## JEE MAIN ANSWER KEY & SOLUTIONS

**SUBJECT :- MATHEMATICS**
**CLASS :- 12th** **PAPER CODE :- CWT-8**
**CHAPTER :- INDEFINITE INTEGRATION**

## ANSWER KEY

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (C) | **2.** | (B) | **3.** | (B) | **4.** | (B) | **5.** | (A) | **6.** | (B) | **7.** | (B) |
| **8.** | (C) | **9.** | (A) | **10.** | (B) | **11.** | (C) | **12.** | (B) | **13.** | (C) | **14.** | (A) |
| **15.** | (B) | **16.** | (C) | **17.** | (B) | **18.** | (A) | **19.** | (A) | **20.** | (B) | **21.** | 2010 |
| **22.** | 5 | **23.** | 3 | **24.** | 3 | **25.** | 5 | **26.** | 7 | **27.** | 3 | **28.** | 3 |
| **29.** | 1 | **30.** | 5 | | | | | | | | | | |

## SOLUTIONS

**1.** (C)

**Sol** We have $f(x) = (x + 1)^3$

Now, $\int f(x)\,dx$

$= \int (x+1)^3\,dx = \frac{(x+1)^4}{4} + C \Rightarrow g(x)$

$= \frac{(x+1)^4}{4}$

Hence $g(3) - g(1) = \frac{4^4}{4} - \frac{2^4}{4} = 64 - 4 = 60$

**2.** (B)

**Sol.** $F(x) = \int \frac{3x+2}{\sqrt{x-9}}dx$ ; let $x - 9 = t^2$

$\Rightarrow$ $dx = 2t\,dt$

$\therefore$ $F(x) = \int \left(\frac{3(t^2+9)+2}{t}\cdot 2t\right)dt$

$= 2\int (29 + 3t^2)dt = 2[29t + t^3]$

$F(x) = 2\left[29\sqrt{x-9} + (x-9)^{3/2}\right] + C$

given $F(10) = 60 = 2[29 + 1] + C$

$\Rightarrow$ $C = 0$

$\therefore$ $F(x) = 2\left[29\sqrt{x-9} + (x-9)^{3/2}\right]$

$F(13) = 2[29 \times 2 + 4 \times 2]$

$= 4 \times 33 = 132$

**3.** (B)

**Sol.** $\int \frac{(3x^4-1)}{x^2(x^3+1+x^{-1})^2}dx$

$= \int \frac{(3x^2 - x^{-2})}{(x^3+1+x^{-1})^2}dx$

Substute $x^3 + 1 + \frac{1}{x} = t$

$\Rightarrow \left(3x^2 - \frac{1}{x^2}\right)dx = dt$

$\therefore \int \frac{dt}{t^2} = \frac{-1}{t} + c$

$= \frac{-1}{x^3 + 1 + \frac{1}{x}} + C$

$= \frac{-x}{x^4 + x + 1} + C$

**4.** (B)

**Sol.** $I = \int \frac{(e^x + \cos x + 1) - (e^x + \sin x + x)}{e^x + \sin x + x}dx$

$= \ln(e^x + \sin x + x) - x + C$

$\therefore$ $f(x) = e^x + \sin x + x$ and $g(x) = -x$

$f(x) + g(x) = e^x + \sin x$

**5.** (A)

**Sol.** Let $\int \frac{ax^2 + 2bx + c}{(Ax^2 + 2Bx + C)^2}dx = \frac{f(x)}{g(x)}$

where $f(x)$ and $g(x)$ are polynomials ;

($B^2 \neq AC \Rightarrow Ax^2 + 2Bx + C$ is not a perfect square)

differentiate w.r.t. x

$\frac{ax^2 + 2bx + c}{(Ax^2+2Bx+C)^2} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$ ...(1)

Hence $g(x) \cong Ax^2 + 2Bx + C$

If $N^r$ of RHS in equation (1), has to be a quadratic function then f(x) must be linear function

i.e. $f(x) = px + q$

$\therefore$ $p(Ax^2 + 2Bx + C) - (px + q)(2Ax + 2B) \equiv ax^2 + 2bx + c$

Comparing coefficient of $x^2$

$Ap - 2Ap = a \Rightarrow -Ap = a \Rightarrow p = \frac{-a}{A}$

coefficient of x

2Bp – (2Bp + 2Aq) = 2b ⇒ – Aq = b

⇒ q = $\frac{-b}{A}$

constant term

pC –2Bq = c

substituting the value of p and q

$\frac{-aC}{A}+\frac{2Bb}{A}=c$ ⇒ 2Bb – aC = Ac

⇒ 2Bb = Ac + aC

Hence 2Bb = Ac + aC

**6.** (B)

**Sol.** I = $\int e^x \sin(e^x \cdot e^{-1})\,dx$

let $e^{x-1} = t$ ; $e^{x-1}\,dx = dt$

$\int e \sin t\,dt$ = – e cos t + C = – e cos $e^{x-1}$ + C

**7.** (B)

**Sol.** Given $\int f(x)\,dx = g(x)$ ⇒ $g'(x) = f(x)$

now $\frac{d}{dx}\left(\ln(1+g^2(x))\right) = \frac{2g(x)g'(x)}{1+g^2(x)}$

$= \frac{2f(x)g(x)}{1+g^2(x)}$

**8.** (C)

**Sol.** $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$

$= \int \frac{\left(\cos^2 x+\cos^4\right)\cos x}{\sin^2 x+\sin^4 x}dx$

$= \int\left(\frac{\left(\cos^2 x\right)+\left(\cos^2 x\right)^2}{\sin^2 x+\sin^4 x}\right)\cos x\ dx$

substitute t = sin x

$= \int \frac{2+t^4-3t^2}{t^2+t^4}.dt$

$= \int \frac{2+t^4-4t^2+t^2}{t^2+t^4}.dt$

$= \int\left(\frac{\left(t^4+t^2\right)+\left(2-4t^2\right)}{t^2x+t^4}\right)dt$

$= \int\left(1+\frac{2-4t^2}{t^2+t^4}\right)dt$

$= \int\left(1-\frac{2\left(2t^2-1\right)}{t^2\left(t^2+1\right)}\right)dt$

$= \int\left(1-\frac{4}{t^2+1}+\frac{2}{t^2\left(t^2+1\right)}\right)dt$

$= \int\left(1-\frac{4}{t^2+1}+2\left(\frac{1}{t^2}-\frac{1}{t^2+1}\right)\right)dt$

$= \left(t-4\tan^{-1}t-\frac{2}{t}-2\tan^{-1}t\right)+C$

$= \left(t-6\tan^{-1}t-\frac{2}{t}\right)+C$

$= \left(\sin x-6\tan^{-1}(\sin x)-2\,\mathrm{cosec}\,x\right)+C$

**9.** (A)

**Sol.** I = $\int x^3(x^2-1)^{1/4}\ dx$

Hint: $x^2 - 1 = t^4$ ; $x\,dx = 2t^3\,dt$

I = $\int 2(t^4+1)\,t\,.t^3\ dt$

$= 2.\int(t^8+t^4)\ dt = \frac{2}{9}t^9 + \frac{2t^5}{5}$

$= \frac{2}{9}(x^2-1)^{9/4} + \frac{2}{5}(x^2-1)^{5/4}$

$= \frac{2}{45}(x^2-1)^{5/4}(5x^2+4)$

**10.** (B)

**Sol.** put x cosx – sinx = t

( – x sinx + cosx – cosx ) dt

I = $\int \frac{-dt}{t}$ = – *ln* ( x cosx – sinx)

**11.** (C)

**Sol.** Let $\tan^{-1} x = \theta$ or $x = \tan\theta$

I = $\int e^{\theta}\left(\theta^2+2\theta\right)d\theta$

$= e^{\theta}\theta^2 + C$

I = $e^{\tan^{-1}x}$ $(\tan^{-1}x)^2$ + C

I = $e^{\tan^{-1}x}\left(\sec^{-1}\sqrt{1+x^2}\right)^2+C$

**12.** (B)

**Sol.** $I_n = \int \cot^n x\, dx = \int \cot^{n-2} x.(\text{cosec}^2 x - 1)\, dx$

$I_n = -\frac{u^{n-1}}{n-1} - I_{n-2}$

or $I_n + I_{n-2} = -\frac{u^{n-1}}{n-1}$

(Put n = 2, 3, 4, ..... 10)

$I_2 + I_0 = -\frac{u}{1}$

$I_3 + I_1 = -\frac{u^2}{2}$

$I_4 + I_2 = -\frac{u^3}{3}$

$\vdots \quad \vdots \quad \vdots$

$I_{10} + I_9 = -\frac{u^9}{9}$

adding $I_0 + I_1 + 2(I_2 + I_3 + ....+ I_8) + I_9 + I_{10}$

$= -\left(u + \frac{u^2}{2} + ......... + \frac{u^9}{9}\right)$

**13.** (C)

**Sol.** $f'(x) = \frac{1}{1+\cos x}$ ; integrating, f (x)

$= \int \frac{dx}{2\cos^2 \frac{x}{2}} = \frac{1}{2}\int \sec^2 \frac{x}{2}\, dx$

$= \frac{1}{2} \cdot 2 \cdot \tan\frac{x}{2} + C = \tan\frac{x}{2} + C$

f (0) = 3 $\Rightarrow$ C = 3 ;

$f(x) = \tan\frac{x}{2} + 3$ ; $f\left(\frac{\pi}{2}\right) = 4$

**14.** (A)

**Sol.** $\int \frac{\sin(\ln(2+2x))}{x+1} dx$

$= \int \frac{\sin(\ln 2(1+x))}{x+1} dx$

= Let 1 + x = u

dx = du

$= \int \frac{\sin(\ln(2u))}{u} du$

Let 2u = v

2 du = dv

$= \int \frac{\sin(\ln v)}{v} dv$

Let ln v = w

$\frac{1}{v} dv = dw$

$= \int \sin w\ dw$

= – cos w + c

= – cos (ln v) + c

= – cos (ln 2(1 + x)) + C

**15.** (B)

**Sol.** $12\left[\frac{1}{4}\tan^{-1}\frac{x+3}{4} + \frac{1}{2\cdot 6} \ln\left|\frac{x-9}{x+3}\right|\right]$

$= 3\tan^{-1}\left(\frac{x+3}{4}\right) + \ln\left|\frac{x-9}{x+3}\right| \Rightarrow \lambda = 3,\ \mu = 1$

**16.** (C)

**Sol.** $I = \int \frac{dx}{x(x^{2007}+1)} = \int \frac{x^{2007}+1-x^{2007}}{x(x^{2007}+1)} dx$

$= \int \left(\frac{1}{x} - \frac{x^{2006}}{1+x^{2007}}\right) dx$

$= \ln x - \frac{1}{2007} \ln(1 + x^{2007})$

$= \frac{\ln x^{2007} - \ln(1+x^{2007})}{2007} = \frac{1}{2007}$

$\ln\left(\frac{x^{2007}}{1+x^{2007}}\right) + C$

p + q + r = 6021

**17.** (B)

**Sol.** $I = \int \frac{\sin 2x + \sin 4x - \sin 6x}{1+\cos 2x + \cos 4x + \cos 6x} dx$

$= \int \frac{2\sin 3x \cos x - 2\sin 3x \cos 3x}{2\cos^2 x + 2\cos x \cos 5x} dx$

$= \int \frac{\sin 3x (\cos x - \cos 3x)}{\cos x (\cos x + \cos 5x)} dx$

$= \int \frac{2\sin 3x \sin 2x \sin x}{2\cos x \cos 2x \cos 3x} dx$

$= \int \tan x \tan 2x \tan 3x\ dx$

$\because$ 3x = 2x + x $\Rightarrow$ tan 3x = $\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$\Rightarrow$ tan 3x – tan 3x tan 2x tan x = tan 2x + tan x

$\Rightarrow$ tan 3x tan 2x tan x = tan 3x – tan 2x – tan x

$\therefore$ I = $\int (\tan 3x - \tan 2x - \tan x)dx$

$= \frac{1}{3} ln|\sec 3x| - \frac{1}{2} ln|\sec 2x| - ln|\sec x| + C$

**18.** (A)

**Sol.** Let I = $\int \frac{dx}{x(1+x)(ln(x+1) - ln x)^{11}}$

Put $ln(x+1) - ln\, x = t \Rightarrow \frac{dx}{x(1+x)} = -dt$

So, I = $-\int \frac{dt}{t^{11}} = \frac{1}{10}\left(\frac{1}{t^{10}}\right)$

$= \frac{1}{10(ln(x+1) - ln\, x)^{10}} + C$

**19.** (A)

**Sol.** Let I = $\frac{1}{2}\int e^{\sin^2 x}(1+\cos^2 x)\sin 2x\, dx$

Put, $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$,

So, I = $\frac{1}{2}\int e^t(1+1-t)\, dt$

$= \frac{1}{2}\int \underbrace{e^t}_{II} \underbrace{(2-t)}_{I} dt = \frac{1}{2}\left[(2-t)e^t + \int e^t dt\right]$

(I.B.P.)

$= \frac{1}{2}\left[(2-t)e^t + e^t\right] + C$

$= \frac{1}{2}(3-t)e^t + C = \frac{1}{2}(3 - \sin^2 x)e^{\sin^2 x} + C.$

**20.** (B)

**Sol.** Let I = $\int \frac{1}{(x^2-4)\sqrt{x+1}} dx$

put $x + 1 = t^2$ & $dx = 2t\, dt$

$I = \int \frac{2t\, dt}{\left\{(t^2-1)^2 - 4\right\}\sqrt{t^2}}$

$= 2\int \frac{dt}{(t^2-1-2)(t^2-1+2)}$

$= 2\int \frac{dt}{(t^2-3)(t^2+1)}$

Let $t^2 = y$

$\frac{1}{(t^2-3)(t^2+1)} = \frac{1}{(y-3)(y+1)}$

$\frac{1}{(y-3)(y+1)} = \frac{A}{y-3} + \frac{B}{y+1}$ ........(i)

1 = A(y + 1) + B (y – 3) .......(ii)

Put y = –1, 3 in (ii)

B = $-\frac{1}{4}$ & A = $\frac{1}{4}$

$\therefore \frac{1}{(y-3)(y+1)} = \frac{1}{4(y-3)} - \frac{1}{4(y+1)}$

$\frac{1}{(t^2-3)(t^2+1)} = \frac{1}{4(t^2-3)} - \frac{1}{4(t^2+1)}$

$= 2\int \left\{\frac{1}{4(t^2-3)} - \frac{1}{4(t^2+1)}\right\} dt$

$= \frac{1}{2}\int \frac{1}{t^2 - (\sqrt{3})^2} dt - \frac{1}{2}\int \frac{1}{t^2+1^2} dt$

$I = \frac{1}{2} \times \frac{1}{2\sqrt{3}} \log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}(t) + C$

$I = \frac{1}{4\sqrt{3}} \log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x+1}}{1}\right) + C$

**21.** 2010

**Sol.** L.H.S.

$\int \frac{(\sin x)^{2008} - (\cos x)^{2008}}{(\sin x)^{2008}\left(\frac{\sin x}{\cos x} + \left(\frac{\cos x}{\sin x}\right)^{2009}\right)} dx$

$= \int \frac{\sin x \cos x\left((\sin x)^{2008} - (\cos x)^{2008}\right)}{(\sin x)^{2010} + (\cos x)^{2010}} dx$

$= \int \frac{\left((\sin x)^{2009}\cos x - (\cos x)^{2009}\sin x\right)}{(\sin x)^{2010} + (\cos x)^{2010}}dx$

put $(\sin x)^{2010} + (\cos x)^{2010} = t$

$= \frac{1}{2010}\int \frac{dt}{t}$

$= \frac{1}{2010}\ ln\left|(\sin x)^{2010} + (\cos x)^{2010}\right| + c$

$\Rightarrow$ k = 2010

**22.** 5

**Sol.** We have $\int \frac{1 - 7\cos^2 x}{\sin^7 x \cos^2 x}dx =$

$\int \frac{\sec^2 x}{\sin^7 x}dx - 7\int \frac{1}{\sin^7 x}dx = I_1 - I_2$

Now, $I_1 = \int \left(\frac{1}{\sin^7 x}\right)\sec^2 x\, dx = \frac{\tan x}{\sin^7 x}$
(I) (II)
**(By parts)**

$+ 7\int \frac{\tan x}{\sin^8 x}\cos x\ dx = \frac{\tan x}{\sin^7 x} + I_2$

$\therefore I_1 - I_2 = \frac{\tan x}{\sin^7 x} + C$, where C is constant of integration.

Hence g(x) = tan x

So, $g'(x) = \sec^2 x$ and $g''(x) = 2\sec^2 x \tan x$

$\therefore$ g'(0) = 1 and $g''\left(\frac{\pi}{4}\right) = 4$.

Hence $g'(0) + g''\left(\frac{\pi}{4}\right) = 1 + 4 = 5$

**23.** 3

**Sol.** $\because$ $f'(x^2) = \frac{1}{x} \Rightarrow f'(x) = \frac{1}{\sqrt{x}}$, x > 0

$\Rightarrow$ $f(x) = 2\sqrt{x} + c$ (c = integration constant)

$\because$ f(1) = 1 $\Rightarrow$ c = – 1

$\therefore$ $f(x) = 2\sqrt{x} - 1$, x > 0

and $g'(\sin^2 x - 1) = \cos^2 x + p\ \forall x \in R$

$\Rightarrow$ $g'(-\cos^2 x) = \cos^2 x + p$

$\Rightarrow$ $g'(x) = p - x,\ \forall x \in [-1, 0]$

$\Rightarrow$ $g(x) = px - \frac{x^2}{2} + k$

(where k = integration constant)

$\because$ $g(-1) = 0 \Rightarrow 0 = -p - \frac{1}{2} + k \Rightarrow$

$k = \frac{1}{2} + p$

$\therefore$ $g(x) = px - \frac{x^2}{2} + \frac{1}{2} + p$

$\therefore$ h(x)

$= \begin{cases} 2\sqrt{x} - 1 & , x > 0 \\ px - \frac{x^2}{2} + \frac{1}{2} + p & , -1 \le x \le 0 \end{cases}$

$\therefore$ At x = 0

L.H.L = R.H.L = f(0)

$\Rightarrow -1 = \frac{1}{2} + p \Rightarrow p = \frac{-3}{2}$

Hence 2p = – 3

$\therefore$ Absolute value of 2p is 3]

**24.** 3

**Sol.** Suppose $g(x) = \int \frac{f(x)dx}{x^2(x+1)^3}$ ....(1)

$= \int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}\right)dx$

$= A\ ln\, x - \frac{B}{x} + C\ ln\,(1 + x) - \frac{D}{1+x} - \frac{E}{2(x+1)^2}$

since g (x) is a rational function hence logarithmic functions must not be there

$\Rightarrow$ A = C = 0

g (x)

$= \int \left(\frac{B}{x^2} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}\right)dx$ ...(2)

comparing $N^r$ of (1) and (2)

$f(x) = B(x + 1)^3 + Dx^2(x + 1) + Ex^2$

$f(x) = (B + D)x^3 + (3B + D + E)x^2 + 3Bx + B$

$\therefore$ f (x) is quadratic function, hence B + D = 0

also f (0) = 1 gives B = 1 $\Rightarrow$ D = – 1

$\therefore$ $f(x) = (2 + E)x^2 + 3x + 1$

$f'(x) = 2(2 + E)x + 3$

f ' (0) = 3 Ans. ]

**25.** 5

**Sol.** $\int \frac{(2x+3)}{(x^2+3x)(x^2+3x+2)+1}dx$

$x^2 + 3x = t$

$\int \frac{dt}{t(t+2)+1} = \int \frac{dt}{(t+1)^2} = \frac{-1}{t+1} + k =$

$k - \frac{1}{x^2+3x+1}$

f (x) = $x^2 + 3x + 1$

compare $x^2 + 3x + 1$ with $ax^2 + bx + c$

therefore, a = 1, b = 3 and c = 1

so, a+b+c =1+3+1 = 5

**26.** 7

**Sol.** $\int e^x \left(\frac{1-x^n}{1-x}\right)dx = e^x P(x) + C$

Let $P(x) = a_0 + a_1x + a_2x^2 + ....... + a_{n-1}x^{n-1}$

$= \int e^x(1+x+x^2+......+x^{n-1})dx = e^x (a_0 +$

$a_1x + a_2x^2 + ...... + a_{n-1}x^{n-1}) + c$

$P(0) = a_0 = 620$

Differentiating both sides

$e^x (1 + x + x^2 + ...... x^{n-1})$

$= e^x\left(a_1 + 2a_2x + 3a_3x^2 + ......+(n-1)_{n-1}x^{n-2}\right)+$

$(a_0 + a_1x + a_2x^2 + ...... + a_{n-1}x^{n-1})e^x$

Comparing coefficient of same power of x

$a_{n-1} = 1$

$a_0 = 620$

$a_1 + a_0 = 1 \Rightarrow a_1 = -619$

$a_1 + 2a_2 = 1 \Rightarrow a_2 = 310$

$a_2 + 3a_3 = 1 \Rightarrow a_3 = -103$

$a_3 + 4a_4 = 1 \Rightarrow a_4 = + 26$

$a_4 + 5a_5 = 1 \Rightarrow a_5 = -5$

$a_5 + 6a_6 = 1 \Rightarrow a_6 = 1$

$\therefore$ n – 1 = 6 $\Rightarrow$ n = 7 . ]

**27.** 3

**Sol.** Put $e^x = t$

$I_n = \int \frac{t^n\,dt}{\left(1+t+\frac{t^2}{2!}+......+\frac{t^n}{n!}\right)}$

$= n!\int \frac{1+t+\frac{t^2}{2!}+......+\frac{t^n}{n!}-\left(1+t+\frac{t^2}{2!}+......+\frac{t^{n-1}}{(n-1)!}\right)}{\left(1+t+\frac{t^2}{2!}+......+\frac{t^n}{n!}\right)}dt$

$= n!\left(t-\int\frac{1+t+\frac{t^2}{2!}+......+\frac{t^{n-1}}{(n-1)!}}{1+t+\frac{t^2}{2!}+......+\frac{t^n}{n!}}dt\right)$

Let $1 + t + \frac{t^2}{2!} + ...... + \frac{t^n}{n!} = v$ ; dv

$= 1 + t + \frac{t^2}{2!} + ...... + \frac{t^{n-1}}{(n-1)!}dt$

$\therefore I_n = n!\left(t - ln\left(1+t+\frac{t^2}{2!}+......+\frac{t^n}{n!}\right)\right) + c$

$= n!\left(e^x - \ln\left(1+e^x+\frac{e^{2x}}{2!}+......+\frac{e^{nx}}{n!}\right)\right) + c$

$\therefore g(x) = \lim_{n\to\infty} ln\left(1+e^x+\frac{e^{2x}}{2!}+......+\frac{e^{nx}}{n!}\right)$

$= ln\left(e^{e^x}\right) = e^x$

$\therefore$ Number of solutions of $e^x = x^2$ is 3. **Ans.** ]

**28.** 3

**Sol.** $f(x) = \frac{e^{3x}}{e^{4x}+8e^{2x}+4}$ and

$g(x) = \frac{e^x}{e^{4x}+8e^{2x}+4}$

Integral = $\int (f(x) - 2g(x))dx$

$= \int \frac{(e^{3x}-2e^x)}{e^{4x}+8e^{2x}+4}dx$

Let $e^x = t$

$I = \int \frac{(t^2-2)}{t^4+8t^2+4}dt = \int \frac{\left(1-\frac{2}{t^2}\right)dt}{t^2+8+\frac{4}{t^2}}$

$$= \int \frac{\left(1-\frac{2}{t^2}\right)dt}{\left(t+\frac{2}{t}\right)^2+4}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{t+\frac{2}{t}}{2}\right) + c = \frac{1}{2}\tan^{-1}\left(\frac{e^x+2e^{-x}}{2}\right) + c$$

$$\therefore \quad h(x) = \frac{1}{2}\tan^{-1}\left(\frac{e^x+2e^{-x}}{2}\right)$$

$$\therefore \quad h(0) = \frac{1}{2}\tan^{-1}\left(\frac{3}{2}\right) \Rightarrow 2\tan\left(2h(0)\right)$$

= 3 **Ans.** ]

**29.** 1

**Sol.** $\because \ f_2(x) - f_1(x) = \int 1\,dx = x + C$

$\because \ f_2(1) - f_1(1) = 1 + C = 4 - 2 \Rightarrow C = 1$

$\therefore \ f_2(x) - f_1(x) = x + 1$

$\therefore \ h(x) = x - 1$

$\therefore \ h'(x) = 1 \Rightarrow h'(5) = 1$ **Ans.** ]

**30.** 5

**Sol.** Using L-Hospital's rule twice

$$\frac{1}{f''(x)} = \frac{x^{1-x}}{1+x(1+\ln x)^2}$$

$$\therefore f''(x) = \frac{x(1+\ln x)^2+1}{x^{1-x}}$$

$= x^{x-1}(x(1+\ln x)^2 + 1)$

$f''(x) = x^x(1+\ln x)^2 + x^{x-1}$

$$\therefore \int f''(x)\,dx$$

$$= \int \underbrace{x^x(1+\ln x)}_{II}\underbrace{(1+\ln x)}_{I}\,dx + \int x^{x-1}\,dx$$

$f'(x) = x^x(1+\ln x) + A$

$\Rightarrow f(x) = x^x + Ax + B$

$\lim_{x\to 0^+} f(x) = 1 + B = 1 \Rightarrow B = 0$

$f(1) = 1 + A = 2 \Rightarrow A = 1$

$\Rightarrow f(x) = x^x + x.$

Now, $\frac{f(3)}{f(2)} = \frac{30}{6} = 5.$ ]