JEE MAIN ANSWER KEY & SOLUTIONS

SUBJECT :- MATHEMATICS
CLASS :- 12th **PAPER CODE :- CWT-5**
CHAPTER :- LIMIT CONTINUITY DIFFERENTIABILITY

ANSWER KEY

1.	(A)	**2.**	(B)	**3.**	(C)	**4.**	(A)	**5.**	(A)	**6.**	(C)	**7.**	(A)
8.	(C)	**9.**	(C)	**10.**	(A)	**11.**	(A)	**12.**	(A)	**13.**	(B)	**14.**	(D)
15.	(D)	**16.**	(B)	**17.**	(B)	**18.**	(B)	**19.**	(C)	**20.**	(A)	**21.**	**6**
22.	**5**	**23.**	**4**	**24.**	**1**	**25.**	**8**	**26.**	**3**	**27.**	**0**	**28.**	**0**
29.	2	**30.**	**4**										

SOLUTIONS

1. (A)

Sol. $$\lim_{x\to\infty}\frac{\cot^{-1}\left(\frac{\log_a x}{x^a}\right)}{\sec^{-1}\left(\frac{a^x}{\log_a x}\right)}$$;

as $\lim_{x\to\infty}\left(\frac{\log_a x}{x^a}\right) \to 0$

and $\left(\frac{a^x}{\log_a x}\right) \to \infty$ (using L'opital rule)

$\therefore$ $l = \frac{\pi/2}{\pi/2} = 1$

2. (B)

Sol. Given, $\lim_{x\to 0}\frac{\sin(ax)+bx}{x^3} = 36$

$\Rightarrow \lim_{x\to 0}\frac{\left(ax-\frac{a^3\cdot x^3}{3!}+\frac{a^5\cdot x^5}{5!}-........\infty\right)+bx}{x^3}$ = 36

$\Rightarrow \lim_{x\to 0}\frac{\left((a+b)x-\frac{a^3}{6}\cdot x^3+\frac{a^5}{120}x^5.........\infty\right)}{x^3}$ = 36.

So, a + b = 0.

Also, $\frac{-a^3}{6} = 36 \Rightarrow a = -6$ and b = 6. **Ans.**

3. (C)

Sol. As, (x + sin x – x cos x – tan x)

= x (1 – cos x) + sin x $\left(1-\frac{1}{\cos x}\right)$

= x (1 – cos x) – tan x (1 – cos x) = (x – tan x) · (1 – cos x)

So, $\lim_{x\to 0}\frac{\left(\frac{x-\tan x}{x^3}\right)\left(\frac{1-\cos x}{x^2}\right)}{x^{n-5}}$

= exist and non-zero,

so, n = 5. **Ans.**

4. (A)

Sol. We have

$\lim_{n\to\infty}\left(\frac{1^2+2^2+3^2+......+n^2}{n^3+1}\right)$

$= \lim_{n\to\infty}\left(\frac{n\cdot(n+1)\cdot(2n+1)}{6\cdot(n^3+1)}\right) = \frac{2}{6} = \frac{1}{3}$.

Also, $\beta = \lim_{x\to 0}\frac{\sin 2x}{\sin 8x}$

$= \lim_{x\to 0}\frac{2x\cdot\left(\frac{\sin 2x}{2x}\right)}{3x\cdot\left(\frac{\sin 8x}{8x}\right)} = \frac{2}{8} = \frac{1}{4}$.

Now, $\alpha + \beta = \frac{1}{3}+\frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$.

Also, $\alpha\beta = \left(\frac{1}{3}\right)\cdot\left(\frac{1}{4}\right) = \frac{1}{12}$.

So, required quadratic equation is

$x^2-\frac{7}{12}\cdot x+\frac{1}{12}=0 \Rightarrow 12x^2 - 7x + 1 = 0$.

Ans.

5. (A)

Sol. $f(1^-) = f(1)$ $\Rightarrow$ a + 9 = 2 + b

$\Rightarrow$ b – a = 7 .....(1)

Also L.H.D. of f at x = 1 = slope of line 2x – y + b = 0

$\Rightarrow$ 3 + 4 + a = 2

$\Rightarrow$ a = –5 $\Rightarrow$ b = 2.

Therefore (a + b) = –3 **Ans.**

6. (C)

Sol. f(x) = sgn $\left((3\cos^{-1}x-\pi)(x-2)\right)$

= sgn (x) is discontinuous at x = 0

$\Rightarrow$ f(x) is discontinuous at x = cos $\frac{\pi}{3}$ only

Since x = 2 $\notin D_f$ & no graph of f exist in viccinity of x = 2.

$\therefore$ Only one point of discontinuity i.e. 1 **Ans.**

7. (A)

Sol. g(x) = $\dfrac{\sin\frac{\pi[x]}{4}}{[x]}$

obv. cont. at x = 3/2

at x = 2, f(2⁻) = sin $\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

f(2) = $\dfrac{\sin\frac{\pi}{2}}{2} = \frac{1}{2}$

Hence discontinuous at x = 2

8. (C)

Sol. f (x) = $\begin{cases} 4x^2-2x & \text{if } -\frac{1}{2}\le x<0 \\ ax^2-bx & \text{if } 0\le x<\frac{1}{2}\end{cases}$

f ' (0⁻) = $\lim\limits_{x\to0} 8x-2=-2$

f ' (0⁺) = $\lim\limits_{x\to0} 2ax-b=-b$

hence b = 2 $\forall$ x $\in$ R

obvious f is continuous $\forall$ a $\in$ R $\Rightarrow$ **(C)**

9. (C)

Sol. The equation

$\left(\sin\theta-\frac{\sqrt{3}}{2}\right)x^2+\left(\cos\theta-\frac{1}{2}\right)x+\left(\tan\theta-\sqrt{3}\right)$

= 0 must be an identity in x.

$\therefore$ sin θ = $\frac{\sqrt{3}}{2}$ and cos θ = $\frac{1}{2}$ and tan θ = $\sqrt{3}$ $\Rightarrow$ θ = $\frac{\pi}{3}$ or $\frac{-5\pi}{3}$ in [– 2π, 2π]. **Ans.**]

10. (A)

Sol. for continuous of f $x_0^2 = ax_0 + b$ ....(1)

for derivability, $2x_0 = a$ ....(2)

$\therefore$ $a = 2x_0$ and $b = -x_0^2$ $\Rightarrow$ **(A)**]

11. (A)

Sol. $\lim\limits_{h\to0}\left(\frac{f(x+h)}{f(x)}\right)^{\frac{1}{h}}$ = $e^{(\tan x)\, f(-x)}$ (Given)

$e^{\lim\limits_{h\to0}\frac{1}{h}\left(\frac{f(x+h)-f(x)}{f(x)}\right)}$ = $e^{(\tan x)\, f(-x)}$

$e^{\frac{f'(x)}{f(x)}}$ = $e^{(\tan x)\, f(-x)}$

$\therefore$ $\frac{f'(x)}{f(x)}$ = (tan x) f (– x)

Put x = 0 $\Rightarrow$ $\frac{f'(0)}{f(0)}$ = 0

$\therefore$ f ' (0) = 0

$\therefore$ f ' (0) = 0 **Ans.**]

12. (A)

Sol. f ′ (x) = $\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$

= $\lim\limits_{h\to0}\frac{f\left(x.\left(1+\frac{h}{x}\right)\right)-f(x)}{h}$

= $\lim\limits_{h\to0}\frac{f(x)+f\left(1+\frac{h}{x}\right)-f(x)}{h}$

= $\lim\limits_{h\to0}\frac{f\left(1+\frac{h}{x}\right)}{x\frac{h}{x}}$

= $\frac{1}{x}\lim\limits_{t\to0}\frac{f(1+t)-f(1)}{t}$ (note that f(1) = 0) = $\frac{f'(1)}{x}$ Ans.

13. (B)

Sol. If function is differentiable in R, then it must be differentiable at x_0 = a.

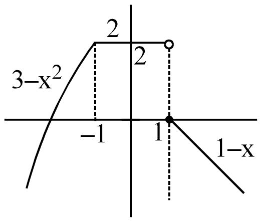
f '(a⁺) = $\lim\limits_{h\to0}\frac{(a+h+1)|h|-0}{h}$ = a + 1

f '(a⁻) = $\lim\limits_{h\to0}\frac{(a-h+1)|-h|}{-h}$ = – (a + 1)

$\therefore$ f '(a⁺) = f '(a⁻) $\Rightarrow$ a = – 1 **Ans.**]

14. (D)

Sol.



15. (D)

Sol. By definition of continuity of f (x) at x = 1,

$f(1^-) = f(1^+) = f(1) \Rightarrow p + q = -2 + \frac{\pi}{4}$ ……(i)

Also, by definition of differentiability of f(x) at x = 1,

$f'(1^-) = f'(1^+) \Rightarrow 3p = \frac{1}{2} \Rightarrow p = \frac{1}{6}$ ……(ii)

∴ Using (ii) in (i), we get

$q = -2 - \frac{1}{6} + \frac{\pi}{4} \Rightarrow q = \frac{\pi}{4} - \frac{13}{6}$ **Ans.**

16. (B)

Sol. $f(0^-) = (P + 1) + 1 = P + 2$

$f(0) = q$

$f(0^+) = \lim_{x \to 0^+} \frac{\sqrt{1+x} - 1}{x}$

$= \lim_{x \to 0^+} \frac{1}{\sqrt{1-x} + 1} = \frac{1}{2}$

$q = \frac{1}{2}$; P + 2 = q $\Rightarrow P = \frac{-3}{2}$

[for continuous f^n : $f(0^-) = f(0) = f(0^+)$]

17. (B)

Sol. f (x) = [x] tan (πx)

$f'(k^+) = \lim_{h \to 0} \frac{f(k+h) - f(k)}{h}$

$= \lim_{h \to 0} \frac{[k+h]\tan(\pi(k+h)) - k\tan(\pi k)}{h}$

$= \lim_{h \to 0} \frac{k\tan(\pi k + \pi h)}{h} = \lim_{h \to 0} \frac{k\tan(\pi h)}{h}$

$= k\pi$

18. (B)

Sol. $\lim_{n \to \infty} \left(\sqrt{2n^2 + n} - \lambda\sqrt{2n^2 - n} \right) = \frac{1}{\sqrt{2}}$

Here, λ > 0.

So, $\lim_{n \to \infty} \frac{(2n^2 + n) - \lambda^2(2n^2 - n)}{\left(\sqrt{2n^2 + n} + \lambda\sqrt{2n^2 - n}\right)}$

$= \lim_{n \to \infty} \frac{2n^2(1 - \lambda^2) + n(1 + \lambda^2)}{n\left[\sqrt{2 + \frac{1}{n}} + \lambda\sqrt{2 - \frac{1}{n}}\right]}$

For existence of limit λ = 1 as λ > 0

and *l* = $\frac{2}{\sqrt{2} + \sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \lambda = 1$. **Ans.**

19. (C)

Sol. $\lim_{x \to \frac{3\pi}{4}} \left(\frac{\tan^3 x - 2\tan x - 1}{\tan^5 x - 2\tan x - 1} \right) \left(\frac{0}{0} \text{ form} \right) =$

$\lim_{x \to \frac{3\pi}{4}} \left(\frac{(\tan x + 1)(\tan^2 x - \tan x - 1)}{(\tan x + 1)(\tan^4 x - \tan^3 x + \tan^2 x - \tan x - 1)} \right)$

$= \frac{(-1)^2 - (-1) - 1}{(-1)^4 - (-1)^3 + (-1)^2 - (-1) - 1} = \frac{1}{3}$

Ans.

20. (A)

Sol. $\lim_{x \to 2} f(x)$ must exist = f (2)

$f(2) = \lim_{x \to 2} \frac{\tan 2\pi x + \sin\frac{\pi x}{2} + \tan\frac{\pi x}{2}}{(x+6)(x-2)}$

put x = 2 + h

$= \frac{1}{8} \lim_{x \to 2} \frac{\tan 2\pi h - \sin\frac{\pi h}{2} + \tan\frac{\pi h}{2}}{h}$

$= \frac{1}{8}\left[2\pi - \frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{\pi}{4}$ Ans.

21. **6**

Sol. Clearly number of points of discontinuity of f (x) in $\left[0, \frac{\pi}{2}\right]$ will be 10

and number of points of non-differentiability of g(x) = (x)

= $(x-1)(x-2)\left|(x-1)(x-2).......(x-2m)\right|$ is (2m – 2).

$\therefore$ 2m – 2 = 10 $\Rightarrow$ m = 6 **Ans.**

22. **5**

Sol. $I = \lim_{p\to\infty} p\,\ell n\left(\frac{e(1+(1/p))}{(1+(1/x))^p}\right)$ ($\infty \times 0$ form)

$= \lim_{p\to\infty} p\left(1+\ell n\left(1+\frac{1}{p}\right)-p\ell n\left(1+\frac{1}{p}\right)\right)$

put x = $\frac{1}{t}$; as p $\to \infty$, t $\to$ 0

Hence $I = \lim_{t\to 0}\frac{1}{t}\left(1+\ell n(1+t)-\frac{\ell n(1+t)}{t}\right)$

$= \lim_{t\to 0}\left[\ell n(1+t)^{1/t}+\frac{t-\ell n(1+t)}{t^2}\right]$

$= 1 + \lim_{y\to 0}\left(\frac{e^y-1-y}{y^2}\right)$ where $ln(1 + t) = y$; 1 + t = e^y, hence t = e^4 – 1

$= 1 + \frac{1}{2} = \frac{3}{2} = \frac{m}{n}$

$\Rightarrow$ (m + n) = 5 **Ans.**

23. **4**

Sol. f(x) =

$\lim_{n\to\infty} ln\left(e^{\frac{\cos x}{2}}\cdot e^{\frac{3\cos x}{2^2}}\cdot e^{\frac{5\cos x}{2^3}}\ ......\ e^{\frac{(2n+1)\cos x}{2^n}}\right)$

$= \lim_{n\to\infty}\left(\frac{\cos x}{2}+\frac{3\cos x}{2^2}+\frac{5\cos x}{2^3}+......+\frac{(2n+1)\cos x}{2^n}\right)$

f (x) =

$\frac{\cos x}{2}+\frac{3\cos x}{2^2}+\frac{5\cos x}{2^3}+......\infty$

$\frac{1}{2}f(x) = \frac{\cos x}{2^2}+\frac{3\cos x}{2^3}+......+\infty$

$\frac{1}{2}f(x) = \frac{\cos x}{2}+\frac{2\cos x}{2^2}+\frac{2\cos x}{2^3}+......\infty$

f (x) = cos x + cos x + $\frac{\cos x}{2}$ + $\frac{\cos x}{2^2}$ + ... ∞

= cos x + $\frac{\cos x}{1-\frac{1}{2}}$ = 3 cos x

g(x) = $\left[\frac{1}{3}f(x)\right]$ = [cos x]

Hence, number of values of x in [0, 2π] where [cos x] is discontinuous is 4, i.e. 0, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, 2π. **Ans.**

24. **1**

Sol. $\lim_{x\to 0^+}\frac{e^{(x-2)ln\sqrt{5^x}+\frac{1}{3x}\log_2 2^{6x}}-e^2}{(x-2)\tan x}$

$\Rightarrow \lim_{x\to 0^+}\frac{e^{(x-2)ln\left(5^{x/2}\right)+2}-e^2}{(x-2)\tan x}$

$\lim_{x\to 0^+}\frac{e^2\left(e^{ln\left(5^{\left(\frac{x(x-2)}{2}\right)}\right)}-1\right)}{(x-2)\tan x}$

$= \lim_{x\to 0^+} e^2\frac{\left(5^{\left(\frac{x(x-2)}{2}\right)}-1\right)}{\frac{2x(x-2)}{2}\cdot\frac{\tan x}{x}} = \frac{1}{2}e^2\, ln\,5$

$\lim_{x\to 0^-}\lambda\frac{e^2\left(\left(\sqrt{5}\right)^x-1\right)\left(e^{x^2}-1\right)}{x\cdot\frac{2\sin x}{x}\cdot\frac{(1-\cos x)}{x^2}\cdot x^2} = \lambda\, e^2\cdot$

$ln\sqrt{5} = \lambda\frac{e^2}{2}\, ln\,5$

$\lambda\, e^2\cdot\frac{1}{2}\, ln\,5 = \frac{1}{2}e^2\, ln\,5 \Rightarrow \lambda = 1$

25. **8**

Sol. We have, $\lim_{x\to 0} \frac{1}{x}\left(\frac{f(-x)}{2x^3} - 1\right) = -3$

$\Rightarrow \lim_{x\to 0}\left(\frac{ax^4-(b+2)x^3+cx^2-dx+e}{x^4}\right)$

$= -6$

$\therefore$ $a = -6,\ b = -2,\ c = d = e = 0$

So, $|f(1)| = |a + b| = 8$ **Ans.**

26. **3**

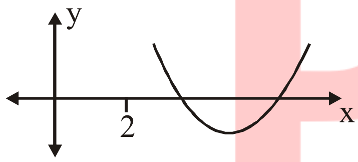
Sol. As, $\lim_{n\to\infty} \frac{1+cn^2}{(2n+3+2\sin n)^2} = \frac{c}{4}$

$\Rightarrow \frac{c}{4} = \frac{1}{2} \quad \Rightarrow \quad c = 2.$

Now both roots lies in $[2, \infty)$

$\therefore D \ge 0, \quad \frac{-b}{2a} \ge 2$ and

$f(2) \ge 0$, where $f(x) = x^2 - 2px + p^2 - 1$



$D = 4p^2 - 4p^2 + 4 > 0 \qquad \frac{-b}{2a} = p \ge 2 \quad ......(1)$

$f(2) = 4 - 4p + p^2 - 1 \ge 0 \Rightarrow p^2 - 4p + 3 \ge 0$

$\Rightarrow \quad (p-1)(p-3) \ge 0 \qquad ......(2)$

$(1) \cap (2) \Rightarrow \quad p \ge 3$

$\therefore \quad p_{min.} = 3$

27. **0**

Sol. Define $f(x) = \begin{cases} -2x; & -\infty < x < \frac{-3}{2} \\ 2 - \frac{2x}{3}; & \frac{-3}{2} \le x \le \frac{3}{4} \\ 2x; & \frac{3}{4} < x < \infty \end{cases}$

So, $f(x)$ is continuous for all $x \in R$.

28. **0**

Sol. $p = 2,\ q = 3$

$\therefore\ \sin^{-1}(\sin 5) + \cos^{-1}(\cos 5)$

$= (5 - 2\pi) + (2\pi - 5) = 0$ **Ans.**

29. 2

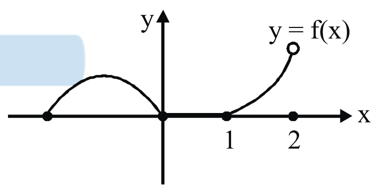
Sol. $p = -8;\ q = 10$

$\Rightarrow (p + q) = 2$. **Ans.**]

30. **4**

Sol. $f(x) = \begin{cases} -x(x+1); & -1 \le x < 0 \\ 0; & 0 \le x \le 1 \\ x(x-1); & 1 < x < 2 \\ 0; & x = 2 \end{cases}$

from the graph and methmatically also



$l = 1$ (at $x = 2$)

$m = 3$ (at $x = 0, 1, 2$)

$\therefore$ $l + m = 4$. **Ans.**