JEE MAIN : CHAPTER WISE TEST PAPER-8

SUBJECT :- MATHEMATICS | **DATE**..
CLASS :- 12th | **NAME**..
CHAPTER :- INDEFINITE INTEGRATION | **SECTION**..

(SECTION-A)

1. Let f (x) be a cubic polynomial with leading coefficient unity such that f (0) = 1 and all the roots of f ' (x) = 0 are also roots of f (x) = 0. If $\int f(x)\,dx$ = g (x) + C, where g(0) = $\frac{1}{4}$ and C is constant of integration, then g (3) – g (1) is equal to
(A) 27 (B) 48 (C) 60 (D) 81

2. Let F(x) be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ w.r.t. x. If F(10) = 60 then the value of F(13), is
(A) 66 (B) 132 (C) 248 (D) 264

3. Primitive of $\frac{3x^4-1}{(x^4+x+1)^2}$ w.r.t. x is :
(A) $\frac{x}{x^4+x+1}$ + c (B) – $\frac{x}{x^4+x+1}$ + c
(C) $\frac{x+1}{x^4+x+1}$ + c (D) – $\frac{x+1}{x^4+x+1}$ + c

4. If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x}dx$ = $ln\left(f(x)\right)$ + g(x) + C where C is the constant of integration and *f* (x) is positive, then *f* (x) + *g* (x) has the value equal to
(A) e^x + sin x + 2x (B) e^x + sin x
(C) e^x – sin x (D) e^x + sin x + x

5. If I = $\int \frac{ax^2+2bx+c}{\left(Ax^2+2Bx+C\right)^2}dx$ (where $B^2 \neq AC$) is a rational function then which one of the following condition must be necessary?
(A) 2Bb = Ac + aC (B) Aa + Bb = Cc
(C) Bb = aC + cA (D) $\frac{A}{c}+\frac{C}{a}=\frac{2B}{b}$

6. $\int e^x \sin e^{x-1} dx$ is equal to
(A) – cos e^{x-1} + C (B) – e cos e^{x-1} + C
(C) cos e^{x-1} + C (D) – $\frac{1}{e}$ cos e^{x-1} + C

7. Let *g* (x) be an antiderivative for *f* (x). Then $ln\left(1+(g(x))^2\right)$ is an antiderivative for
(A) $\frac{2f(x)g(x)}{1+\left(f(x)\right)^2}$ (B) $\frac{2f(x)g(x)}{1+\left(g(x)\right)^2}$
(C) $\frac{2f(x)}{1+\left(f(x)\right)^2}$ (D) none

8. $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}$ dx
(A) sin x – 6 $\tan^{-1}$ (sin x) + c
(B) sin x – 2 $\sin^{-1}$ x + c
(C) sin x – 2 $(\sin x)^{-1}$ – 6 $\tan^{-1}$ (sin x) + c
(D) sin x – 2 $(\sin x)^{-1}$ + 5 $\tan^{-1}$ (sin x) + c

9. $\int x^3 (x^2-1)^{1/4}$ dx
(A) $\frac{2\left(x^2-1\right)^{5/4}}{45}$ $(4+5x^2)$ + C
(B) $\frac{2\left(x^2-1\right)^{5/4}}{45}$ $(5+4x^2)$ + C
(C) $\frac{\left(x^2-1\right)^{5/4}}{45}$ $(4+x^2)$ + C
(D) $\frac{\left(x^2-1\right)^{4/5}}{45}$ $(4x^2+1)$ + C

10. $\int \frac{x\sin x}{(x\cos x-\sin x)}dx$
(A) – *ln* $\left(\frac{1}{x\cos x-\sin x}\right)$ + C
(B) – *ln* (x cosx – sinx) + C
(C) *ln* (x sinx – cosx) + C
(D) *ln* $\left(\frac{1}{x\sin x-x}\right)$ + C

11. $\int \frac{e^{\tan^{-1}x}}{(1+x^2)}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2 + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] dx$ (x > 0)

(A) $e^{\tan^{-1}x}.\tan^{-1}x + C$

(B) $\frac{e^{\tan^{-1}x}.\left(\tan^{-1}x\right)^2}{2} + C$

(C) $e^{\tan^{-1}x}.\left(\sec^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

(D) $e^{\tan^{-1}x}.\left(\operatorname{cosec}^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

12. If $I_n = \int \cot^n x \, dx$, then $I_0 + I_1 + 2(I_2 + I_3 + \ldots\ldots + I_8) + I_9 + I_{10}$ equals to :
(where u = cot x)

(A) $u + \frac{u^2}{2} + \ldots\ldots + \frac{u^9}{9}$

(B) $-\left(u + \frac{u^2}{2} + \ldots\ldots + \frac{u^9}{9}\right)$

(C) $-\left(u + \frac{u^2}{2!} + \ldots\ldots + \frac{u^9}{9!}\right)$

(D) $\frac{u}{2} + \frac{2u^2}{3} + \ldots\ldots + \frac{9u^9}{10}$

13. Let $f : \left[0, \frac{\pi}{2}\right] \to R$ be continuous and satisfy $f'(x) = \frac{1}{1+\cos x}$ for all $x \in \left(0, \frac{\pi}{2}\right)$. If f (0) = 3 then $f\left(\frac{\pi}{2}\right)$ has the value equal to

(A) $\frac{13}{4}$ (B) 2 (C) 4 (D) None

14. The value of the integral $\int \frac{\sin(\ln(2+2x))}{x+1} dx$ is

(A) – cos *ln* (2x + 2) + C

(B) $\ln\left(\sin\frac{2}{x+1}\right) + C$

(C) $\cos\left(\frac{2}{x+1}\right) + C$

(D) $\sin\left(\frac{2}{x+1}\right) + C$

15. Suppose $A = \int \frac{dx}{x^2+6x+25}$ and $B = \int \frac{dx}{x^2-6x-27}$.

If $12(A + B) = \lambda \cdot \tan^{-1}\left(\frac{x+3}{4}\right) + \mu \cdot \ln\left|\frac{x-9}{x+3}\right| + C$, then the value of $(\lambda + \mu)$ is

(A) 3 (B) 4 (C) 5 (D) 6

16. Let $\int \frac{dx}{x^{2008}+x} = \frac{1}{p}\ln\left(\frac{x^q}{1+x^r}\right) + C$

where p, q, r ∈ N and need not be distinct, then the value of (p + q + r) equals

(A) 6024 (B) 6022
(C) 6021 (D) 6020

17. $\int \frac{\sin 2x + \sin 4x - \sin 6x}{1+\cos 2x + \cos 4x + \cos 6x} dx$ is equal to (where C is indefinite integration constant.)

(A) $\frac{1}{3}$ *ln* | sec 3x | + $\frac{1}{2}$ *ln* | sec 2x | + *ln* | sec x | + C

(B) $\frac{1}{3}$ *ln* | sec 3x | – $\frac{1}{2}$ *ln* | sec 2x | – *ln* | sec x | + C

(C) C – $\frac{1}{3}$ *ln* | sec 3x | + $\frac{1}{2}$ *ln* | sec 2x | + *ln* | sec x |

(D) C – $\frac{1}{3}$ *ln* | sec 3x | – $\frac{1}{2}$ *ln* | sec 2x | – *ln* | sec x |

18. $\int \frac{dx}{x(x+1)(\ln(x+1)-\ln x)^{11}}$ equals

(A) $\frac{1}{10(\ln(x+1)-\ln x)^{10}} + C$

(B) $\frac{(\ln(x+1)-\ln x)^{10}}{10} + C$

(C) $\frac{1}{11(\ln(x+1)-\ln x)^{11}} + C$

(D) $\frac{(\ln(x+1)-\ln x)^{11}}{11} + C$

where C is constant of integration.

19. $\int e^{\sin^2 x} \sin x \left(\cos x + \cos^3 x\right) dx$ is equal to

(A) $\frac{1}{2} e^{\sin^2 x}\left(3 - \sin^2 x\right) + C$

(B) $\frac{1}{2} e^{\sin^2 x}\left(1 - \frac{1}{2}\cos^2 x\right) + C$

(C) $e^{\sin^2 x}\left(3\cos^2 x + 2\sin^2 x\right) + C$

(D) $e^{\sin^2 x}\left(2\cos^2 x + 3\sin^2 x\right) + C$

where C is constant of integration.

20. Evaluate: $\int \frac{1}{(x^2-4)\left(\sqrt{x+1}\right)} dx$

(A) $\frac{1}{4\sqrt{3}} \log\left|\frac{\sqrt{x+1}+\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| + \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{1}\right)$

(B) $\frac{1}{4\sqrt{3}} \log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x+1}}{1}\right)$

(C) $\frac{1}{4\sqrt{3}} \log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| + \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x+1}}{1}\right)$

(D) None of these

(SECTION-B)

21. If the value $\int \frac{1-(\cot x)^{2008}}{\tan x + (\cot x)^{2009}} dx = \frac{1}{k} \ln |\sin^k x + \cos^k x| + C$, then find k.

22. Suppose $\int \frac{1 - 7\cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{g(x)}{\sin^7 x} + C$, where C is arbitrary constant of integration.

Then find the value of $g'(0) + g''\left(\frac{\pi}{4}\right)$

23. Let $f'(x^2) = \frac{1}{x}$ for $x > 0$, $f(1) = 1$ and $g'(\sin^2 x - 1) = \cos^2 x + p \; \forall x \in R$, $g(-1) = 0$.

If $h(x) = \begin{cases} f(x), & x > 0 \\ g(x), & -1 \le x \le 0 \end{cases}$

is a continuous function then find the absolute value of 2p.

24. Let $f(x)$ is a quadratic function such that $f(0) = 1$ and $\int \frac{f(x)\,dx}{x^2(x+1)^3}$ is a rational function, find the value of $f'(0)$.

25. If $\int \frac{(2x+3)}{x(x+1)(x+2)(x+3)+1} dx = k - \frac{1}{f(x)}$ where $f(x)$ is of the form $ax^2 + bx + c$, then find the value of $(a + b + c)$.

26. If $\int e^x \left(\frac{1-x^n}{1-x}\right) dx = e^x \cdot P(x) + C$ where $n \in N$ and 'C' is constant of integration and $P(0) = 620$ then find the value of n.

27. If $I_n = \int \frac{e^{(n+1)x}\,dx}{\left(1 + e^x + \frac{e^{2x}}{2!} + \ldots\ldots + \frac{e^{nx}}{n!}\right)}$

$= \lambda_n\left(e^x - \ln\left(f_n(x)\right)\right) + C$

where $f_n(0) = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}$ and C is constant of integration and $g(x) = \lim_{n\to\infty} \ln\left(f_n(x)\right)$, then find the number of real solutions of the equation $g(x) = 4x^2$.

28. Let $f(x) = \frac{1}{e^x + 8e^{-x} + 4e^{-3x}}$ and $g(x) = \frac{1}{e^{3x} + 8e^x + 4e^{-x}}$. If $\int (f(x) - 2g(x))dx = h(x) + c$,

where c is constant of integration and $\lim_{x\to\infty} h(x) = \frac{\pi}{4}$, then find the value of $2\tan\left(2h(0)\right)$.

29. Let $f_1(x)$ and $f_2(x)$ be two real valued function such that $f_1(1) = 2$, $f_2(1) = 4$. Given that $f_1(x)$ is primitive of $g(x)$ and $f_2(x)$ is antiderivative of $g(x) + 1$. If $h(x)$ is inverse function of $f_2(x) - f_1(x)$ then find $h'(5)$.

30. Let $\lim_{h\to 0} \frac{h^2}{f(x+2h) - 2f(x+h) + f(x)} = \frac{x^{1-x}}{1 + x(1+\ln x)^2}$. If $\lim_{x\to 0^+} f(x) = 1$ and $f(1) = 2$, then find the value of $\frac{f(3)}{f(2)}$.