JEE MAIN : CHAPTER WISE TEST PAPER-5

SUBJECT :- MATHEMATICS **DATE..**
CLASS :- 12th **NAME..**
CHAPTER :- LIMIT CONTINUITY DIFFERENTIABILITY **SECTION..**

(SECTION-A)

1. The value of $\underset{x\to\infty}{\text{Limit}} \dfrac{\cot^{-1}\left(x^{-a}\log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}$ (a > 1) is equal to

(A) 1 (B) 0
(C) $\pi/2$ (D) does not exist

2. If $\lim\limits_{x\to 0} \dfrac{\sin(ax)+bx}{x^3}$ = 36, then

(A) a = 6, b = – 6 (B) a = – 6, b = 6
(C) a = 6, b = 6 (D) a = – 6, b = – 6

3. If $\lim\limits_{x\to 0} \dfrac{x+\sin x - x\cos x - \tan x}{x^n}$ exists and is non-zero finite value, then the value of n is

(A) 3 (B) 4 (C) 5 (D) 6

4. If $\alpha = \lim\limits_{n\to\infty}\left(\dfrac{1}{n^3+1}+\dfrac{4}{n^3+1}+\dfrac{9}{n^3+1}+......+\dfrac{n^2}{n^3+1}\right)$

and $\beta = \lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 8x}$, then

the quadratic equation whose roots are α, β is

(A) $12x^2 - 7x + 1 = 0$
(B) $x^2 + 19x - 120 = 0$
(C) $x^2 - 17x + 66 = 0$
(D) $x^2 - 7x + 12 = 0$

5. If $f(x) = \begin{cases} x^3+2x^2+ax+6, & x<1 \\ 2x+b, & x\ge 1\end{cases}$

is differentiable for all $x \in R$, then the value of (a + b) is equal to

(A) – 3 (B) 7 (C) 5 (D) 2

6. If f(x) = sgn (3)x $\cos^{-1}x - 6\cos^{-1}x - \pi x + 2\pi$), then the number of points of discontinuity of f(x) is/are

[**Note :** sgn k denotes signum function of k.]

(A) 2 (B) 0 (C) 1 (D) 3

7. Let $f(x) = \dfrac{\sin\frac{1}{4}\pi[x]}{[x]}$. Then which one of the following does **NOT** hold good?

[**Note:** [y] denotes greatest integer function less than or equal to y.]

(A) f (x) is not continuous at any point.
(B) f (x) is continuous at $x = \dfrac{3}{2}$.
(C) f (x) is discontinuous at x = 2.
(D) f (x) is differentiable at $x = \dfrac{4}{3}$.

8. Let $f(x) = \begin{cases} 4x^2+2[x]x & \text{if } -\frac{1}{2}\le x<0 \\ ax^2-bx & \text{if } 0\le x<\frac{1}{2}\end{cases}$

where[x] denotes the greatest integer function. Then

(A) f(x) is continuous in $\left(-\frac{1}{2},\frac{1}{2}\right)$ iff a = 4 and b = 0.
(B) f(x) is continuous and differentiable in $\left(-\frac{1}{2},\frac{1}{2}\right)$ iff a = 4, b = 2.
(C) f(x) is continuous and differentiable in $\left(-\frac{1}{2},\frac{1}{2}\right)$ for all a, provided b = 2.
(D) for no choice of a and b, f(x) is differentiable in $\left(-\frac{1}{2},\frac{1}{2}\right)$

9. Let f(x) =

$\operatorname{sgn}\left(\left(\sin\theta-\frac{\sqrt{3}}{2}\right)x^2+\left(\cos\theta-\frac{1}{2}\right)x+(\tan\theta-\sqrt{3})\right)$.

If f(x) is identically zero for every $x \in R$, then the number of values of θ in $[-2\pi, 2\pi]$, is

[**Note:** sgn k denotes the signum function of k.]

(A) 0 (B) 1 (C) 2 (D) 3

10. If $f(x) = \begin{cases} x^2 & \text{if } x\le x_0 \\ ax+b & \text{if } x>x_0\end{cases}$ derivable $\forall$ $x \in R$ then the values of a and b are respectively

(A) $2x_0, -x_0^2$ (B) $-x_0, 2x_0^2$
(C) $-2x_0, -x_0^2$ (D) $2x_0^2, -x_0$

11. Let f(x) be a differentiable function such that

$$\lim_{h\to 0}\left(\frac{f(x+h)}{f(x)}\right)^{\frac{1}{h}} = e^{(\tan x)\, f(-x)} \text{ and } f(0)$$

=1, then the value of $f'(0)$ is equal to

(A) 0 (B) 1 (C) e (D) – e

12. Let f(x) be a differentiable function which satisfies the equation

f(xy) = f(x) + f(y) for all x > 0, y > 0 then

$f'(x)$ is equal to

(A) $\frac{f'(1)}{x}$ (B) $\frac{1}{x}$

(C) $f'(1)$ (D) $f'(1).(\ln x)$

13. The number of values of a for the which the function $f(x) = (x+1)|x-a|$ is differentiable $\forall x \in R$, is

(A) 0 (B) 1

(C) 2 (D) more than 2

14. The function f (x) =

$$\begin{cases} 3-x^2 & \text{for } x \le -1 \\ 2 & \text{for } -1 < x < 1 \\ 1-x & \text{for } x \ge 1 \end{cases} \text{ is}$$

(A) decreasing in $(-\infty, -1)$

(B) differentiable at x = – 1

(C) continuous at x = 1 but discontinuous at x = – 1

(D) continuous at x = – 1 but discontinuous at x = 1

15. If the function f(x) =

$$\begin{cases} px^3 + q, & 0 \le x \le 1 \\ 2\cos(\pi x) + \tan^{-1} x, & 1 < x \le 2 \end{cases}$$

is differentiable in [0, 2], then

(A) $p = \frac{1}{6}$, $q = \frac{13}{6} - \frac{\pi}{6}$

(B) $p = \frac{1}{6}$, $q = \frac{\pi}{4} + \frac{13}{6}$

(C)) $p = \frac{1}{6}$, $q = \frac{\pi}{4} - \frac{7}{3}$

(D) $p = \frac{1}{6}$, $q = \frac{\pi}{4} - \frac{13}{6}$

16. The value of p and q for which the function f

$$(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R, is

(A) $p = \frac{5}{2}$, $q = \frac{1}{2}$ (B) $p = \frac{-3}{2}$, $q = \frac{1}{2}$

(C) $p = \frac{1}{2}$, $q = \frac{3}{2}$ (D) $p = \frac{1}{2}$, $q = \frac{-3}{2}$

17. If f (x) = [x] tan (πx) then $f'(k^+)$ is equal to (where k is some integer and [x] denote greatest integer less than or equal to x)

(A) $(k-1)\pi\,(-1)^k$ (B) $k\pi$

(C) $k\pi\,(-1)^{k+1}$ (D) $(k-1)\pi \cdot (-1)^{k+1}$

18. If $\lim_{n\to\infty}\left(\sqrt{2n^2+n} - \lambda\sqrt{2n^2-n}\right) = \frac{1}{\sqrt{2}}$

where $\lambda > 0$, then λ is equal to

(A) –1 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

19. $\lim_{x\to\cot^{-1}(-1)}\left(\frac{\tan^3 x - 2\tan x - 1}{\tan^5 x - 2\tan x - 1}\right)$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$

20. If the function defined by f (x)

$$= \frac{\tan 2\pi x + \sin\frac{\pi x}{2} + \tan\frac{\pi x}{2}}{x^2 + 4x - 12} \text{ is continuous at}$$

x = 2 then *f* (2)

(A) equals π/4 (B) equals 3π/8

(C) equal 2π (D) is non existent

(SECTION-B)

21. If number of points of discontinuity of the function $f(x) = [2 + 10\sin x]$, in $x \in \left[0, \frac{\pi}{2}\right]$ is same as number of points of non-differentiability of the function

$g(x) = (x-1)(x-2)\left|(x-1)(x-2)\ldots\ldots(x-2m)\right|$, $(m \in N)$ in $x \in (-\infty, \infty)$

then find the value of m.

[**Note :** [k] denotes largest integer less than or equal to k.]

22. Let $\lim\limits_{p\to\infty} p\, ln\left(e\left(1+\frac{1}{p}\right)^{1-p}\right)$ equal $\frac{m}{n}$ where m and n are relatively prime positive integer. Find (m + n).

23. Let f (x)

$$= \lim_{n\to\infty} ln\left(\sqrt{e^{\cos x}\sqrt{e^{3\cos x}\sqrt{e^{5\cos x}\ldots\ldots\sqrt{e^{(2n+1)\cos x}}}}}\right).$$

If $g(x) = \left[\frac{1}{3}f(x)\right]$, then find the number of points in $[0, 2\pi]$ where g(x) is discontinuous.

[**Note:** [y] denotes greatest integer function less than or equal to y.]

24. Consider, f (x)

$$= \begin{cases} \dfrac{e^{\left((x-2)ln\sqrt{5^x}+\frac{1}{3x}\log_2\left((4^x)^3\right)\right)}-e^2}{(x-2)\tan x}, & x > 0 \\ \lambda\cdot\dfrac{\left(\left(\sqrt{5}\right)^x-1\right)\left(e^{x^2+2}-e^2\right)}{(2\sin x-\sin 2x)}, & x < 0 \end{cases}$$

If $\lim\limits_{x\to 0} f(x)$ exists then find the value of λ.

25. Let f be a biquadratic function of x such that $\lim\limits_{x\to 0}\left(\frac{f(-x)}{2x^3}\right)^{\frac{1}{x}} = \frac{1}{e^3}$, then find the value of $|f(1)|$.

26. Let $\lim\limits_{n\to\infty}\frac{1+cn^2}{(2n+3+2\sin n)^2} = \frac{1}{2}$. If $c \le \alpha \le \beta$ where α and β are the roots of the quadratic equation $x^2 - 2px + p^2 - 1 = 0$, then find the minimum integral value of p.

27. Find the number of points of discontinuity of the function f (x)

$$= |x| + \left|\frac{x}{3}-1\right| + \left||x| - \left|\frac{x}{3}-1\right|\right|$$

in $x \in (-\infty, \infty)$.

28. If the value of

$$\lim_{x\to 2}\frac{\sqrt{x^2-2x+9}-\sqrt{9+2x-x^2}}{\sqrt{(x-1)^2+3}-\sqrt{5-(x-1)^2}}$$ equals $\frac{p}{q}$ where p and q are relatively prime positive integers, then find the value of $\sin^{-1}(\sin(p+q)) + \cos^{-1}(\cos(p+q))$.

29. Let f (x) =

$$\begin{cases} x\left(\dfrac{10\,e^{\left(\frac{\{x\}+\{-x\}}{|x|}\right)}-7}{3+e^{\frac{1}{|x|}}}\right), & \text{for } x<0 \\ 0, & \text{for } x=0 \\ x\cdot\dfrac{8-8e^{|x|+\{x\}}}{|x|+\{x\}}, & \text{for } x>0 \end{cases}$$

If $f'(0^+) = p$ and $f'(0^-) = q$, then find the value of (p + q).

[**Note:** {k} denotes fractional part of k.]

30. Consider the function $f(x) = x[x]\{x\}$ in interval [–1, 2]. If *l* denotes the number of point of discontinuity and m denotes the number of points of non-differentiability of function, then find the value of (*l* + m).

[**Note:** [k] and {k} denotes greatest integer less than or equal to k and fractional part of k respectively.]