

JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS

CLASS :- 12th

CHAPTER :- METHOD OF DIFFERENTIATION

PAPER CODE :- CWT-6

ANSWER KEY											
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8.	(B)	9.	(D)	10.	(D)	11.	(B)	12.	(C)	13.	(B)
15.	(C)	16.	(B)	17.	(A)	18.	(D)	19.	(D)	20.	(D)
22.	0	23.	1	24.	3	25.	17	26.	4	27.	8
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SOLUTIONS

1. (D)

Sol. $x = \sin\theta ; x = \sin^2\phi$
 $y = \sin^{-1}(\sin\theta \cos\phi + \sin\phi \cos\theta) = \sin^{-1}((\sin(\theta + \phi)) = \theta + \phi = \sin^{-1}x + \sin^{-1}\sqrt{x}$

$$Dy = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x(1-x)}} \text{ assuming } x^2 +$$

$$x \leq 1 \text{ i.e. } 0 \leq x < \frac{\sqrt{5}-1}{2}$$

Note: $\sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$
 $= \sin^{-1}x + \sin^{-1}y \quad (\text{if } x^2 + y^2 \leq 1)$
 $= \pi - (\sin^{-1}x + \sin^{-1}y) \quad (\text{if } x^2 + y^2 \geq 1)$

2. (B)

Sol. $f[g(x)] = x$
 $\Rightarrow f'[g(x)] \cdot [g'(x)] = 1$
 $\Rightarrow f'[g(2)] g'(2) = 12$
 $\Rightarrow f'(a) g'(2)$
putting $x = 2$
given, $f'(a) = \frac{a^{10}}{1+a^2} \Rightarrow g'(2) = \frac{1+a^2}{a^{10}}$

3. (A)

Sol. Let $\sqrt{a^2 - 1} = C_1$ and $a + \sqrt{a^2 - 1} = C_2 \Rightarrow$
 $C_2 - C_1 = a$
 $\therefore y = \frac{x}{C_1} - \frac{2}{C_1} \tan^{-1}\left(\frac{\sin x}{C_2 + \cos x}\right)$
 $= \frac{1}{C_1} - \frac{2}{C_1} \cdot \frac{C_2 \cos x + 1}{(C_2 + \cos x)^2 + \sin^2 x}$
 $\frac{dy}{dx} \Big|_{x=\frac{\pi}{2}} = \frac{1}{C_1} - \frac{2}{C_1} \cdot \frac{1}{C_2^2 + 1}$

$$= \frac{1}{C_1} \left[\frac{C_2^2 + 1 - 2}{C_2^2 + 1} \right] = \frac{1}{C_1} \cdot \frac{C_2^2 - 1}{C_2^2 + 1}$$

$$\therefore \frac{dy}{dx} \Big|_{x=\frac{\pi}{2}}$$

$$= \frac{2a^2 + 2a\sqrt{a^2 - 1} - 2}{\sqrt{a^2 - 1} (2a^2 - 1 + 2a\sqrt{a^2 - 1} + 1)}$$

$$= \frac{2a^2 + 2a\sqrt{a^2 - 1} - 2}{\sqrt{a^2 - 1} (2a(a + \sqrt{a^2 - 1}))}$$

$$= \frac{2(a^2 - 1) + 2a\sqrt{a^2 - 1}}{\sqrt{a^2 - 1} (2a(a + \sqrt{a^2 - 1}))}$$

$$= \frac{2\sqrt{a^2 - 1}(\sqrt{a^2 - 1} + a)}{(\sqrt{a^2 - 1})(2a)(a + \sqrt{a^2 - 1})} = \frac{1}{a} \text{ Ans.}$$

4. (D)

Sol. $y = \frac{(a+x) + \sqrt{a-x} \cdot \sqrt{a+x}}{(a-x) + \sqrt{a-x} \cdot \sqrt{a+x}}$
 $y = \frac{\sqrt{a+x} (\sqrt{a+x} + \sqrt{a-x})}{\sqrt{a-x} (\sqrt{a+x} + \sqrt{a-x})} = \left(\frac{a+x}{a-x}\right)^{1/2}$
 $\frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{a-x}{a+x}} \left(\frac{(a-x) + (a+x)}{(a-x)^2} \right)$
 $= \frac{1}{2} \sqrt{\frac{a-x}{a+x}} \times \frac{2a}{(a-x)^2};$
 $\therefore \frac{dy}{dx} \Big|_{x=0} = \frac{1}{a} \text{ Ans.}$

5. (C)

Sol. $\tan y = \frac{x \sin \alpha}{1 - x \cos \alpha}$

On differentiating wrt x, we get

$$\frac{dy}{dx} = \frac{\sin \alpha}{(1 - x \cos \alpha)}$$

$$\Rightarrow m = \sin \alpha, n = -\cos \alpha$$

$$m^2 + n^2 = 1$$

6. (B)

Sol. $\frac{x+a}{2} = b \cot^{-1}(b \ln y);$

$$\cot\left(\frac{x+a}{2b}\right) = b \ln y$$

$$\therefore -\operatorname{cosec}^2\left(\frac{x+a}{2b}\right) \frac{1}{2b} = \frac{b}{y} y'$$

$$\therefore -\frac{1}{2b^2} \left(1 + \cot^2\left(\frac{x+a}{2b}\right)\right) = \frac{y'}{y}$$

$$\therefore -\frac{1}{2b^2} \left((1 + (b \ln y))^2\right) = \frac{y'}{y};$$

$$\therefore -\frac{1}{2b^2} \left(2(b \ln y) \frac{b}{y} y'\right) = \frac{yy'' - y'^2}{y^2}$$

$$\therefore -\ln y y' = y y'' - y'^2;$$

$$\therefore y y'' = y'^2 - y' y \ln y$$

$$\therefore y y'' + y y' \ln y = y'^2 - y' y \ln y + y y' \ln y$$

$$= y'^2 \text{ Ans.}$$

7. (B)

Sol. $y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2} \Rightarrow x^2 dy + dx = 0$

$$\Rightarrow \frac{x^2}{\sqrt{1+x^4}} dy + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{\frac{1}{x^4}+1}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} + 3 = 3$$

8.

(B)

We have $\ln(3 \sin x - 4 \cos x + 7 + 5y) = (\sin x)y \dots \dots \dots (1)$

Put $x = \pi$ in equation (1), we get $\ln(11 + 5y) = 0 \Rightarrow 11 + 5y = 1 \Rightarrow 5y = -10$

$\therefore y = -2$. So, $(\pi, -2)$ lies on the given curve. Now on differentiating both the sides of equation (1) w.r.t. x, we get

$$\frac{1}{(3 \sin x - 4 \cos x + 7 + 5y)} \times \left(3 \cos x + 4 \sin x + 5 \frac{dy}{dx}\right)$$

$$= (\sin x) \frac{dy}{dx} + (\cos x) y$$

As $(\pi, -2)$ satisfy it, we get

$$1 \times \left(-3 + 0 + 5 \frac{dy}{dx}\right) = 2$$

$$\frac{dy}{dx} = \frac{5}{5} = 1. \text{ Ans.}]$$

9. (D)

Sol. $I = \lim_{x \rightarrow 2} \frac{-2x f'(x^2)}{(-1)} \Rightarrow$

$$I = \frac{-4 f'(4)}{(-1)} = 20 \text{ Ans.}]$$

10. (D)

Sol. Use L'Hospital's rule

$$\lim_{y \rightarrow 0} \left(\cos y - 1 + \frac{y^2}{2} \right) \frac{1}{y^4}$$

$$= \lim_{y \rightarrow 0} \left(\frac{2 \cos y - 2 + y^2}{2y^4} \right)$$

$$\lim_{y \rightarrow 0} \left(\frac{-2 \sin y + 2y}{8y^3} \right) = \lim_{y \rightarrow 0} \left(\frac{y - \sin y}{4y^3} \right)$$

$$= \lim_{y \rightarrow 0} \left(\frac{1 - \cos y}{12y^2} \right) = \frac{1}{24}]$$

11. (B)

Sol. $f(x)$

$$= \sqrt{(x-1)+4-4\sqrt{x-1}} + \sqrt{(x-1)+9-6\sqrt{x-1}}$$

$$= |\sqrt{x-1}-2| + |\sqrt{x-1}-3|$$

at $x = 1.5$

$$f(x) = (2 - \sqrt{x-1}) + (3 - \sqrt{x-1})$$

$$= 5 - 2\sqrt{x-1}$$

$$f'(x) = \frac{-2}{2\sqrt{x-1}}$$

$$f'(1.5) = \frac{-1}{\sqrt{\frac{3}{2}-1}} = -\sqrt{2}$$

12. (C)

Sol. $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$

$$\therefore f(x) = \sqrt{(\sqrt{x-2} + \sqrt{2})^2} + \sqrt{(\sqrt{x-2} - \sqrt{2})^2} = |\sqrt{x-2} + \sqrt{2}| + |\sqrt{x-2} - \sqrt{2}|$$

for $\sqrt{x-2}$ to exist $x \geq 2$

Also, $\sqrt{x-2} + \sqrt{2} > 0$ (always true, think why?)

but $\sqrt{x-2} - \sqrt{2} \geq 0$ only if $x \geq 4$

< 0 only if $x < 4$

\therefore now $f(x)$ becomes

$$f(x) = \sqrt{x-2} + \sqrt{2} - \sqrt{x-2} + \sqrt{2} \text{ for } 2 \leq x < 4$$

$$= \sqrt{x-2} + \sqrt{2} + \sqrt{x-2} - \sqrt{2} \text{ for } x \geq 4$$

$$\therefore f(x) = 2\sqrt{2}, \text{ for } 2 \leq x < 4$$

$$= 2\sqrt{x-2}, \text{ for } 4 \leq x < \infty$$

$\because f$ is continuous $[2, 4) \cup [4, \infty)$ (verify)

$$\therefore f'(x) = 0, 2 \leq x < 4$$

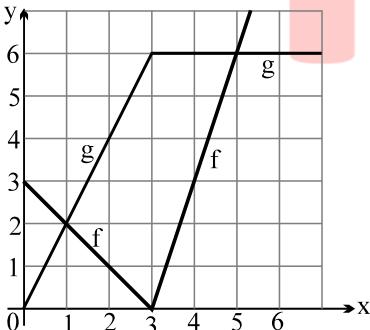
$$= \frac{1}{\sqrt{x-2}}, 4 \leq x < \infty$$

$$\therefore f'(102_+) = \frac{1}{\sqrt{102-2}} = \frac{1}{10}$$

$$\therefore 10 f'(102_+) = 1 \text{ Ans.]}$$

13.

(B)



Sol.

$$\begin{aligned} P'(x) &= f(x)g'(x) + g(x)f'(x) \\ P'(2) &= f(2)g'(2) + g(2)f'(2) \\ &= (1)(2) + 4(-1) \\ &= -2 \end{aligned}$$

$$Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

$$Q'(2) = \frac{(4)(-1) - (1)(2)}{16} = -\frac{6}{16} = -\frac{3}{8}$$

$$C'(x) = f'(g(x))g'(x)$$

$$C'(2) = f'(4) \cdot 2 = 3 \cdot 2 = 6$$

$$\therefore (P'(2) - C'(2)) Q'(2) = (-2 - 6) \left(\frac{-3}{8} \right)$$

$$= (-8) \left(\frac{-3}{8} \right) = 3$$

14.

Sol.

(D)

Let $f(x) = px^2 + qx + r$

$f(1) = f(-1)$ gives $p + q + r = p - q + r$

hence $q = 0$

Hence $f(x) = px^2 + r$

$$f'(x) = 2px \quad \dots(1)$$

Given a, b, c are in A.P.

hence $2pa, 2pb, 2pc$ will also be in A.P.

or $f'(a), f'(b), f'(c)$ will also be in A.P.

$\Rightarrow (D)$]

15.

Sol.

(C)

$$x \sin \alpha \cos y + x \cos \alpha \sin y = \sin y$$

$$x \sin \alpha \cot y + x \cos \alpha = 1$$

$$\cot y = \frac{1 - x \cos \alpha}{x \sin \alpha}$$

$$\tan y = \frac{x \sin \alpha}{1 - x \cos \alpha}$$

$$\sec^2 y \cdot \frac{dy}{dx} = \frac{\sin \alpha (1 - x \cos \alpha) - x \sin \alpha (-\cos \alpha)}{(1 - x \cos \alpha)^2}$$

$$\sec^2 y \cdot \frac{dy}{dx} = \frac{\sin \alpha - x \sin \alpha \cos \alpha + x \sin \alpha \cos \alpha}{(1 - x \cos \alpha)^2}$$

$$\sec^2 y \frac{dy}{dx} = \frac{\sin \alpha}{(1 - x \cos \alpha)^2}$$

$$m = \sin \alpha n = -\cos \alpha$$

$$m^2 + n^2 = 1$$

16.

Sol.

(B)

$$g' = \frac{dx}{dy} \text{ hence } 1 = 2x \frac{dx}{dy} - \frac{dx}{dy} \Rightarrow \frac{dx}{dy} =$$

$$\frac{1}{2x-1}; \text{ when } f(x) = 14$$

$$\Rightarrow x^2 - x - 20 = 0 \Rightarrow x = 5 \text{ or } -4 \text{ (rejected)}$$

$$\Rightarrow g' = \frac{1}{2.5-1} = \frac{1}{9}]$$

17.

Sol.

(A)

$$y = (A + Bx)e^{mx} + (m-1)^{-2} \cdot e^x$$

$$y \cdot e^{-mx} = (A + Bx) + (m-1)^{-2} \cdot e^{(1-x)x}$$

$$e^{-mx} \cdot y_1 - my + e^{-mx} = B - (m-1)^{-1} \cdot e^{-(m-1)x}$$

$$e^{-mx} \cdot y_2 - y_1 e^{-mx} \cdot m - m[e^{-mx} \cdot y_1 - y e^{-mx}] = e^{-(m-1)x}$$

$$e^{-mx} \cdot y_1 - m_2 y_1 e^{-mx} + my \cdot e^{-mx} = e^{-(m-1)x}$$

$$y_2 - 2my_1 + my = e^x \text{ Ans.]}$$

18.

Sol.

(D)

$$\ln(2a - a^2) \geq 0 \Rightarrow 2a - a^2 \geq 1$$

$$0 \geq (a-1)^2 \text{ or } (a-1)^2 \leq 0 \Rightarrow a = 1, \text{ hence } f(x) \text{ can be defined only when } a = 1.$$

Now $f'(x) = 12x^2 - 12x \cos 2 + 3 \sin 2 \sin 6$

$$f'\left(\frac{1}{2}\right) = 3 - 6 \cos 2 + 3 \sin 2 \sin 6 = 3(1 + \sin 2 \sin 6) - 6 \cos 2.$$

Note that $\cos 2 < 0$ and $1 + \sin 2 \sin 6 > 0$

$\Rightarrow D]$

- 19.** (D)
Sol. Degree of $f(x) = n$; degree of $f'(x) = n - 1$
degree of $f''(x) = (n - 2)$
hence $n = (n - 1) + (n - 2) = 2n - 3$
 $\therefore n = 3$
hence $f(x) = ax^3 + bx^2 + cx + d$,
 $(a \neq 0)$
 $f'(x) = 3ax^2 + 2bx + c$
 $f''(x) = 6ax + 2b$
 $\therefore ax^3 + bx^2 + cx + d = (3ax^2 + 2bx + c)(6ax + 2b)$
 $\therefore 18a^2 = a \Rightarrow a = \frac{1}{18}$ Ans.]

- 20.** (D)
Sol. We have $g(x) = x^3 \ln(x^2 f(x))$
 \therefore On differentiating b.t.s. w.r.t. x, we get

$$g'(x) = \frac{x^3}{x^2 f(x)} (x^2 f'(x) + 2x f(x)) + 3x^2 \ln(x^2 f(x))$$

Hence $g'(2) = \frac{2}{f(2)}$

$$(4f'(2) + 4f(2)) + 12 \ln(4f(2))$$

 $= 8\left(4 \times -3 + 4 \times \frac{1}{4}\right) + 12 \ln\left(4 \times \frac{1}{4}\right) = 8(-12 + 1) = -88$. Ans.]

- 21.** 6
Sol. $g(x) = f(-x + f(f(x)))$; $f(0) = 0$;
 $f'(0) = 2 g'(x) =$
 $f'(-x + f(f(x))) \cdot [-1 + f'(f(x)) \cdot f'(x)]$
 $g'(0) = f'(f(0)) \cdot [-1 + f'(0) \cdot f'(0)]$
 $= f'(0)[-1 + (2)(2)]$
 $= (2)(3) = 6$ Ans.]

- 22.** 0
Sol. $y = 2 \ln(1 + \cos x)$
 $y_1 = \frac{-2 \sin x}{1 + \cos x}$
 $y_2 = -2 \left[\frac{(1 + \cos x) \cos x - \sin x (-\sin x)}{(1 + \cos x)^2} \right]$
 $= -2 \left[\frac{\cos x + 1}{(1 + \cos x)^2} \right] = \frac{-2}{(1 + \cos x)}$
 $\therefore 2e^{-y/2} = 2 \cdot e^{-\frac{\ln(1+\cos x)^2}{2}} = \frac{2}{(1 + \cos x)}$
 $\therefore y_2 + \frac{2}{e^{y/2}} = 0$ Ans.]

- 23.** 1
Sol. We have: $\log(x + y) - 2xy = 0 \dots (1)$
Diff. w.r.t. x, $\frac{1}{x+y} \left(1 + \frac{dy}{dx}\right) - 2x \frac{dy}{dx} - 2y = 0$
 $\Rightarrow \left(\frac{1}{x+y} - 2x\right) \frac{dy}{dx} = 2y - \frac{1}{x+y}$
 $\Rightarrow \frac{dy}{dx} = \frac{2y(x+y) - 1}{1 - 2x(x+y)}.$
When $x = 0$, from (1), $\log(y) = 0 \Rightarrow y = e^0 = 1$.
 $\frac{dy}{dx} \Big|_{x=0} = \frac{2(1)(0+1)-1}{1-0} = \frac{1}{1} = 1$.
- 24.** 3
Sol. $(a-1)x^2 = x(2b+3)$
The above equation is satisfied by three distinct values of x therefore it is an identity
 $\therefore 2 - 2a = 0 \Rightarrow a = 1$ and $2b + 3 = 0$
 $\Rightarrow b = \frac{-3}{2}$
Now, $f(x) = 2x + 1$,
Let $g(x) = px + q \Rightarrow g'(x) = p$
 $f(g(x)) = 6x - 7 \Rightarrow 2(px + q) + 1 = 6x - 7$
 $\Rightarrow 2px + 2q + 1 = 6x - 7$
 $\Rightarrow 2p = 6 \Rightarrow p = 3$ and $q = -4$
 $\therefore g'(2012) = 3$ Ans.]
- 25.** 17
Sol. We have $\frac{dy}{dx} = 5x^4 (\cos(\ln x) + \sin(\ln x)) + x^5 \left(\frac{-\sin(\ln x)}{x} + \frac{\cos(\ln x)}{x} \right)$,
 $\Rightarrow xy_1 = 5y + x^5 (\cos(\ln x) - \sin(\ln x))$
 $\Rightarrow xy_2 + y_1 = 5y_1 + 5x^4$
 $(\cos(\ln x) - \sin(\ln x)) + x^5 \left(\frac{-\sin(\ln x)}{x} - \frac{\cos(\ln x)}{x} \right)$
 $\Rightarrow x^2 y_2 + xy_1 = 5xy_1 + 5x^5 (\cos(\ln x) - \sin(\ln x)) -$
 $x^5 (\sin(\ln x) + \cos(\ln x))$
 $\Rightarrow x^2 y_2 - 4xy_1 = 5(xy_1 - 5y) - y$
 $\Rightarrow x^2 y_2 - 4xy_1 = 5xy_1 - 26y$
 $\Rightarrow x^2 y_2 - 9xy_1 + 26y = 0$
 $\equiv x^2 y_2 + axy_1 + by = 0$
 $\therefore a = -9$ and $b = 26$
Hence $(a + b) = 17$ Ans.]

26. 4

Sol. When $f(x) = \pi$, then $x = \frac{\pi}{2}$.

[As $f(x)$ is an increasing function on \mathbb{R} , so $f(x)$ is invertible.]

We have to find $\frac{dx}{dy}$ at $y = \pi$.

$$\text{Now } \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} \text{ and } \frac{dy}{dx}$$

$$= 6(2x - \pi)^2 + 2 + \sin x.$$

$$\text{Now } \left. \frac{dy}{dx} \right|_{x=\pi/2} = 0 + 2 + 1 = 3.$$

$$\text{Hence } \left. \frac{dx}{dy} \right|_{y=\pi} = \left. \frac{1}{\frac{dy}{dx}} \right|_{x=\pi/2} = \frac{1}{3} = \frac{p}{q}.$$

$$\therefore p = 1 \text{ and } q = 3$$

$$\text{Hence } (p+q) = 4. \text{ Ans.}$$

27. 8

Sol. $f(x) = 2 \tan^{-1} x$ & $g(x) = x + 2 \Rightarrow f(g(x)) = 2 \tan^{-1}(x+2)$ solution of inequality

$f^2(g(x)) - 5f(g(x)) + 4 > 0$ is

$f(g(x)) < 1$ or $f(g(x)) > 4 \Rightarrow \tan^{-1}(x+2) < \frac{1}{2}$ or $\tan^{-1}(x+2) > 2$

$$\Rightarrow \tan^{-1}(x+2) < \frac{1}{2} \quad [\text{As } \tan^{-1}(x+2) < \frac{\pi}{2}]$$

$$\text{or } x+2 < \tan\left(\frac{1}{2}\right)$$

$$\Rightarrow x \in \left(-10, \tan\left(\frac{1}{2}\right) - 2\right)$$

$$\text{As } \frac{1}{2} < \frac{\pi}{6} \Rightarrow \tan\frac{1}{2} < \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\frac{1}{2} - 2 < \frac{1}{\sqrt{3}} - 2$$

Hence total integer in the range are $\{-9, -8, -7, -6, -5, -4, -3, -2\} \Rightarrow 8 \text{ integer}\}$

28. 48

Sol. As $f(x)$ is derivable at $x = 0$, so $f(x)$ is also continuous at $x = 0$.

$$\begin{aligned} \therefore f(0^+) &= \lim_{h \rightarrow 0} \frac{\ln(1-ch)}{h} \quad \left(\frac{0}{0} \right) \\ &= \lim_{h \rightarrow 0} \frac{-c \times \ln(1-ch)}{-ch} = -c \\ &\Rightarrow -c = 2 \Rightarrow c = -2 \quad \dots\dots\dots(1) \end{aligned}$$

$$\text{Now } f'(0^+) = \lim_{h \rightarrow 0} \frac{\ln(1+2h) - 2}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\ln(1+2h) - 2h}{h^2}$$

$$= \lim_{h \rightarrow 0} \frac{\left(2h - \frac{(2h)^2}{2} + \dots\dots\dots\right) - 2h}{h^2} = -2$$

$$\text{Also } f'(0^-) = \lim_{h \rightarrow 0} \frac{a \cot^{-1}\left(\frac{b-h}{4}\right) - 2}{-h} \quad \left(\frac{0}{0} \right)$$

$$= \lim_{h \rightarrow 0} \frac{\frac{-a}{1+\left(\frac{b-h}{4}\right)^2} \times \frac{-1}{4}}{-1} = \frac{-4a}{b^2+16}$$

$$\text{As } f'(0^-) = f'(0^+), \text{ so } \frac{-4a}{b^2+16} = -2$$

$$\Rightarrow 2a = b^2 + 16$$

$$\therefore b^2 - 2a = -16 \quad \dots\dots\dots(2)$$

Using equation (1) and equation (2)

$$\text{Hence } (b^2 - 2a + c^6) = -16 + 64 = 48 \text{ Ans.}$$

9

$$y^{2/3} - 2xy^{1/3} + 1 = 0$$

$$y^{1/3} = \left(x \pm \sqrt{x^2 - 1} \right)$$

$$\Rightarrow \ln y = 3 \ln \left(x \pm \sqrt{x^2 - 1} \right)$$

$$\frac{y_1}{y} = \frac{\pm 3}{\sqrt{x^2 - 1}} \Rightarrow (x^2 - 1)y_1^2 = 9y^2$$

$$2x y_1^2 + (x^2 - 1)2y_1 y_2 = 18yy_1$$

$xy_1 + (x^2 - 1)y_2 = 9y$ (As y_1 is not equal to 0, because y is not constant)

Dividing by y , we get

$$\therefore x \frac{y_1}{y} + (x^2 - 1) \frac{y_2}{y} = 9 \text{ Ans.}$$

30. 8

$$\ln(x+y) = 2xy$$

$$x = 0, y = 1$$

$$\frac{1+y'}{x+y} = 2(xy' + y)$$

$$\text{Put } x = 0, y = 1$$

$$1 + y' = 2(0 + 1) = 2$$

$$\Rightarrow y' = 1$$

$$\frac{(x+y)y'' - (1+y')^2}{(x+y)^2} = 2(xy'' + 2y')$$

$$x = 0, y = 1, y' = 1$$

$$\frac{y'' - 4}{1} = 2(0 + 2) = 4$$

$$\Rightarrow y''(0) = 8 \quad]$$