JEE MAIN : CHAPTER WISE TEST-10

SUBJECT :- MATHEMATICS **DATE.....**

CLASS :- 12th **NAME.....**

CHAPTER :- DIFFERENTIAL EQUATION **SECTION.....**

(SECTION A)

1. The general solution of the differential equation $\frac{dy}{dx} = (x^3 - 2x \tan^{-1}y)(1 + y^2)$ is-

(A) $2\tan^{-1}x = y^2 - 1 + 2C\ e^{-x^2}$

(B) $2\tan^{-1}y = x^2 - 1 + 2C\ e^{-x^2}$

(C) $2\tan^{-1}y = y^2 - 1 + 2C\ e^{-x^2}$

(D) $2\tan^{-1}x = x^2 - 1 + 2C\ e^{-x^2}$

2. The solution of
$(y + x + 5)dy = (y - x + 1)\,dx$ is

(A) $\log((y+3)^2 + (x+2)^2) + \tan^{-1}\frac{y+3}{x+2} = C$

(B) $\log((y+3)^2 + (x-2)^2) + \tan^{-1}\frac{y-3}{x-2} = C$

(C) $\log((y+3)^2 + (x+2)^2) + 2\tan^{-1}\frac{y+3}{x+2} = C$

(D) $\log((y+3)^2 + (x+2)^2) - 2\tan^{-1}\frac{y+3}{x+2} = C$

3. Solution of differential equation
$(2x \cos y + y^2 . \cos x)dx +$
$(2y . \sin x - x^2 . \sin y)dy = 0$ is –

(A) $x^2.\cos y + y^2 . \sin x = C$

(B) $x . \cos y - y . \sin x = C$

(C) $x^2 . \cos^2 y + y^2 . \sin^2 x = C$

(D) $x \cos y + y \sin x = C$

4. The solution curve of the differential equation,]

$(xdx + ydy)\sqrt{x^2+y^2} = (xdy - ydx)\sqrt{1-x^2-y^2}$ are -

(A) circles of radius 1 through the origin

(B) circles of radius 1/2 through the origin

(C) circles not through the origin

(D) not the circles

5. Solution of $(xy^4 + y)\,dx - xdy = 0$ is

(A) $\frac{x^4}{4} + \left(\frac{x}{y}\right)^3 = c$

(B) $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^2 = c$

(C) $\frac{x^4}{4} + 3\left(\frac{x}{y}\right)^2 = c$

(D) None of these

6. Let $\frac{dy}{dx} + y = f(x)$ where y is a continuous function of x with y(0) = 1 and $f(x) = \begin{cases} e^{-x}, & \text{if } 0 \le x \le 2 \\ e^{-2}, & \text{if } x > 2 \end{cases}$. The value of y(3) is equal to

(A) $\frac{1}{e^2}$ (B) $\frac{e+1}{e^3}$

(C) $\frac{e+2}{e^3}$ (D) $\frac{e-1}{e^3}$

7. The real value of m for which the substitution
$y = u^m$ will transform the differential equation
$2x^4y\frac{dy}{dx} + y^4 = 4x^6$ in to a homogeneous equation is

(A) m = 0 (B) m = 1

(C) m = 3/2 (D) m = 2/3

8. The solution of $\frac{d^3y}{dx^3} - 8\frac{d^2y}{dx^2} = 0$ satisfying y(0) = 1/8, $y_1(0) = 0$ and $y_2(0) = 1$ is -

(A) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$

(B) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} + x + \frac{7}{8}\right)$

(C) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} + x - \frac{7}{8}\right)$

(D) None of these

9. If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3} - 13\frac{dy}{dx}}{y} =$ k, then the value of $\frac{k}{3}$ is

(A) – 4 (B) 2 (C) 3 (D) 4

10. The differential equation of the family of hyperbolas with asymptotes as the line x + y = 1 and x – y = 1 is:

(A) $yy' + x = 0$ (B) $yy' = (x - 1)$

(C) $yy'' + y' = 0$ (D) $y' + xy = 0$

11. The equation to the curve which is such that portion of the axis of x cut off between the origin and the tangent at any point is proportional to the ordinate of that point is

(A) $x = y (C - K \log y)$
(B) $\log x = Ky^2 + C$
(C) $x^2 = y (C - K \log y)$
(D) None of these
[K is constant of proportionality]

12. Solution of the differential equation $\left\{\frac{1}{x} - \frac{y^2}{(x-y)^2}\right\} dx + \left\{\frac{x^2}{(x-y)^2} - \frac{1}{y}\right\} dy = 0$ is

(A) $\ln \left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$

(B) $\frac{xy}{x-y} = ce^{x/y}$

(C) $\ln |xy| = c + \frac{xy}{x-y}$

(D) None of these

13. The degree of the differential equation whose general solution is given by

$y = (c_1 + c_2) \cos (x + c_3) - c_4 e^{x+c_5}$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is -
(A) 5 (B) 4 (C) 1 (D) 2

14. The equation of the curve passing through the point $\left(a, -\frac{1}{a}\right)$ and satisfying the differential equation $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ is -

(A) $(x + a)(1 + ay) = -4a^2y$
(B) $(x + a)(1 - ay) = 4a^2y$
(C) $(x + a)(1 - ay) = -4a^2y$
(D) None of these

15. Solution of the equation $xdy = \left(y + x\frac{f(y/x)}{f'(y/x)}\right) dx$ is -

(A) $f\left(\frac{x}{y}\right) = cy$ (B) $f\left(\frac{y}{x}\right) = cx$

(C) $f\left(\frac{y}{x}\right) = cxy$ (D) None of these

16. The general solution of the differential equation $[2\sqrt{xy} - x]\, dy + y\, dx = 0$ is -

(A) $\log x + \sqrt{\frac{y}{x}} = c$ (B) $\log y - \sqrt{\frac{x}{y}} = c$

(C) $\log y + \sqrt{\frac{x}{y}} = c$ (D) None of these

17. Find the solution of differential equation $\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$ -
(A) $f(x) = y (x - c)$ (B) $f(x) = y (c - x)$
(C) $f(x) = y (x + c)$ (D) None

18. The general solution of $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$ is -
(A) $x^2 - xy + y^2 = c$ (B) $x^2 - xy - y^2 = c$
(C) $x^2 + xy - y^2 = c$ (D) $x^2 + xy^2 = c$

19. If $y' = \frac{x - y}{x + y}$, then its solution is -
(A) $y^2 + 2xy - x^2 = c$ (B) $y^2 + 2xy + x^2 = c$
(C) $y^2 - 2xy - x^2 = c$ (D) $y^2 - 2xy + x^2 = c$

20. The solution of the differential equation $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$ is -

(A) $x + \sqrt{x^2 + y^2} = cx^2$

(B) $y - \sqrt{x^2 + y^2} = cx$

(C) $x - \sqrt{x^2 + y^2} = cx$

(D) $y + \sqrt{x^2 + y^2} = cx^2$

(SECTION B)

21. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax + 3}{2y + f}$ represents a circle, then the value of $|a|$ is -

22. The population of a country increases at a rate proportional to the number of inhabitants. If the population doubles in 30 years, find after how many years the population will triple–

23. The rate at which a substance cools in moving air is proportional to the difference between the temperatures of the substance and that of the air. If the temperature of the air is 290° K and the substance cools from 370° K to 330°K in 10 minutes, when will the temperature be 295°K -

24. If the population of a country doubles in 50 years in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants –

25. If $y' = y + 1$ and $y(0) = 1$ then values of $y(\ell n\ 2)$ is-

26. A rumourspreeds through a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number who have not. Suppose that 100 people initiate the rumour and that a total of 500 people know the rumour after 2 dyas. How long will it take for half the people to hear the rumour.
(log 9 / log 49 = 129/229).

27. If the solution of differential equation $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$ then find the value of m + n, if m & n $\in$N.

28. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

29. The degree of the differential equation, of which $y^2 = 4a(x + a)$ is a solution, is –

30. Let $\alpha(t)$ and $\beta(t)$ be differentiable functions on R such that $\alpha(0) = 2$ and $\beta(0) = 1$.
If $\alpha(t) + \beta'(t) = 1$ and $\alpha'(t) + \beta(t) = 1$ for all $t \in [0, \infty)$, then the value of $\alpha(\ell n\ 2)$ is expressed in the lowest form as $\frac{p}{q}$. Find the value of $(p - q)$.