e^{-x}} = 12$

$\therefore \frac{k}{3} = 4.$

10. (B)

Sol. Solving the equations of the asymptotes the centre is x = 1 and y = 0, since $e = \sqrt{2}$ the equation of the family of the hyperbola is

$\frac{(x-1)^2}{a^2} - \frac{(y)^2}{a^2} = 1$

$\Rightarrow 2(x - 1) - 2y\frac{dy}{dx} = 0$

$\Rightarrow (x - 1) = yy'$ is differential equation.

11. (B)

Sol. Let the curve be y = f(x). The equation of tangent at any point (x, y) is given by $Y - y = f'(x)(X - x)$. So the portion of the axis of x which is cut off between the origin and the tangent at any point is obtained by putting Y = 0. Therefore,

$x - \frac{y}{f'(x)} = Ky \Rightarrow x - y\frac{dx}{dy} = Ky \Rightarrow \frac{dx}{dy} - \frac{x}{y} = -K$

which is a linear equation in x, so its integrating factor is $e^{-\int(1/y)dy} = y^{-1}$. Therefore, multiplying by y^{-1}, we have

$\frac{d}{dy}(xy^{-1}) = -Ky^{-1} \Rightarrow xy^{-1} = -K\log y + C$

$\Rightarrow x = y(C - K\log y)$

where C is arbitrary constant.

12. (A)

Sol. $\left(\frac{dx}{x} - \frac{dy}{y}\right) + \left(\frac{x^2dy - y^2dx}{(x-y)^2}\right) = 0$

$\left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right)}{\left(\frac{1}{y} - \frac{1}{x}\right)^2} = 0$

$\left(\frac{dx}{x} - \frac{dy}{y}\right) + \left[\frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{x} - \frac{1}{y}\right)^2}\right] = 0$

$\ln|x| - \ln|y| - \frac{1}{\left(\frac{1}{x} - \frac{1}{y}\right)} = c$

$\ln\left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$

13. (C)

Sol. We can write $y = A\cos(x + B) - Ce^x$

where $A = c_1 + c_2$, $B = c_3$ and $C = c_4e^{c_5}$

$\frac{dy}{dx} = -A\sin(x + B) - Ce^x$

$\Rightarrow \frac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x$

$\Rightarrow \frac{d^2y}{dx^2} + y = -2Ce^x$

$\Rightarrow \frac{d^3y}{dx^3} + \frac{dy}{dx} = -2Ce^x = \frac{d^2y}{dx^2} + y$

$\Rightarrow \frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

Which is a differential equation of degree 1.
Hence (C) is the correct answer.

14. (C)

Sol. We have, $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$\Rightarrow \quad y\,dx - x\,dy = ay^2\,dx + ady$

$\Rightarrow \quad y(1 - ay)\,dx = (x + a)\,dy$

$\Rightarrow \quad \frac{dx}{x+a} - \frac{dy}{y(1-ay)} = 0$

Integrating, we get

$\log(x + a) - \log y + \log(1 - ay) = \log c$

$\Rightarrow \log\frac{(a+x)(1-ay)}{y} = \log c \Rightarrow (x + a)$

$(1 - ay) = cy.$

Since the curve passes through $\left(a, -\frac{1}{a}\right)$,

$\therefore 2a \times (1 + 1) = -\frac{c}{a} \Rightarrow c = -4a^2.$

So, the equation of curve is

$(x + a)(1 - ay) = -4a^2y.$

Hence (C) is the correct answer

15. (B)

Sol. We have, $x\,dy = \left(y + \frac{xf(y/x)}{f'(y/x)}\right)dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{f(y/x)}{f'(y/x)}$ which is homogeneous.

Put $y = Vx \Rightarrow \frac{dy}{dx} = V + x\frac{dV}{dx}$,

We obtain

$V + x\frac{dV}{dx} = V + \frac{f(V)}{f'(V)}\,dV$

$\Rightarrow \frac{f'(V)}{f(V)}dV = \frac{dx}{x}$

Integrating, we get

$\Rightarrow \log f(V) = \log cx \quad \Rightarrow f\left(\frac{y}{x}\right) = cx.$

Hence (B) is the correct answer.

16. (C)

Sol. We have, $\frac{dy}{dx} = \frac{y}{x - 2\sqrt{xy}}$ which is homogeneous.

Put $y = Vx$ so that $\frac{dy}{dx} = x\frac{dV}{dx} + V$

$\Rightarrow x\frac{dV}{dx} = \frac{V}{1-2\sqrt{V}} - V = \frac{2V^{3/2}}{1-2\sqrt{V}}$

$\Rightarrow \frac{dx}{x} = \frac{1-2\sqrt{V}}{2V^{3/2}}dV = \left(\frac{1}{2V^{3/2}} - \frac{1}{V}\right)dV$

Integrating, we get

$-c + \log x = -V^{-1/2} - \log V$

$= -\sqrt{\frac{x}{y}} - \log y + \log x$

$\Rightarrow \log y + \sqrt{\frac{x}{y}} = c.$

Hence (C) is the correct answer.

17. (C)

Sol. The given equation is

$\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$

$\Rightarrow yf'(x)\,dx - f(x)\,dy = y^2dx$

$\Rightarrow \frac{yf'(x)dx - f(x)dy}{y^2} = dx \Rightarrow d\left[\frac{f(x)}{y}\right] = dx$

On integration, we get

$\frac{f(x)}{y} = x + c \Rightarrow f(x) = y(x + c).$

18. (B)

Sol. Given that, $\frac{dy}{dx} = \frac{2x-y}{x+2y}$ ... (i)

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{2-v}{1+2v}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{2 - v - v(1+2v)}{1+2v}$

$\Rightarrow \int\frac{1+2v}{2(1-v-v^2)}dv = \int\frac{1}{x}dx$

$\Rightarrow \quad \log c - \frac{1}{2}\log(1 - v - v^2) = \log x$

$\Rightarrow \quad 2\log c - \log(1 - v - v^2) = 2\log x$

$\Rightarrow \quad \log c^2 = \log[x^2(1 - v - v^2)]$ where $c^2 = C$

$\Rightarrow \quad C = x^2\left(1 - \frac{y}{x} - \frac{y^2}{x^2}\right)$

$\Rightarrow \quad x^2 - xy - y^2 = C$

19. (A)

Sol. Given, $\frac{dy}{dx}=\frac{x-y}{x+y}$

This is a homogeneous equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Given equation becomes

$v + x\frac{dv}{dx} = \frac{1-v}{1+v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v}{1+v} - v$

$\Rightarrow \frac{1+v}{2-(1+v)^2}\,dv = \frac{dx}{x}$

On integrating both sides

$\int\frac{1+v}{2-(1+v)^2}\,dv = \int\frac{dx}{x}$

Put $(1+v)^2 = t \Rightarrow 2(1+v)\,dv = dt$

$\Rightarrow \frac{1}{2}\int\frac{dt}{2-t} = \int\frac{dx}{x}$

$\Rightarrow -\frac{1}{2}\log(2-t) = \log x + \log c$

$\Rightarrow -\frac{1}{2}\log[2-(1+v)^2] = \log xc$

$\Rightarrow -\frac{1}{2}\log[-v^2-2v+1] = \log xc$

$\Rightarrow \log\frac{1}{\sqrt{1-2v-v^2}} = \log xc$

$\Rightarrow x^2c^2(1-2v-v^2) = 1$

$\Rightarrow x^2c^2\left(1-\frac{2y}{x}-\frac{y^2}{x^2}\right) = 1 \quad \left[\because v=\frac{y}{x}\right]$

$\Rightarrow \frac{x^2c^2(x^2-2yx-y^2)}{x^2} = 1$

$\Rightarrow y^2 + 2xy - x^2 = c$

20. (D)

Sol. Given that, $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$\Rightarrow x\,dy = (\sqrt{x^2+y^2} + y)\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Now, put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$

$\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2} + v - v = \sqrt{1+v^2}$

On integrating both sides

$\int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$

$\Rightarrow \log(v+\sqrt{1+v^2}) = \log x + \log c$

$\Rightarrow y + \sqrt{x^2+y^2} = cx^2$

21. 2

Sol. We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\Rightarrow (ax+3)\,dx = (2y+f)\,dy$

On integrating, we obtain

$a\frac{x^2}{2} + 3x = y^2 + fy + c$

$\Rightarrow -\frac{a}{2}x^2 + y^2 - 3x + fy + c = 0$

This will represent a circle, if

$-\frac{a}{2} = 1$ [$\because$ Coeff. of x^2 = Coeff. of y^2]

and $\frac{9}{4} + f^2 - c > 0$ [Using $g^2 + f^2 - c > 0$]

$\Rightarrow a = -2$ and $9 + 4f^2 - 4c > 0$

22. 48

Sol. Let Population = x, time = t (in years)

Given $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

Where k is a constant of proportionality

or $\frac{dx}{x} = k\,dt$

Integrating, we get

$\ln x = kt + \ln c$

$\Rightarrow \ln\left(\frac{x}{c}\right) = kt \Rightarrow \frac{x}{c} = e^{kt}$

or $x = ce^{kt}$

If initially i.e., when time $t = 0$, $x = x_0$

then $x_0 = ce^0 = c$

$\therefore x = x_0e^{kt}$

Given $x = 2x_0$, $t = 30$

then $2x_0 = x_0e^{30k}$ $\Rightarrow 2 = e^{30k}$

$\therefore \ln 2 = 30k$ ... (1)

To find t, when it triples, $x = 3x_0$

$\therefore 3x_0 = x_0e^{kt}$ $\Rightarrow 3 = e^{kt}$ ... (2)

Dividing (2) by (1) then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$

or $t = 30 \times \frac{\ln 3}{\ln 2} = 30 \times 1.5849 = 48$ years.

23. 40 min.

Sol. Let T be the temperature of the substance at a time t then $-\frac{dT}{dt} \propto (T-290)$

$\Rightarrow \frac{dT}{dt} = -k(T-290)$

Where k is constant of proportionality and negative sign denote rate of cooling.

or $\frac{dT}{(T-290)}$ = – k dt

integrating, we get

$\int \frac{dT}{(T-290)} = -k \int dt$

$\Rightarrow ln$ (T – 290) = – kt + ln c

$\Rightarrow \left(\frac{T-290}{c}\right) = e^{-kt}$

or (T – 290) = ce^{-kt}

If initially i.e., when t = 0 & T = 370

Then (370 – 290) = ce

∴ c = 80

∴ T – 290 = $80e^{kt}$ ... (1)

and for t = 10, T = 330

∴ 330 – 290 = $80e^{-10k}$ ⇒ (40) = $80e^{-10k}$

⇒ 2 = e^{10k}

ln2 = 10 k ... (2)

To find t, when T = 295

from (1), 295 – 290 = $80e^{-kt}$

$\Rightarrow \frac{5}{80} = e^{-kt}$ ⇒ln 16 = kt

or 4 ln 2 = kt ... (3)

Dividing (3) by (2) then 4 = $\frac{t}{10}$

∴ t = 40 minutes.

24. 79

Sol. Let x denote the population at a time t in years.

then $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

when k is a constant of proportionality.

Solving $\frac{dx}{dt}$ = kx, we get

$\int \frac{dx}{x} = \int k\,dt \Rightarrow \log x = kt + c$

⇒ x = $e^{kt + c}$

⇒ x = x_0e^{kt}

Where x_0 is the population at time t = 0.

Since it doubles in 50 years, at t = 50, we must have x = $2x_0$.

Hence $2x_0 = x_0e^{50k}$ ⇒ 50 k = log 2

$\Rightarrow k = \frac{\log 2}{50}$ so that $x = x_0 e^{\frac{\log 2}{50}t}$

To find t, when it triples, x = $3x_0$

$\Rightarrow 3x_0 = x_0 e^{\frac{\log 2}{50}t} \Rightarrow \log 3 = \frac{\log 2}{50} t$

$\Rightarrow t = \frac{50\log 3}{\log 2}$ = 79 years.

25. 3

Sol. $\frac{dy}{dx}$ = y + 1

variable sep.

$\int \frac{dy}{y+1} = \int dx \Rightarrow$ log (y + 1) = x + c, y(0) =1

log 2 = c

So log (y + 1) = x + log 2

⇒ y (ℓn 2)

log (y +1) = 2 log 2 = log 4

y + 1 = 4

⇒ y = 3

26. 0005

Sol. Let y(t) denote the number of people who know the rumour at time t. Maximum value of y(t) is 5000, y(0) = 100 and y(2) = 500. Also

$\frac{dy}{dt}$ (xy (5000 – y) = $\frac{dy}{dt}$ = ky(5000 – y)

Separating variables and integrating we get :

$\int \frac{dy}{y(5000-y)}$ = kt + c

$\Rightarrow \frac{1}{5000} \log \frac{y}{5000-y}$ = kt + c

$\Rightarrow \frac{y}{5000-y}$ = Const. $e^{5000kt} = ke^{5000kt}$

⇒y(1 + ke^{5000kt}) = $5000ke^{5000kt}$; put y(0) = 100 ⇒ 100(1 +k) = 5000 k

⇒ k = 1/49. Using y(2) = 500

⇒ 500 (1 + $ke^{10,000k}$) = $5000ke^{10,000k}$

⇒ 1 = 9 $ke^{10,000k}$⇒ e^{10000k} = 49/9

$\Rightarrow k = \frac{1}{10,000}$ log 49/9

$\therefore$i⇒ $y = \frac{5000/49}{e^{-5000kt}+1/49} = \frac{5000}{49e^{-5000kt}+1}$

To determine how long it will take for half the population to hear rumour,

$2500 = \frac{5000}{1+49e^{-5000kt}}$

⇒ 1 + $49e^{-5000kt}$ = 2 ⇒ $e^{-5000kt}$ = 1/49

5000 kt = log 49 ⇒ t = log 49/5000 k

$\Rightarrow t = \dfrac{2\log 49}{\log 49 - \log 9} = \dfrac{2}{1-\dfrac{\log 9}{\log 49}} = \dfrac{2}{1-\dfrac{129}{229}}$

$= 4.58$ days ≈ 5 days.

27. 7

Sol. $y = Ax^m + Bx^{-n}$

$\Rightarrow \dfrac{dy}{dx} = Amx^{m-1} - nBx^{-n-1}$

$\Rightarrow \dfrac{d^2y}{dx^2} = Am(m-1)x^{m-2} + n(n+1)Bx^{-n-2}$

Putting these values in $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 12y$

We have $= m(m+1)Ax^m + n(n-1)Bx^{-n}$

$= 12(Ax^m + Bx^{-n})$

$\Rightarrow m(m+1) = 12$ or $n(n-1) = 12$

$\Rightarrow m = 3, -4$ or $n = 4, -3$

28. 1

Sol. The parametric form of the given equation is $x = t$, $y = t^2$. The equation of any tangent at t is $2xt = y + t^2$. Differentiating, we get $2t = y_1$. Putting this value in the equation of tangent, we have $2x\, y_1/2 = y + (y_1/2)^2 \Rightarrow 4xy_1 = 4y + y_1^2$

The order of this equation is one.

29. 2

Sol. We have, $y^2 = 4a(x + a)$... (1)

On differentiating w.r.t. x, we get

$2y\dfrac{dy}{dx} = 4a \Rightarrow a = \dfrac{y}{2}\dfrac{dy}{dx}$.

On substituting the value of a in equation (1), we get

$y^2 = 2y\dfrac{dy}{dx}\left[x + \dfrac{y}{2}\dfrac{dy}{dx}\right]$

$\Rightarrow y = 2x\dfrac{dy}{dx} + y\left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow \quad y\left[1-\left(\dfrac{dy}{dx}\right)^2\right] = 2x\,.\dfrac{dy}{dx}$,

which is the required differential equation. The degree of the differential equation is 2.

30. 5

Sol. $\alpha(t) + \beta'(t) = 1$....(1)

$\alpha'(t) + \beta(t) = 1$ .....(2)

Add (1) and (2)

$(\alpha + \beta) + (\alpha' + \beta') = 2$

$\therefore \quad (\alpha + \beta) + (\alpha + \beta)' = 2$

$(\alpha + \beta) = y$

$\therefore \quad \dfrac{dy}{dt} + y = 2 \qquad$ I.F. $= e^t$

$y \cdot e^t = 2e^t + C$

$\left(\alpha(t)+\beta(t)\right)e^t = 2e^t + C$

Put $t = 0$

$2 + 1 = 2 + C$

$C = 1$

$\therefore \quad \alpha(t) + \beta(t) = 2 + e^{-t} \qquad$ .....(3)

|||ly (1) – (2)

$(\alpha - \beta) - (\alpha - \beta)' = 0$

$(\alpha - \beta)' - (\alpha - \beta) = 0$

$\dfrac{dy}{dt} - y = 0 \qquad$ I.F. $= e^{-t}$

$ye^{-t} = C$

$y = Ce^t$

$\alpha(t) - \beta(t) = Ce^t \qquad$ Put $t = 0$

$2 - 1 = C \qquad \therefore \qquad C = 1$

$\alpha(t) - \beta(t) = e^t \qquad$ ......(4)

$\therefore \quad$ Put $t = \ln 2$ in (3) & (4) and add

$2\alpha(\ln 2) = 2 + \dfrac{1}{2} + 2 = 4 + \dfrac{1}{2} = \dfrac{9}{2}$

$\alpha(\ln 2) = \dfrac{9}{4} = \dfrac{p}{q}$

$\therefore \quad p - q = 9 - 4 = 5$