JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12th **PAPER CODE :- CWT-7**

CHAPTER :- APPLICATION OF DERIVATIVES

ANSWER KEY

1.	(B)	**2.**	(D)	**3.**	(C)	**4.**	(D)	**5.**	(B)	**6.**	(C)	**7.**	(A)
8.	(B)	**9.**	(D)	**10.**	(C)	**11.**	(C)	**12.**	(C)	**13.**	(B)	**14.**	(A)
15.	(B)	**16.**	(B)	**17.**	(D)	**18.**	(B)	**19.**	(A)	**20.**	(B)	**21.**	3
22.	4	**23.**	1	**24.**	9	**25.**	7	**26.**	25	**27.**	9	**28.**	4
29.	3	**30.**	3										

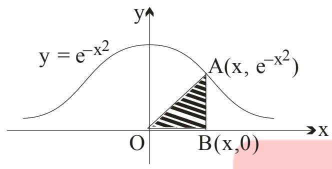
SOLUTIONS

1. (B)

Sol. $A = \dfrac{x e^{-x^2}}{2}$;A'

$= \dfrac{1}{2}\left[e^{-x^2} - 2x^2 \cdot e^{-x^2}\right]$

$\dfrac{e^{-x^2}}{2}\left[1-2x^2\right] = 0 \Rightarrow x = \dfrac{1}{\sqrt{2}}$ gives



A_{max}.

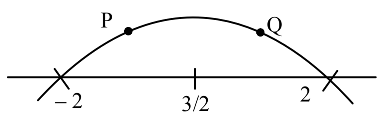
$\therefore A_{max} = \dfrac{e^{-1/2}}{2\sqrt{2}} = \dfrac{1}{\sqrt{8e}}$

2. (D)

Sol. $\dfrac{y_1+y_2}{2} \le$ max. (y)

$y_1 + y_2 \le 2\left(\dfrac{9}{2} - 2 - \dfrac{9}{4}\right)$

$3x_1 + 3x_2 - x_1^2 - x_2^2 \le \dfrac{9}{2}$



3. (C)

Sol. $\dfrac{dy}{dx} = (x-1)(x-2)^2$ so $\dfrac{d^2y}{dx^2} = (x-2) \times (3x - 4)$.

The points of inflection are given by $\dfrac{d^2y}{dx^2} = 0$

So x = 2, x = 4/3 are points of inflection.

4. (D)

Sol. Given, $V = \pi r^2 h$

Differentiating both sides

$\dfrac{dV}{dt} = \pi\left(r^2\dfrac{dh}{dt} + 2r\dfrac{dr}{dt}h\right) = \pi r\left(r\dfrac{dh}{dt} + 2h\dfrac{dr}{dt}\right)$

$\dfrac{dr}{dt} = \dfrac{1}{10}$ and $\dfrac{dh}{dt} = -\dfrac{2}{10}$

$\dfrac{dr}{dt} = \pi r\left(r\left(-\dfrac{2}{10}\right) + 2h\left(\dfrac{1}{10}\right)\right)$

$= \dfrac{\pi r}{5}(-r+h)$

Thus, when r = 2 and h = 3,

$\dfrac{dV}{dt} = \dfrac{\pi(2)}{5}(-2+3) = \dfrac{2\pi}{5}$

5. (B)

Sol. for $x \ge d$, $f(x) = 4x - (a+b+c+d)$

$\Rightarrow f'(x) = 4 \Rightarrow f(x)$ is $\uparrow$

for $c \le x < d$, $f(x) = 2x - a - b - c + d \Rightarrow f'(x) = 2 \Rightarrow f(x)$ is $\uparrow$

for $b \le x < c$, $f(x) = x - a + x - b + c - x + d - x$

$= c + d - a - b \Rightarrow f'(x) = 0 \Rightarrow f(x)$ = constant

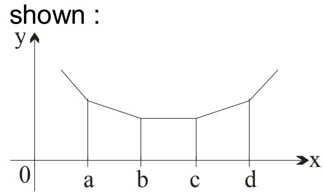
for $a \le x < b$, $f(x) = x - a + b - x + c - x + d - x$

$= b + c + d - a - 2x \Rightarrow f'(x) = -2 \Rightarrow f(x)$ is $\downarrow$

for $x < a$ f(x) is again decreasing .

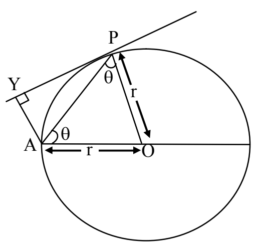
Hence f (x) is least when $b \le x \le c$.

As shown :



6. (C)

Sol.



OP $\perp$ PY $\angle$APY = 90 – θ

OPA = θ $\angle$PAY = θ

Now ΔOPA

$AP^2 = r^2 + r^2 - 2rr\cos(\pi - 2\theta) = 4r^2\cos 2\theta$

$AP = 2r\cos 2\theta \quad PY = AP\sin\theta = r\sin 2\theta$

$AY = AP\cos\theta = 2Y\cos 2\theta$

$\therefore$ Area of $\Delta APY = 1/2\ PY.\ AY$

$= r^2\sin 2\theta\cos 2\theta$

$\frac{d\Delta}{d\theta} = r^2[2\cos 2\theta\cos 2\theta - \sin^2 2\theta] = 0$

$\theta = \frac{\pi}{2}, \frac{\pi}{6}$

$\theta \neq \frac{\pi}{2}$ Δ is maximum at $\theta = \frac{\pi}{6}$

$\Delta_{max} = r^2 . \frac{\sqrt{3}}{2} . \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3\sqrt{3}r^2}{8}$

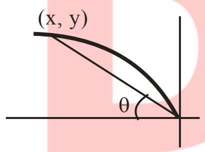
7. (A)

Sol. $y = \sqrt{-x}$; $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sqrt{-x}}{x} = \frac{1}{\sqrt{-x}}$

$\sec^2\theta\frac{d\theta}{dt} = +\frac{1}{2}\frac{1}{(-x)^{3/2}}\frac{dx}{dt} = \frac{1}{16}\cdot(-8)$

(where $x = -4$)



$\sec^2\theta\frac{d\theta}{dt} = -\frac{1}{2}$ ....(1)

Also where $x = -4$, $\tan\theta = \frac{2}{-4} = -\frac{1}{2}$

$\therefore (1 + \tan^2\theta)\frac{d\theta}{dt} = -\frac{1}{2}$

$\frac{5}{4}\frac{d\theta}{dt} = -\frac{1}{2} \Rightarrow \frac{d\theta}{dt} = \frac{-2}{5} = -0.4.$

8. (B)

Sol. $y = x(\ln x - 2)$

$y' = x\left(\frac{1}{x}\right) + (\ln x - 2) = \ln x - 1$

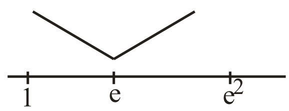
$\frac{dy}{dx} = \ln x - 1 = 0 \Rightarrow x = e$

now $f(1) = -2$

$f(e) = -e$ (least)

$f(e^2) = 0$ (greatest)

$\therefore$ difference $= 0 - (-e) = e$



9. (D)

Sol. $x = \phi(t) = t^5 - 5t^3 - 20t + 7$

$\frac{dx}{dt} = \phi'(t) = 5t^4 - 15t^2 - 20 = 5(t^2 - 4)(t^2 + 1)$

$\neq 0$

If $-2 < t < 2$

$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$

$\frac{dy}{dt} = \psi'(t) = 12t^2 - 6t - 18$

$\frac{dy}{dt} = 0 \Rightarrow t = -1$ or $3/2$

$\frac{d^2y}{dt^2} = \psi''(t) = 24t - 6 = \psi''(-1) = -30$

and $\psi''(3/2) = 30$

$y = f(x)$ is minimum at $t = 3/2$

10. (C)

Sol. $f(x) = \begin{cases} (-1)^{m+n}x^n(x-1)^n & \text{if } x < 0 \\ (-1)^n x^m (x-1)^n & \text{if } 0 \leq x < 1 \\ x^m(x-1)^n & \text{if } x \geq 1 \end{cases}$

$g(x) = x^m(x-1)^n$, then

$g'(x) = mx^{m-1}(x-1)^n + nx^m(x-1)^{n-1}$

$= x^{m-1}(x-1)^{n-1}\{mx - m + nx\} = 0$

$f'(x) = 0 \Rightarrow g'(x) = 0 \Rightarrow x = 0, 1$ or $\frac{m}{m+n}$

$f(0) = 0$, $f(1) = 1$ and

$f\left(\frac{m}{m+n}\right) = (-1)^n\frac{m^m n^n(-1)^n}{(m+n)^{m+n}}$

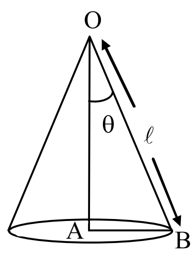
$\Rightarrow \frac{m^m n^n}{(m+n)^{m+n}} > 1 \quad \left[0 < \frac{m}{m+n} < 1\right]$

The maximum value $= \frac{m^m n^n}{(m+n)^{m+n}}$

11. (C)

Sol. Let $OB = \ell$, $OA = \ell\cos\theta$ and $AB = \ell\sin$

θ ($0 \leq \theta \leq \pi/2$). Then



$V = \frac{\pi}{3}(AB)^2(OA) = \frac{\pi}{3}\ell^3\sin^2\theta\cos\theta$

$\Rightarrow \frac{dV}{d\theta} = \frac{\pi}{3}\ell^3 \sin\theta\,(3\cos^2\theta - 1)$

So from $\frac{dV}{d\theta} = 0$, we get $\theta = 0$ or $\cos\theta = \frac{1}{\sqrt{3}}$. Also $V(0) = 0$, $V(\pi/2) = 0$ and

$V\left(\cos^{-1}\frac{1}{\sqrt{3}}\right) = \frac{2\pi\ell^3}{9\sqrt{3}}$

Hence V is maximum when $\cos\theta = 1/\sqrt{3}$,

i.e., $\tan\theta = \sqrt{2}$

12. (C)

Sol. Distance 'd' from the origin is $= \sqrt{x^2 + y^2}$

$= \sqrt{(1 - 2\cos^2 t)^2 + \cos^2 t}$

$d = \sqrt{4\cos^4 t - 3\cos^2 t + 1}$

$= 2\sqrt{\left(\cos^2 t - \frac{3}{8}\right)^2 + \frac{16}{14} - \frac{9}{64}}$

$= 2\sqrt{\left(\cos^2 t - \frac{3}{8}\right)^2 + \frac{7}{64}}$

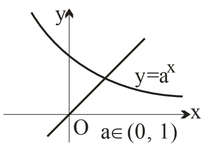
This is a quadratic in $\cos^2 t$, this will be minimum when $\cos^2 t = \frac{3}{8}$

$\Rightarrow \cos t = \frac{\sqrt{3}}{2\sqrt{2}} \Rightarrow \cos t = \frac{\sqrt{6}}{4}$

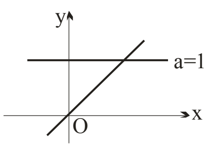
$\Rightarrow a = \frac{\sqrt{6}}{4}$

13. (B)

Sol. for $0 < a \le 1$ the line

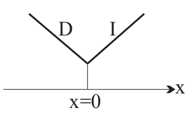


always cuts $y = a^x$



for $a > 1$ say $a = 2$

consider $f(x) = e^x - x$



f (0) is the minimum

$f'(x) = e^x - 1$

$f(x)|_{min} = 1$

$f'(x) > 0$ for $x > 0$ and $f'(x) < 0$ for $x < 0$

$f(x) \ge 1$

$\therefore$ f(x) is incresing ($\uparrow$) for $x > 0$

$e^x - x \ge 1$

and decreasing ($\downarrow$)for $x < 0$

$y = e^x$ always lies above $y = x$ i.e. $e^x - x \ge 1$ for $a > 1$

hence never cuts = a = (0, 1]

14. (A)

Sol. f(x) is defined for all $x > 2$

$f(x) = 2\log(x - 2) - x^2 + 4x + 1$

$f'(x) =$

$\frac{2}{x-2} - 2x + 4 = \frac{2 - 2x(x-2) + 4(x-2)}{x-2} = \frac{-2x^2 + 8x - 6}{x-2}$

$f'(x) = \frac{-2(x^2 + 4x - 3)}{x-2} = \frac{-2(x-1)(x-3)}{x-2}$

For f(x) to be increasing $f'(x) > 0$

$\frac{-2(x-1)(x-3)}{x-2} > 0$

$\frac{(x-1)(x-3)}{x-2} < 0$

$x - 3 < 0$ [$\therefore x \in$ Domain (f)

$\Rightarrow x > 2 \Rightarrow x - 1 > 0$ & $x - 2 > 0$]

$x < 3 \Rightarrow x \in (2,3)$ [$\because x > 2$]

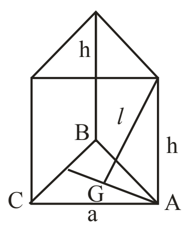
15. (B)

Sol. $AG = \frac{2}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$. Now $l^2 = \frac{a^2}{3} + h^2$;

area (A) $= \frac{\sqrt{3}}{4}a^2$

$V(h) = \frac{\sqrt{3}}{4}a^2 h = \frac{\sqrt{3}}{4}h(l^2 - h^2)3;$

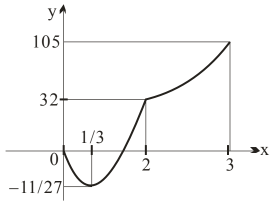
$V'(h) = 0 \Rightarrow h = \frac{l}{\sqrt{3}}$; $V_{max} = \frac{l^3}{2}$]



16. (B)

Sol. $\frac{dy}{dx} = \left[\begin{array}{ll} 12x^2-2x+2 & \text{for } 2<x\le 3 \\ 12x^2+2x-2 & \text{for } 0\le x<2 \end{array}\right.$

$= 2\,(2x + 1)\,(3x - 1)$



Continuous but not derivable at $x = 2$ &

$\frac{dy}{dx} = 0$ at $x = 1/3$,

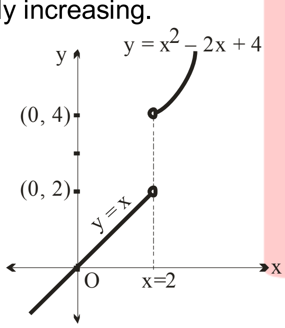
Decreasing in (0, 1/3) & Increasing for $(1/3, 2) \cup (2, 3)$

$\Rightarrow$ Minima occurs at $x = 1/3$. $f(1/3) = -11/27$]

17. (D)

Sol. Let $g(x) = x$, where $x < 2$, then g(x) is strictly increasing.

let $h(x) = x^2 - 2x + 4$, where $x > 2$, then h(x) is also strictly increasing.



Since, f(x) is strictly increasing at $x = 2$,

so $\lim\limits_{x\to 2^-} x \le k \le \lim\limits_{x\to 2^+}\left(x^2 - 2x + 4\right) \Rightarrow$

$2 \le k \le 4 \Rightarrow k \in [2, 4]$

18. (B)

Sol. $f'(x) = 2a^2x^2 - 5ax + 3 = (ax - 1)(2ax - 3) = 0$

$x = 1/a, 3/2a$

If $a > 0$ then local maxima occurs at $x = 1/a$ and minima at $x = 3/2a$

$\because$ maxima occurs $x = 1/a = 1/3 \Rightarrow a = 3$

minima coccurs $x = \frac{3}{2a} = 1/2$

$f\left(\frac{1}{2}\right) > 0 \Rightarrow 3/8 + b > 0,\ b > -3/8$

If $a < 0$ then maxima shall occur at $x = 3/2a$ and minima at $x = 1/2a$

$\frac{3}{2a} = \frac{1}{3} \Rightarrow a = \frac{9}{2} > 0$ not admissible

Hence $b > -3/8$

19. (A)

Sol. $y = \frac{ax-b}{(x-1)(x-4)} = \frac{ax-b}{x^2-5x+4}$ ....(i)

$\frac{dy}{dx} = \frac{(x^2-5x+4)a - (ax-b)(2x-5)}{(x^2-5x+4)^2}$ ....(ii)

$\left(\frac{dy}{dx}\right)_P = \frac{(4-10+4)a-(2a-b)(4-5)}{(4-10+4)^2} = \frac{-b}{4}$

Since P is a turning Point of the curve (i)

$\therefore \left(\frac{dy}{dx}\right)_P = 0 \Rightarrow \frac{-b}{4} = 0 \Rightarrow b = 0$ ....(iii)

$\because$ P(2,–1) lies on $y = \frac{ax-b}{(x-1)(x-4)}$

$\therefore -1 = \frac{2a-b}{(2-1)(2-4)}$

$\Rightarrow -1 = \frac{2a-b}{-2} \Rightarrow 2a - b = 2$ ....(iv)

From (iii) & (iv)

$a = 1, b = 0$

$\therefore a + b = 1 + 0 = 1$

20. (B)

Sol. $f(\theta) = \frac{(a^2-b^2)\cos\theta}{a - b\sin\theta} = \frac{a^2-b^2}{a\sec\theta - b\tan\theta}$

$f(\theta) = \frac{a^2-b^2}{h(\theta)}$, $h(\theta) = a\sec\theta - b\tan\theta$

$f(\theta)$ is maximum and minimum according as $h(\theta)$ is min. or max. respectively

$h(\theta) = a\sec\theta - b\tan\theta$

$h'(\theta) = a\sec\theta\tan\theta - b\sec^2\theta$

for max. and min. of $h(\theta)$ put $h'(\theta) = 0$

$\sec\theta\,(a\tan\theta - b\sec\theta) = 0$

$\sin\theta = b/a$ ....(1) as $[\sec\theta \ne 0]$

$h''(\theta) = \sec\theta\tan\theta\,(a\tan\theta - b\sec\theta)$

$+ (a\sec^2\theta - b\sec\theta\tan\theta)\sec\theta$

$= a\sec^3\theta + a\sec\theta\tan^2\theta - 2b\sec^2\theta\tan\theta$

$h''\theta = \frac{a + a\sin^2\theta - 2b\sin\theta}{\cos^3\theta}$

$= \frac{a + a\,b^2/a^2 - 2b.\,b/a}{\cos^3\theta}$

$= \dfrac{a^2 - b^2}{a\cos^3\theta} > 0$

$\begin{cases} \sin\theta = b/a \\ a > b > 0 \text{ and } \sin\theta = \dfrac{b}{a} \end{cases}$ is +ve

So, $h(\theta)$ is minimum when $\sin\theta = \dfrac{b}{a}$

$f(\theta)$ is maximum when $\sin\theta = \dfrac{b}{a}$

max. $f(\theta) = \dfrac{(a^2-b^2).\dfrac{\sqrt{a^2-b^2}}{a}}{(a-b).b/a}$

$\Rightarrow \dfrac{(a^2-b^2)\left(\sqrt{a^2-b^2}\right)}{(a^2-b^2)}$

max. $f(\theta) = \sqrt{a^2-b^2}$

21. 3

Sol. Let $f(x) = \tan^{-1} x$, $x \in [a, b]$

$\therefore$ Using LMVT, we get

$\dfrac{\tan^{-1} b - \tan^{-1} a}{b-a} = \dfrac{1}{1+c^2}$, where $0 < a < c < b < \sqrt{3}$

So, $1 < \left(\dfrac{b-a}{\tan^{-1} b - \tan^{-1} a}\right) < 4$

As, $\dfrac{1}{4} < \dfrac{1}{1+c^2} < 1$

$\Rightarrow$The greatest possible integral value is 3.

22. 4

Sol. We have $f(x) = (b^2 - 3b + 2)(\cos^2 x - \sin^2 x) + (b - 1)x + \sin 2$

$\therefore f'(x) = (b-1)(b-2)(-2\sin 2x) + (b-1)$

Now, $f'(x) \neq 0$ for every $x \in R$,

So $(b-1)\left(1 - 2(b-2)\sin 2x\right) \neq 0\ \forall x \in R \therefore b \neq 1$

Also, $\left|\dfrac{1}{2(b-2)}\right| > 1 \Rightarrow b \in \left(\dfrac{3}{2}, 2\right) \cup \left(2, \dfrac{5}{2}\right)$

Now, when $b = 2$, $f(x) = x + \sin 2 \Rightarrow f'(x) = 1\ (\neq 0)$.

Hence, $b \in \left(\dfrac{3}{2}, \dfrac{5}{2}\right) \Rightarrow b_1 = \dfrac{3}{2}$

and $b_2 = \dfrac{5}{2}$

$\Rightarrow (b_1 + b_2) = \dfrac{3}{2} + \dfrac{5}{2} = \dfrac{8}{2} = 4$

23. 1

Sol. $f(x) = \dfrac{10}{x^{12} + 2 + 3x^4 + \dfrac{3}{x^4} + \dfrac{1}{x^{12}}} =$

$\dfrac{10}{x^{12} + 3x^4 + \dfrac{3}{x^4} + \dfrac{1}{x^{12}} + 2} = \dfrac{10}{\left(x^4 + \dfrac{1}{x^4}\right)^3 + 2}$

$\because x^4 + \dfrac{1}{x^4} \geq 2 \Rightarrow \left(x^4 + \dfrac{1}{x^4}\right)^3 + 2 \geq 10$

$\therefore f(x) \leq \dfrac{10}{10} = 1$

24. 9

Sol. $f(x) = \dfrac{x^3}{3} - (m-3)\dfrac{x^2}{2} + mx - 2013$

$\therefore f'(x) = \left(x^2 - (m-3)x + m\right) \geq 0$, $\forall x \in [0, \infty)$

Case-I: When $D \leq 0$

$\Rightarrow m \in [1, 9]$ ....(I)

Case-II: When $D \geq 0$

$\Rightarrow m \in [-\infty, 1] \cup [9, \infty)$ ....(i)

$\dfrac{-b}{2a} \leq 0 \Rightarrow (m-3) \leq 0$

$\Rightarrow m \leq 3$ .....(ii)

And $f'(0) \geq 0$

$\Rightarrow m \geq 0$ ......(iii)

$\therefore (1) \cap (ii) \cap (iii)$

$\Rightarrow m \in [0, 1]$ ....(II)

So, finally (I) $\cap$ (II)

$\Rightarrow m \in [0, 9] \equiv [0, k]$

$\therefore k = 9$

25. 7

Sol. $f(x) = x - \sin x$

$f'(x) = 1 - \cos x$

$f'(x) = 0 \Rightarrow 1 - \cos x = 0 \Rightarrow \cos x = 1$

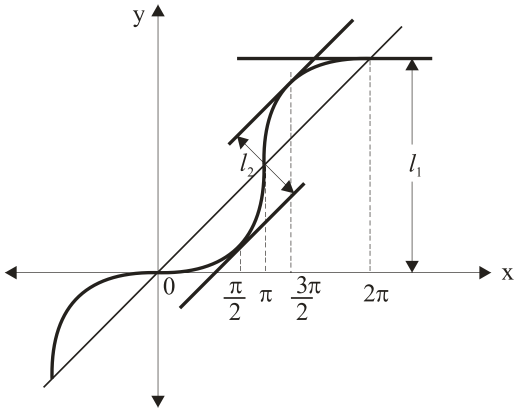
$x = 0, 2\pi$

$y = 0$ and $y = 2\pi$ are two parallel tangents and distance between them $l_1 = 2\pi$

Also other two parallel tangents are parallel to the line y = x

$\therefore$ f '(x) = 1 $\Rightarrow$ 1 – cos x = 1

$\Rightarrow$ cos x = 0 $\Rightarrow$ x = $\frac{\pi}{2}, \frac{3\pi}{2}$. Tangent at x = $\frac{\pi}{2}$



$y-\left(\frac{\pi}{2}-1\right) = 1\left(x-\frac{\pi}{2}\right) x - y = 1$

$l_2 = 2.\left|\frac{1}{\sqrt{2}}\right| = \sqrt{2}$

$\therefore$ $[l_1 + l_2] = \left[2\pi+\sqrt{2}\right] = 7$

26. 25

Sol. $9\sec^2\theta + 4\text{cosec}^2\theta = 13 + 9\tan^2\theta + 4\cot^2\theta$

By using A.M. and G.M.

$13 + 9\tan^2\theta + 4\cot^2\theta \geq 13 + 2$

$\sqrt{9\tan^2\theta.4\cot^2\theta}$

So $13 + 9\tan^2\theta + 4\cot^2\theta \geq 25$.

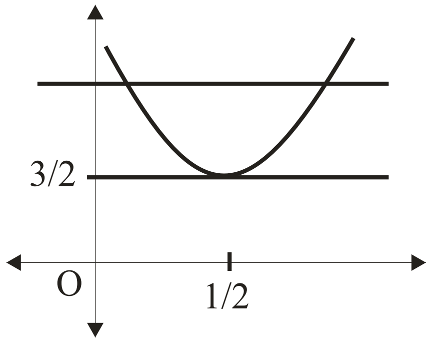
27. 9

Sol. $y^2 - \frac{15y}{2} + \lambda = 0$ $\begin{cases} y = c_1 \\ y = c_2 \end{cases}$

$y = 2x + \frac{1}{8x^2}$

$y' = 2 - \frac{2}{8x^3} \Rightarrow 2\left(\frac{8x^3-1}{8x^3}\right) \Rightarrow$

$$\begin{cases} \text{For } x \in \left(0, \frac{1}{2}\right), & y' < 0 \Rightarrow y \text{ is } \downarrow \\ \text{For } x \in \left(\frac{1}{2}, \infty\right), & y' > 0 \Rightarrow y \text{ is } \uparrow \end{cases}$$



$\therefore$ y = $\frac{3}{2}$ satisfies the equation $y^2 - \frac{15y}{2} + \lambda = 0$

$\Rightarrow \frac{9}{4} - \frac{45}{4} + \lambda = 0 \Rightarrow \lambda = 9$

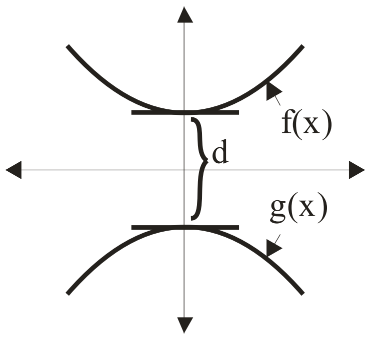
For $\lambda = 9$, y = $\frac{3}{2}$ and y = 6.

28. 4

Sol. f (x) = 1 + $3x^2 + 5x^4 + 7x^6 + \ldots\ldots + 21x^{20}$, $x \in R$

$g(x) = -x^2 + 4\cos^2\theta - 4\sin\theta - 7, \theta \in R$

$f(x)|_{min.} = 1$, $g(x)|_{max.} = 4\cos^2\theta - 4\sin\theta - 7$ at x = 0



shortest distance between f(x) and g(x) is

$d = 1 - (4\cos^2\theta - 4\sin\theta - 7)$

$= 1 - (4 - 4\sin^2\theta - 4\sin\theta - 7)$

$= 4\sin^2\theta + 4\sin\theta + 4$

$= (2\sin\theta + 1)^2 + 3$

$d_1 = 3$, $d_2 = 12$

$\therefore \frac{d_2}{d_1} = 4$

29. 3

Sol. f (x) =

$$\begin{cases} \frac{1}{x}; & \text{if } x^2 > 1 \Rightarrow x < -1 \text{ or } x > 1 \\ ax^3 + bx^2; & \text{if } 0 \le x^2 < 1 \Rightarrow -1 < x < 1 \\ \frac{1/x + ax^3 + bx^2}{2}; & \text{if } x^2 = 1 \end{cases}$$

f is continuous

$\therefore$ at x = 1 $\quad$ 1 = a + b $\quad$ ......(i)

And at x = –1 $\quad$ –1 = –a + b $\quad$ ......(ii)

$\therefore$ b = 0 and a = 1

$\therefore$ points A and B are = (–1, 3) and (1, –1)

$g'(x) = \lambda(x-1)(x+1)$

$g(x) = \lambda\left(\frac{x^3}{3} - x\right) + c$

$g(-1) = \frac{2\lambda}{3} + c = 3$ $\quad$ ......(iii)

$g(1) = -\frac{2\lambda}{3} + c = -1$

c = 1 and λ = 3

$\therefore$ $g(x) = x^3 - 3x + 1$

$\therefore$ g(2) = 3

30. 3

Sol. f(x) is increasing

$\therefore$ $f'(x) > 0 \ \forall x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow 3\sec^2 x + (3a+1)\sec x + a > 0$

$\Rightarrow 3\sec^2 x + \sec x + 3a\sec x + a > 0$

$\Rightarrow (3\sec x + 1)(\sec x + a) > 0 \ \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ sec x + a > 0

$\Rightarrow$ min. value $a + 1 \ge 0$ $\Rightarrow a \ge -1$

$\therefore$ minimum possible integral value of a = –1

$\therefore$ $a^2 - a + 1 = 3$