JEE MAIN : CHAPTER WISE TEST-7

SUBJECT :- MATHEMATICS **DATE..**
CLASS :- 12th **NAME..**
CHAPTER :- APPLICATION OF DERIVATIVES **SECTION...**

(SECTION A)

1. Point 'A' lies on the curve $y = e^{-x^2}$ and has the coordinate (x, e^{-x^2}) where x > 0. Point B has the coordinates (x, 0). If 'O' is the origin then the maximum area of the triangle AOB is

(A) $\frac{1}{\sqrt{e}}$ (B) $\frac{1}{\sqrt{8e}}$
(C) $\frac{1}{\sqrt{2e}}$ (D) $\frac{1}{\sqrt{4e}}$

2. If P (x_1, y_1), Q (x_2, y_2) be any two points on the curve $y = 3x - 2 - x^2$ for 1 < x < 2, then maximum value of $3x_1 + 3x_2 - x_1^2 - x_2^2$ is-

(A) 9 (B) 4
(C) 2 (D) $\frac{9}{2}$

3. The inflection points on the graph of function y = $\int_0^x (t-1)(t-2)^2 dt$ are

(A) x = –1 (B) x = 3/2
(C) x = 4/3 (D) x = 1

4. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in cm^3/min, when the radius is 2 cm and the height is 3 cm is

(A) -2π (B) $-\frac{8\pi}{5}$
(C) $-\frac{3\pi}{5}$ (D) $\frac{2\pi}{5}$

5. If a < b < c < d & x ∈ R then the least value of the function, f(x) = $|x-a| + |x-b| + |x-c| + |x-d|$ is

(A) c – d + b – a (B) c + d – b – a
(C) c + d – b + a (D) c – d + b + a

6. From a fixed point A on the circumference of a circle of radius r, the perpendicular AY is let fall on the tangent at P. The maximum area of the triangle APY is-

(A) r^2 (B) $\frac{3\sqrt{3}}{4} r^2$
(C) $\frac{3\sqrt{3}}{8} r^2$ (D) $\sqrt{3}\, r^2$

7. A particle move from right to left along the parabola y = $\sqrt{-x}$ in such a way that x-coordinate (measured in meters) decreases at the rate of 8 m/sec. At the moment when x = – 4 the rate at which the angle of inclination θ of the line joining the particle to the origin is changing, is

(A) – 0.4 (B) – 0.2
(C) – 0.5 (D) – 1

8. Difference between the greatest and the least values of the function f (x) = x(ln x – 2) on [1, e^2] is

(A) 2 (B) e (C) e^2 (D) 1

9. The function y = f(x) is represented parametrically by $x = t^5 - 5t^3 - 20t + 7$ and $y = 4t^3 - 3t^2 - 18t + 3$, (–2 < t < 2). The minimum of y = f(x) occurs at

(A) t = – 1 (B) t = 0
(C) t = 1/2 (D) t = 3/2

10. A function f is defined by $f(x) = |x|^m |x-1|^n$ $\forall$ x ∈ R. The maximum value of the function is (m, n ∈ N) -

(A) 1 (B) $m^n n^m$
(C) $\frac{m^m n^n}{(m+n)^{m+n}}$ (D) $\frac{(mn)^{mn}}{(m+n)^{m+n}}$

11. If θ is the angle (semi-vertical) of a cone of maximum volume and given slant height, then tan θ is given by

(A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\sqrt{3}$

12. A particle is moving on the curve defined parametrically by the system of equation $x = 1 - 2\cos^2 t$ and y = cos t if the particle is closest to the origin when t = $\cos^{-1}$(a) for $0 \le t \le \pi/2$, then the value of 'a' is

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{\sqrt{6}}{4}$ (D) $\frac{\sqrt{6}}{8}$

13. Number of positive integral values of 'a' for which the curve y = a^x intersects the line y = x is

(A) 0 (B) 1

(C) 2 (D) More than 2

14. Find the interval in which f(x) = 2 log (x–2) – x^2 + 4x + 1 is increasing ?

(A) (2,3) (B) [2,3]

(C) $(3,\infty)$ (D) (–2,3)

15. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is l . The altitude of the prism for which the volume is greatest :

(A) $\frac{l}{2}$ (B) $\frac{l}{\sqrt{3}}$ (C) $\frac{l}{3}$ (D) $\frac{l}{4}$

16. The point(s) of minimum of the function, $f(x) = 4x^3 - x\,|x-2|$, $x \in [0, 3]$ is :

(A) x = 0 (B) x = 1/3

(C) x = 1/2 (D) x = 2

17. The complete set of values of k for which the function

$$f(x) = \begin{cases} x, & -\infty < x < 2 \\ k, & x = 2 \\ x^2 - 2x + 4, & 2 < x < \infty \end{cases}$$

is strictly increasing at x = 2, is

(A) [2, 4) (B) (2, 4]

(C) (2, 4) (D) [2, 4]

18. Set of value of b for which local extrema of the function f(x) are positive where $f(x) = \frac{2}{3}a^2x^2 - \frac{5a}{2}x^2 + 3x + b$ and maxima occurs at x = 1/3 is

(A) $(-4, \infty)$ (B) $\left(-\frac{3}{8}, \infty\right)$

(C) $\left(-10, \frac{3}{8}\right)$ (D) None of these

19. If $y = \frac{ax-b}{(x-1)(x-4)}$ has a turning point P(2,–1), find the value of a + b.

(A) 1 (B) 0 (C) 3 (D) 4

20. a > b > 0 and $f(\theta) = \frac{(a^2-b^2)\cos\theta}{a - b\sin\theta}$, then the maximum value of $f(\theta)$ is

(A) $\sqrt{a^2+b^2}$ (B) $\sqrt{a^2-b^2}$

(C) a – b (D) a + b

(SECTION B)

21. Find the greatest possible integral value of $\frac{b-a}{\tan^{-1}b - \tan^{-1}a}$, where $0 < a < b < \sqrt{3}$.

22. If the range of all real values of b for which the function $f(x) = (b^2 - 3b + 2)(\cos^2 x - \sin^2 x) + (b-1)x + \sin 2$ does not possess any critical points on R is (b_1, b_2), then find the value of $(b_1 + b_2)$.

23. Maximum value of the expression $\frac{10x^{12}}{x^{24} + 2x^{12} + 3x^{16} + 3x^8 + 1}$ is equal to

24. If all the real values of m for which the function $f(x) = \frac{x^3}{3} - (m-3)\frac{x^2}{2} + mx - 2013$ is strictly increasing in $x \in [0, \infty)$ is [0, k], then find the value of k.

25. If l_1 and l_2 are the least and greatest distance between parallel tangents drawn to the curve f(x) = x – sin x where $x \in \left(\frac{-\pi}{6}, \frac{13\pi}{6}\right)$, then find the value $[l_1 + l_2]$.

[**Note** : [k] denotes greatest integer less than or equal to k.]

26. Least value of the expression $9\sec^2\theta + 4\text{cosec}^2\theta$, is-

27. If $y^2 - \frac{15y}{2} + \lambda = 0$ cuts the graph of the function $y = 2x + \frac{1}{8x^2}$, x > 0 at three distinct points, then find the value of λ.

28. Let $f(x) = 1 + 3x^2 + 5x^4 + 7x^6 + \ldots\ldots + 21 \cdot x^{20}$, $x \in R$

and $g(x) = -x^2 + 4\cos^2\theta - 4\sin\theta - 7$, $\theta \in R$

If d is the shortest distance between f(x) & g(x) and d_1, d_2 are the least and greatest value of d respectively, then find $\left(\frac{d_2}{d_1}\right)$.

29. Let $f(x) = \lim_{n\to\infty} \frac{x^{2n-1} + ax^3 + bx^2}{x^{2n} + 1}$ is continuous for all $x \in R$. If points A(–a, 3) and $B((b+1), -1)$ are points of relative maximum and minimum of a cubic polynomial y = g(x), then find the value of g(2).

30. If $f(x) = 3\tan x + (3a + 1)\, ln\, |(\sec x + \tan x)| + ax$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$ then corresponding to minimum possible integral value of a, find the value of $(a^2 - a + 1)$.