JEE MAIN : CHAPTER WISE TEST PAPER-6

SUBJECT :- MATHEMATICS   DATE...

CLASS :- 12th   NAME..

CHAPTER :- METHOD OF DIFFERENTIATION   SECTION......................................#...

(SECTION-A)

1. If $y = \sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right)$ & $\dfrac{dy}{dx} =$

$\dfrac{1}{2\sqrt{x(1-x)}} + p$, then $p =$

(A) 0   (B) $\sin^{-1} x$

(C) $\sin^{-1}\sqrt{x}$ (D) none of these

2. Let g is the inverse function of f and $f'(x) = \dfrac{x^{10}}{\left(1+x^2\right)}$. If $g(2) = a$ then $g'(2)$ is equal to

(A) $\dfrac{5}{2^{10}}$ (B) $\dfrac{1+a^2}{a^{10}}$

(C) $\dfrac{a^{10}}{1+a^2}$ (D) $\dfrac{1+a^{10}}{a^2}$

3. If $y =$

$\dfrac{x}{\sqrt{a^2-1}} - \dfrac{2}{\sqrt{a^2-1}}\tan^{-1}\left(\dfrac{\sin x}{a+\sqrt{a^2-1}+\cos x}\right)$

where $a \in (-\infty, -1) \cup (1, \infty)$

then $y'\left(\dfrac{\pi}{2}\right)$ equals

(A) $\dfrac{1}{a}$ (B) $\dfrac{2}{a}$

(C) $\dfrac{1}{2a}$ (D) a

4. If $f(x) = \dfrac{a+\sqrt{a^2-x^2}+x}{\sqrt{a^2-x^2}+a-x}$ where a > 0 and x < a, then $f'(0)$ has the value equal to

(A) $\sqrt{a}$ (B) a

(C) $\dfrac{1}{\sqrt{a}}$ (D) $\dfrac{1}{a}$

5. If $x\sin(\alpha + y) = \sin y$ and $y' = \dfrac{m}{(x^2+2nx+1)}$, then

(A) m − n = 1   (B) m + n = 1

(C) m² + n² = 1   (D) m = n

6. If $\dfrac{x+a}{2} = b\cot^{-1}(b\ \ell n\ y)$, b > 0 then, value of $yy'' + yy'\ \ell n\ y$ equals

(A) y'   (B) y'^2 (C) 0   (D) 1

7. If $y = \dfrac{1}{x}$, then the value of $\dfrac{dy}{\sqrt{1+y^4}} + \dfrac{dx}{\sqrt{1+x^4}} + 3$ is equal to-

(A) 0   (B) 3   (C) 4   (D) −3

8. If $\ell n(3\sin x - 4\cos x + 7 + 5y) = (\sin x)\ y$, then $y'(\pi)$ is equal to

(A) $\dfrac{5}{3}$ (B) 1   (C) $\dfrac{3}{5}$ (D) none

9. If f is differentiable in (0, 6) & $f'(4) = 5$ then $\underset{x\to 2}{\text{Limit}}\ \dfrac{f(4)-f\left(x^2\right)}{2-x} =$

(A) 5   (B) 5/4   (C) 10   (D) 20

10. $\underset{x\to\infty}{\text{Lim}}\ x^4\cdot\left(\cos\dfrac{1}{x} - 1 + \dfrac{1}{2x^2}\right)$ has the value equal to :

(A) 1/6   (B) 1/12   (C) 1/8   (D) 1/24

11. If $f(x) = \sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}}$ then $f'(x)$ at x = 1.5 is

(A) 0   (B) $-\sqrt{2}$ (C) $-\sqrt{3}$ (D) −4

12. If $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$, then the value of $10f'(102^+)$

(A) is − 1   (B) is 0

(C) is 1   (D) does not exist

13. If f and g are the functions whose graphs are shown, let $P(x) = f(x)\ g(x)$, $Q(x) = \dfrac{f(x)}{g(x)}$ and $C(x) = f(g(x))$. The value of $\left(P'(2) - C'(2)\right)Q'(2)$ equals

(A) 3/2   (B) 3   (C) − 3   (D) − 6

PG #1
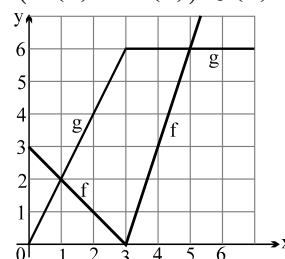
14. Let f (x) be a polynomial function of second degree. If f (1) = f (–1) and a, b, c are in A.P., then f '(a), f '(b) and f '(c) are in
(A) G.P. (B) H.P. (C) A.G.P. (D) A.P.

15. If x sin (α + y) = sin y and

$\sec^2 y \dfrac{dy}{dx} = \dfrac{m}{(x^2 + 2nx + 1)}$. Then

(A) m – n = 1 (B) m + n = 1
(C) m² + n² = 1 (D) m = n

16. Let $f(x) = x^2 - x - 6$, $x > \dfrac{1}{2}$ & let g be the inverse of f . The value of g′ when f(x) = 14 is

(A) 1/4 (B) 1/9 (C) $\dfrac{1}{3\sqrt{5}}$ (D) $\dfrac{2}{3\sqrt{5}}$

17. If $y = (A + Bx) e^{mx} + (m - 1)^{-2} e^x$ then $\dfrac{d^2 y}{dx^2}$ –

$2m \dfrac{dy}{dx} + m^2 y$ is equal to :
(A) eˣ (B) e^{mx} (C) e^{-mx} (D) $e^{(1-m)x}$

18. Given: $f(x) = 4x^3 - 6x^2 \cos 2a + 3x \sin 2a$. sin 6a + $\sqrt{\ell n (2a - a^2)}$ then
(A) f(x) is not defined at x = 1/2
(B) f ′ (1/2) < 0
(C) f ′(x) is not defined at x = 1/2
(D) f ′ (1/2) > 0

19. A non zero polynomial with real coefficients has the property that f (x) = f ' (x) · f ''(x). The leading coefficient of f (x) is

(A) $\dfrac{1}{6}$ (B) $\dfrac{1}{9}$ (C) $\dfrac{1}{12}$ (D) $\dfrac{1}{18}$

20. Let $g(x) = x^3 \ln(x^2 f(x))$ where f(x) is a differentiable positive function on (0, ∞) satisfying $f(2) = \dfrac{1}{4}$ and f '(2) = – 3, then g'(2) equals
(A) 77 (B) – 77 (C) 88 (D) – 88

(SECTION-B)

21. Suppose that f (0) = 0 and f ' (0) = 2, and let g (x) = $f(-x + f(f(x)))$. The value of g ' (0) is equal to

22. Let $y = \ln(1 + \cos x)^2$ then the value of $\dfrac{d^2 y}{dx^2} + \dfrac{2}{e^{y/2}}$ equals

23. If y is a function of x and log (x + y) – 2xy = 0, then y'(0) is equal to -

24. Let the equation $(a - 1)x^2 = x(2b + 3)$ be satisfied by three distinct values of x, where a, b ∈ R. If $f(x) = (a - 1)x^3 + (2b + 3)x^2 + 2x + 1$, and $f(g(x)) = 6x - 7$ where g (x) is a linear function then find the value of g ' (2012).

25. If $y = x^5 (\cos(\ln x) + \sin(\ln x))$, then find the value of (a + b) in the relation x²y₂ + axy₁ + by = 0.

26. Let $f(x) = (2x - \pi)^3 + 2x - \cos x$. If the value of $\dfrac{d}{dx}(f^{-1}(x))$ at x = π can be expressed in the form of $\dfrac{p}{q}$ (where p and q are natural numbers in their lowest form), then find the value of (p + q).

27. Let $f(x) = 2\tan^{-1} x$ and g(x) be a differentiable function satisfying

$g\left(\dfrac{x + 2y}{3}\right) = \dfrac{g(x) + 2g(y)}{3}$ ∀ x, y ∈ R

and g'(0) = 1 , g(0) = 2.

Find the number of integers satisfying $f^2(g(x)) - 5f(g(x)) + 4 > 0$ where x ∈ (– 10, 10).

28. Let $f(x) = \begin{cases} a \cot^{-1}\left(\dfrac{b+x}{4}\right), & \dfrac{-2}{3} < x < 0 \\ 2, & x = 0 \\ \dfrac{\ln(1 - cx)}{x}, & 0 < x < \dfrac{2}{3} \end{cases}$

If the function f (x) is differentiable at x = 0, then find the value of (b² – 2a + c⁶).

29. If $2x = \left(y^{\frac{1}{3}} + y^{\frac{-1}{3}}\right)$, then find the value of

$\dfrac{(x^2 - 1)}{y} \cdot \dfrac{d^2 y}{dx^2} + \dfrac{x}{y} \cdot \dfrac{dy}{dx}$.

30. If y is a function of x and $\ln(x + y) = 2xy$, then find the value of y" (0).