JEE MAIN : CHAPTER WISE TEST PAPER-9

SUBJECT :- MATHEMATICS **DATE.....**

CLASS :- 12th **NAME.....**

CHAPTER :- DEFINITE INTEGRATION **SECTION.....**

(SECTION-A)

1. The value of the integral $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2$ dx where p, q are integers, is equal to

(A) $-\pi$ (B) 0 (C) π (D) 2π

2. $\lim_{n\to\infty} \int_0^2 \left(1+\frac{t}{n+1}\right)^n dt$ is equal to

(A) 0 (B) e^2

(C) $e^2 - 1$ (D) does not exist

3. Let a > 0 and let $f(x)$ is monotonic increasing such that f (0) = 0 and $f(a) = b$ then $\int_0^a f(x)\,dx + \int_0^b f^{-1}(x)\,dx$ equals

(A) a + b (B) ab + b

(C) ab + a (D) ab

4. $\lim_{n\to\infty} \frac{n}{(n!)^{1/n}}$ is equal to

(A) e (B) $\frac{1}{e}$

(C) 1 (D) $\int_0^1 \ln x\,dx$

5. The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos\left(\frac{1}{\sin\left(\frac{1}{\sin x}\right)}\right)\cdot\cos\left(\frac{1}{\sin x}\right)\cdot\cos x\cdot\frac{1}{\sin^2 x\cdot\sin^2\left(\frac{1}{\sin x}\right)}dx$ equals

(A) sin(cosec 1) – sin (cosec 2)

(B) cos (cosec 1) – cos (cosec 2)

(C) 2(cos (cosec 1) – cos (cosec 2))

(D) sin(sin 2) – sin (sin 1)

6. Let $f(x) = \int_x^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 x f(x)\,dx$ is equal to

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{4}{3}$ (D) $\frac{2}{3}$

7. Consider a function of the form $f(x) = \alpha e^{2x} + \beta e^x - \gamma x$, where α, β, γ are independent of x and f(x) satisfies the following conditions f(0) = – 1, f '(ln 2) = 30 and $\int_0^{\ln 4} (f(x)+\gamma x)\,dx = 24$. The value of $(\alpha + \beta + \gamma)$ is equal to

(A) 3 (B) 4 (C) 6 (D) 8

8. Let function f(x) satisfies the equation $f(x) + 3\int_{-1}^{1} (xy - x^2y^2) f(y)\,dy = x^3$. Then the value of f (5) equals

(A) 123 (B) 125 (C) 625 (D) 131

9. $\lim_{x\to 0} \frac{\int_0^x \sin t^2 dt}{x(1-\cos x)}$ equals

(A) $\frac{1}{3}$ (B) 2 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

10. Suppose f is continuous and satisfies $f(x) + f(-x) = x^2$ then the integral $\int_{-1}^{1} f(x)\,dx$ has the value equal to

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{4}{3}$ (D) zero

11. The value of the definite integral $\int_0^{\pi/2} \sqrt{\tan x}\,dx$, is

(A) $\sqrt{2}\pi$ (B) $\frac{\pi}{\sqrt{2}}$

(C) $2\sqrt{2}\pi$ (D) $\frac{\pi}{2\sqrt{2}}$

12. The value of the definite integral $\int_{-1}^{1} \frac{x\tan^{-1} x\,dx}{(1+e^{\text{arc}\tan x})}$ is

(A) $\frac{\pi}{4} - \frac{1}{2}$ (B) $\frac{\pi}{4} - \ln 2$

(C) $\frac{\pi}{4} + \frac{1}{2}$ (D) $\frac{\pi}{8} - \frac{1}{2}$

13. Positive value of 'a' so that the definite integral $\int_a^{a^2} \frac{dx}{x+\sqrt{x}}$ achieves the smallest value is

(A) $\tan^2\left(\frac{\pi}{8}\right)$ (B) $\tan^2\left(\frac{3\pi}{8}\right)$

(C) $\tan^2\left(\frac{\pi}{12}\right)$ (D) 0

14. $\lim\limits_{x\to+\infty} \frac{\int_0^x \left(\tan^{-1}t\right)^2 dt}{\sqrt{x^2+1}}$ has the value =

(A) zero (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{\pi^2}{4}$

15. The value of the integral

$I = \int_0^{2\alpha} \frac{\sqrt[4]{\sin(3\alpha-x)}}{\sqrt[4]{\sin(3\alpha-x)}+\sqrt[4]{\sin(\alpha+x)}}dx$ is

(A) $\frac{\alpha}{2}$ (B) 0 (C) 2α (D) α

16. The true set of values of 'a' for which the inequality $\int_a^0 (3^{-2x} - 2.3^{-x})\,dx \geq 0$ is true is:

(A) [0 , 1] (B) $(-\infty, -1]$

(C) $[0, \infty)$ (D) $(-\infty, -1] \cup [0, \infty)$

17. Let $A = \int_0^1 \frac{e^t\,dt}{1+t}$ then $\int_{a-1}^{a} \frac{e^{-t}\,dt}{t-a-1}$ has the value

(A) Ae^{-a} (B) $-Ae^{-a}$

(C) $-Aae^{-1}$ (D) Ae^{a}

18. If g (x) is the inverse of f (x) and f (x) has domain $x \in [1, 5]$, where f (1) = 2 and f (5) = 10 then the values of $\int_1^5 f(x)\,dx + \int_2^{10} g(y)\,dy$ equals

(A) 48 (B) 64 (C) 71 (D) 52

19. The value of $\lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{n-k}{n^2}\cos\frac{4k}{n}$ equals

(A) $\frac{1}{4}\sin 4 + \frac{1}{16}\cos 4 - \frac{1}{16}$

(B) $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

(C) $\frac{1}{16}(1-\sin 4)$

(D) $\frac{1}{16}(1-\cos 4)$

20. For $n \in N$, the value of the definite integral $\int_0^{n\pi+V} \sqrt{\frac{1+\cos 2x}{2}}dx$ where $\frac{\pi}{2} < V < \pi$, is

(A) 2n + 1 – cosV (B) 2n – sin V

(C) 2n + 2 – sin V (D) 2n + 1 – sin V

(SECTION-B)

21. Let $f(x) = \alpha x^2 + \beta x + \gamma$, where $\alpha, \beta, \gamma \in N$ and $\int_0^1 f(x)\,dx = \frac{11}{6}$, then find the number of points of non-derivability of $g(x) = f(|x|)$ in [–5, 5].

22. If $I_1 = \int_0^1 x^{100}(1-x)^{50}\,dx$ and $I_2 = \int_0^1 x^{99}(1-x)^{50}\,dx$. Find the value of $\frac{151 I_1}{50 I_2}$.

23. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200}.(1-x)^7\,dx$ is equal to $\frac{1}{k}$, where $k \in N$, then the value of $\frac{k}{26}$ is

24. If $f(x) = 20x + 49x^2 + \int_0^1 (xy + x^2y^2) f(y)\,dy$ where x and y are variables, independent of each other, then the value of $\int_0^1 x f(x)dx = \frac{40k}{3}$, find the value of k.

25. If $\int_{-1}^{1}\left(\left(x - \frac{1}{1+x^2}\right) + \left|x+1-\frac{1}{1+x^2}\right|\right)dx = a + b\pi$ where $a, b \in Q$ then find the value of 2(a + b).

26. Let f be a derivable function which satisfies the relation

$f(x) = \int_0^1 \sin\left(\frac{\pi}{2}x\right) f(t)\,dt + \int_0^x x f(t)\,dt$ and $f(1) \neq 0$

If $\frac{f'(1)}{f(1)} = \frac{p}{q}$ where p and q are relatively prime numbers then find the value of (p + q).

27. Let $f(x) = \begin{cases} x e^{-x}; & 0 \le x < 1 \\ \frac{-1}{e}(x-2); & 1 \le x \le 2 \end{cases}$ and $g(x) = x^2 - \big(f(2+t) - f(t)\big)x - 4$

such that $g(-x) = g(x)\ \forall\ x, t \in R$. If $\int_0^{42} f(x)dx = m - \frac{n}{2e}$, $m, n \in N$ then find the value of $\left(\frac{n}{m}\right)$.

28. If $\int_0^1 \tan^{-1}\left(\frac{\tan x - 3\cot x}{4}\right)dx = \frac{3-\pi}{2} + \alpha$, $(\alpha \in R)$ then find the value of $\int_0^1 \tan^{-1}\left(\frac{\tan x}{3}\right)dx - \alpha$.

29. If $f : R \to R$ be a derivable function such that $\big(f(x)\big)^7 = x - f(x)$, then find the value of $\int_0^{\sqrt{2}} f^{-1}(x)\,dx$.

30. If f is continuous function defined on R such that

$$\lim_{h\to 0}\frac{f(h)}{h} = \frac{1}{2} \text{ and}$$

$$\lim_{x\to 0}\frac{\int_0^x f(x-t+2)\big(1-\cos f(x-t)\big)dt}{x\,ln(1+x^2)} = \frac{1}{6},$$

then find the value of $f(2)$.